UNDERSTANDING OF ALGEBRAIC NOTATION AND ITS RELATIONSHIP WITH COGNITIVE DEVELOPMENT AMONG FORM FOUR STUDENTS

Ong Saw Hoon

A Project Paper Submitted to the Faculty of Education, University of Malaya in Partial Fulfillment of the Requirements for the Degree of Master of Education

2000
Kajian ini bertujuan untuk mengenalpasti interpretasi abjad yang digunakan dalam ungkapan dan persamaan algebra oleh 139 orang pelajar Tingkatan Empat di sebuah sekolah menengah. Ia juga menentukan taburan tahap kefahaman notasi algebra dan taburan tahap perkembangan kognitif mereka. Ia menyiasat perhubungan antara tahap kefahaman notasi algebra dengan tahap perkembangan kognitif ini. Untuk mencapai tujuan-tujuan tersebut, dua set instrumen kajian digunakan: Ujian Algebra dan Ujian Penaakulan Longeot.

Dapatian kajian ini menunjukkan bahawa lebih daripada 80% pelajar menggunakan tahap interpretasi abjad yang rendah dengan berjaya. Sebagai bandingan, kurang daripada 30% menggunakan tahap interpretasi abjad yang tinggi dengan berjaya.

Sembilan jenis kesilapan pelajar telah dikenalpastikan semasa mereka menginterpretasikan abjad. Kesilapan menggabungkan elemen-elemen nombor dan algebra adalah yang lebih kerap dan dibuat oleh 50.4% pelajar. Kajian ini juga mengenalpastikan kesilapan menuliskan hasildarab dan bukan hasiltambah abjad-abad. Kesilapan ini tidak pernah dijumpai dalam kajian-kajian lalu dan 29.5% pelajar-pelajar membuat kesilapan ini.

Hasil kajian ini menunjukkan bahawa terdapat lebih ramai pelajar (61.2%) di tahap kefahaman notasi algebra yang rendah berbanding dengan 38.8% di tahap
yang tinggi. Ramai pelajar berada di paras perkembangan kognitif konkrit (71.9%) berbanding dengan paras perkembangan kognitif formal (28.1%). Tambahan juga, terdapat perhubungan di antara tahap perkembangan kognitif pelajar dengan tahap kefahaman notasi algebra mereka. Pelajar yang beroperasi pada tahap perkembangan kognitif formal mencapai tahap kefahaman notasi algebra yang lebih tinggi sedangkan pelajar yang beroperasi pada tahap perkembangan kognitif konkrit mencapai tahap kefahaman notasi algebra yang lebih rendah. Perbezaan tahap kefahaman notasi algebra di antara pelajar di tahap perkembangan kognitif konkrit dan tahap perkembangan kognitif formal adalah signifikan.

Kajian lanjutan dalam kefahaman notasi algebra atau dalam aspek-aspek lain di bidang algebra dicadangkan.
ABSTRACT

This study attempts to identify the interpretation of letters used in algebraic expressions and equations by a sample of 139 Form Four students in an urban secondary school. It also determines the distribution of their levels of understanding of algebraic notation and their cognitive levels. It further investigates the relationship between the levels of understanding of algebraic notation and cognitive levels. To achieve these aims, two sets of instruments were used: the Algebra Test and the Longeot Reasoning Test.

The findings of the study show that more than 80% of the students could successfully use the lower level of interpretations of letters. In contrast, less than 30% were successful in using the higher level of interpretations of letters.

Nine error types made by the students in their interpretations of letters were identified. The error of conjoining numerical and algebraic elements was the most prevalent, with 50.4% of the students making this error. The error of writing down of product instead of sum is peculiar to the study, as it is not noted in past studies. This error was made by 29.5% of the students.

The findings also reveal that a majority of the students (61.2%) were in the lower levels of understanding of algebraic notation compared to 38.8% in the higher levels. Most of the students were at concrete level of cognitive development (71.9%) compared to formal level of cognitive development (28.1%). In addition, there was a relationship between students’ levels of cognitive development and their levels of understanding of algebraic notation. Students at formal cognitive levels
achieved higher levels of understanding of algebraic notation while those at concrete cognitive levels achieved lower levels of understanding of algebraic notation. The difference in the levels of understanding of algebraic notation between students at concrete and formal cognitive levels was statistically significant.

It is suggested this study be extended to further studies in the understanding of algebraic notation, specifically or in other aspects of algebra, generally.
ACKNOWLEDGEMENTS

To the many people who have contributed towards this study, I take this opportunity to express my heartfelt thanks and appreciation.

Firstly, I would like to thank my supervisor, Pn. Lee Siew Eng, for her invaluable advice and the generous sacrifice of her time and effort.

To the Ministry of Education, Malaysia for granting me a scholarship to pursue my Master of Education program at the University of Malaya, my special thanks. My acknowledgement also goes to the Education Planning and Research Division of the Ministry of Education and the Education Department of Selangor for granting me the permission to carry out this study. My sincere thanks to the principals, teachers and students of the schools who gave their full cooperation in the study.

I would like to convey my deepest appreciation to Pn. Ooi Chooi Lin, Ms. S. C. Lee, Pn. Anis Sabrina Abu Bakar, Pn. Saroja Theavy Balakrishnan and Pn. Lim Bee Leng for refining my two instruments for the study. Special mention of thanks to Ms. Tan Mun Wai, Ms. Wong Mei Leng, Ms. Wong Soo Eet and Ms. Low Lee Kim for their insightful comments and suggestions as well as for reading my report.

I also extend my thanks and appreciation to my good friend, Ms. Siew Lai Teoh-Lilley, for taking great effort to put into my hands important books and articles for my study from her college in the United States of America.
achieved higher levels of understanding of algebraic notation while those at concrete
cognitive levels achieved lower levels of understanding of algebraic notation. The
difference in the levels of understanding of algebraic notation between students at
concrete and formal cognitive levels was statistically significant.

It is suggested this study be extended to further studies in the understanding
of algebraic notation, specifically or in other aspects of algebra, generally.
To my family members for lovingly putting up with my idiosyncrasies throughout the whole course and standing by me all the way, I dedicate to them this research report.

Most of all, all praise and glory to God Almighty, the Giver of every good and perfect gift, and with Whom there is no variation or shadow due to change.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>CHAPTER 1</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Background of the Study</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Rationale of the Study</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Research Questions</td>
<td>6</td>
</tr>
<tr>
<td>1.4</td>
<td>Operational Definitions</td>
<td>7</td>
</tr>
<tr>
<td>1.5</td>
<td>Significance of the Study</td>
<td>9</td>
</tr>
<tr>
<td>1.6</td>
<td>Limitations of the Study</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td><strong>CHAPTER 2</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>REVIEW OF RELATED LITERATURE</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Uses of Variables in Algebra</td>
<td>12</td>
</tr>
<tr>
<td>2.3</td>
<td>Definitions of Variables</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>Interpretation of Letters</td>
<td>15</td>
</tr>
<tr>
<td>2.5</td>
<td>Conceptual Difficulties in Algebra</td>
<td>23</td>
</tr>
</tbody>
</table>
2.6 Cognitive Development of Secondary School Students 28

2.7 Cognitive Development and Achievement in Mathematics 32

CHAPTER 3 METHODOLOGY

3.1 Introduction 36

3.2 Selection of Subjects 36

3.3 Research Instruments 37

3.3.1 Algebra Test 37
3.3.1.1 Validity of the Algebra Test 42
3.3.1.2 Scoring Procedure for the Algebra Test 43
3.3.1.3 Pilot Test and Reliability of the Algebra Test 44

3.3.2 Longeot Reasoning Test 45
3.3.2.1 Scoring Procedure for the Longeot Reasoning Test 49
3.3.2.2 Reliability and Validity of the Longeot Reasoning Test 50

3.4 Data Collection Procedure 51

CHAPTER 4 DATA ANALYSIS AND DISCUSSION

4.1 Introduction 53

4.2 Interpretation of Letters 53

4.2.1 Meanings Attached to Letters 54

4.2.2 Difficulties in Interpretation of Letters 55

4.3 Levels of Understanding of Algebraic Notation 61
4.4 Cognitive Levels

4.5 Achievement of Algebra Test and Cognitive Levels

4.5.1 Comparison of Students at Concrete and Formal Levels

4.5.2 Comparisons of Cognitive Levels on Achievement in Algebra Test

4.6 Relationship Between Cognitive Levels and Levels of Understanding of Algebraic Notation

4.6.1 Levels of Understanding of Algebraic Notation by Cognitive Levels

4.6.2 Comparisons of Levels of Understanding of Algebraic Notation Between Students at Concrete and Formal Levels

CHAPTER 5 SUMMARY AND CONCLUSIONS

5.1 Introduction

5.2 Summary of Findings

5.3 Implications and Suggestions

5.4 Recommendations for Further Research

5.5 Conclusion

REFERENCES

APPENDICES

Appendix A Ujian Algebra

Appendix B Algebra Test (English Version)
<table>
<thead>
<tr>
<th>Appendix</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix C</td>
<td>Ujian Penaakulan Longeot</td>
<td>94</td>
</tr>
<tr>
<td>Appendix D</td>
<td>Longeot Reasoning Test (English version)</td>
<td>108</td>
</tr>
<tr>
<td>Appendix E</td>
<td>Panel for Validation of Instruments</td>
<td>110</td>
</tr>
<tr>
<td>Table</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2.1</td>
<td>Percentage of Children at Each Algebra Level</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Distribution of the Students According to Gender and PMR Mathematics Grades</td>
<td>37</td>
</tr>
<tr>
<td>3.2</td>
<td>Items of Algebra Test According to Their Levels</td>
<td>38</td>
</tr>
<tr>
<td>3.3</td>
<td>An Analysis of the Topic of Algebra in the Malaysian Mathematics Syllabus and the Algebra Test</td>
<td>39</td>
</tr>
<tr>
<td>3.4</td>
<td>Items in Accordance to Their Interpretation of Letters</td>
<td>41</td>
</tr>
<tr>
<td>3.5</td>
<td>Initial and Final Versions of Some Items in the Algebra Test</td>
<td>43</td>
</tr>
<tr>
<td>3.6</td>
<td>Criteria for Classification of Students into the Levels of Understanding of Algebraic Notation</td>
<td>44</td>
</tr>
<tr>
<td>3.7</td>
<td>Time Allocation for the Longest Reasoning Test</td>
<td>47</td>
</tr>
<tr>
<td>3.8</td>
<td>Modification of Item 12</td>
<td>48</td>
</tr>
<tr>
<td>3.9</td>
<td>Classification of Longest Reasoning Test</td>
<td>49</td>
</tr>
<tr>
<td>3.10</td>
<td>Classification Scheme for Concrete and Formal Levels</td>
<td>49</td>
</tr>
<tr>
<td>4.1</td>
<td>Distribution of Students According to Their Successful Interpretations of Letters</td>
<td>54</td>
</tr>
<tr>
<td>4.2</td>
<td>Distribution of Types of Errors</td>
<td>56</td>
</tr>
<tr>
<td>4.3</td>
<td>Items and Error of Numerical Substitution</td>
<td>57</td>
</tr>
<tr>
<td>4.4</td>
<td>Distribution of Levels of Understanding of Algebraic Notation of Students in Algebra Test</td>
<td>62</td>
</tr>
<tr>
<td>4.5</td>
<td>Distribution of Cognitive Levels of Students</td>
<td>63</td>
</tr>
<tr>
<td>4.6</td>
<td>t-test Comparison Between Students at Concrete and Formal Levels in Algebra Test</td>
<td>65</td>
</tr>
</tbody>
</table>
4.7 One-way Analysis of Variance for Algebra Test by Cognitive Levels 67
4.8 Distribution of Levels of Understanding of Algebraic Notation and Cognitive Levels of Students 69
4.9 Cross-tabulation of Levels of Understanding of Algebraic Notation With Cognitive Levels of Students 70