CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Algebra forms the basis in mathematics learning. It is endorsed as the essence of mathematics by Dunne and Jennings (1996). According to Fey (1989),

> Concepts, principles, and methods of algebra are the core of secondary school mathematics. As a prerequisite for study in every branch of mathematics, science, and technology, algebra is the first high school mathematics course. Most college-intending students spend at least one year beyond algebra to develop the algebraic skills and understandings required by trigonometry, analytic geometry, calculus, and statistics.
>
> (p. 199)

Not surprisingly, algebra holds the central position in school mathematics. The central role is further enhanced in the goals for algebra instruction in school mathematics by several mathematics educators, as noted by Thorpe (1989), with the proposed goals as:

1. To develop student skills in the solution of equations, finding numbers that meet specified conditions.
2. To teach students to use symbols to solve real problems, such as mixture problems, rate problems, and so forth.
3. To prepare students to follow derivations in other subjects, for example, in physics and engineering.
4. To enable students to become sufficiently at ease with algebraic formula so that they can read popular scientific literature intelligently.

5. To enable students to grow in their natural powers of seeing the mathematical elements in a situation, reasoning with these elements to come to relevant conclusions, and carrying out the process with confidence and responsibility.

In addition, Thorpe contends that algebra instruction promotes the understanding of concepts and encourages thinking. Therefore, algebra is not only vital to the learning of school mathematics but also aids in the learning of other science courses.

According to Booth (1984), fundamental to the study of school algebra is the idea of algebra as generalised arithmetic. From this generalised mathematics perspective, algebraic notation is used to describe arithmetical patterns, laws and relationships in a precise way (Costello, 1991). Therefore, an understanding of algebraic notation is important to determine the root of the learning difficulties involved in algebra.

The primary concept of the algebraic notation is the variable. To develop an understanding of algebra, students are faced with several levels of meaning for the concept of variable. The different levels of meaning attached to the use of letters are dependent upon the context of the problem and the learners' cognitive development to understand and make use of a particular meaning (Driscoll, 1983). However, it is the matching of these levels of meaning with the levels of the learners' readiness that makes the learning and teaching of algebra especially difficult. This is shown by the 'Concepts in Secondary Mathematics and Science' [CSMS] research project linking students' levels of understanding of algebraic letters to Piagetian stages of cognitive development (Kuchemann, 1981a).

In addition, studies have shown that there are cognitive difficulties involved in the learning of algebra (Booth, 1984; Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1996; Sfard & Linchevski, 1994). Kieran (1992) identified the dilemma students faced in algebra. On one hand, they had to treat symbolic representations as mathematical objects and to operate upon these objects with processes that usually do not yield numerical solutions. On the other hand, they had to modify their former interpretations of certain symbols and to represent the relationships of word-problem situations with operations that are often the inverses of those that they used for solving similar problems in arithmetic.

Therefore, this study investigates the students' interpretations of the letters used in algebraic notation and the levels of understanding of algebraic notation. It also aims to ascertain if there is any significant relationship between the levels of understanding of algebraic notation with the students' levels of cognitive development.

1.2 Rationale of the Study

Algebra is an important component in the secondary school mathematics curriculum. This topic is introduced in the Form Two secondary school Mathematics, known as Integrated Secondary School Mathematics Curriculum or *Matematik Kurikulum Bersepadu Sekolah Menengah* (KBSM). Its importance is clearly seen in the weightage given to algebra and its application in the mathematics paper in the lower secondary public examination known as *Penilaian Menengah Rendah* (PMR). About 22% of the questions in its Mathematics Paper I for the year

1999 are on the understanding of algebraic notation. The same can also be said of the mathematics paper in the upper secondary public examination, *Sijil Pelajaran Menengah* (SPM). Factorizing algebraic expressions, simplifying algebraic expressions, solving algebraic equations, solving linear inequalities, formulating relationships and graphing are examples of a few of the topics in which understanding of algebra is tested. These topics and their applications contribute to approximately 32% of the questions in the Mathematics Paper II for the year 1999.

Algebra is also a difficult topic in the secondary school mathematics curriculum. This is because the learning of algebra involves higher abstract thinking (Fong & Chong, 1995). In addition, algebra is a language that uses variables. However, the use of variables and other notational conventions in algebra seems to complicate students' meaningful learning of the concepts involved (Anderson, 1995). From his own experience, Rosnick (1981) admitted that letters in equations were his initial downfall. He identified mathematics "as being little more than obscurity by abstractions and dehumanisation by symbolisation" (p. 418). As the use of variable is the principal characteristic of algebra, an understanding of the concept of variable is essential for developing the ability to understand and manipulate algebraic notations. Another reason for algebra to be a difficult topic is that algebra requires analysis. Viete Zetetics, as cited by Charbonneau (1996), used the terms "algebra" and "analysis" synonymously.

The fact that algebra is a difficult topic in the mathematics curriculum can also be seen from the results of the National Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1982). The results indicated that algebra was a major stumbling block for many students in secondary school. The students' performance covered several aspects of algebra: symbolisation, algebraic representation or translation and algebraic manipulation. The results showed that a consistent cluster of only 15% to 20% of the 17-year-old population had a mastery of algebraic skills and concepts while about 30% to 40% of the population had an intuitive knowledge of algebraic processes. This meant that the latter group of students would fail when specific well-defined procedures were required. In addition, it was noted that algebraic skills to solve problems were a major area of difficulty for most students. Carpenter et al. noted that "performance in algebra has not reached the point of mastery for most students with two years of algebra and a year of geometry. Students have difficulty dealing with the manipulation of mathematical symbols in expressions and equations" (p. 530).

In view of the fact that algebra is both an important and difficult topic in secondary school mathematics, it warrants a study to determine the levels of understandings of algebraic notation among Malaysian Form Four students. Furthermore, the categories of interpretation of letters used by the students and also the difficulties encountered by them in the interpretation of letters have implications for instructional strategies for mathematics teachers.

As the learning of algebra involves abstract concepts and requires formal operational ability of students to understand fully, a study on the students' level of cognitive development is therefore useful to determine the readiness of our students for this topic. From the data of the five-year CSMS research programme in Britain, Kuchemann (1981b) noted that "there is a mismatch between children's understanding of mathematics and the cognitive demand of much of what they are taught" (p.301). Similarly, Shemesh, Eckstein, and Lazarowitz (1992) raised the question on how to bridge the gap between the cognitive ability required of the students and the cognitive level actually attained by them. Therefore the present study intends to explore if there is any significant relationship between the students' levels of understanding of algebraic notation and their cognitive levels. It is hoped that the findings could be used to address three basic curricular issues (Carpenter, 1980): firstly, the sequencing of algebraic topics; secondly, matching instruction to appropriate levels of developments; and thirdly, choosing instructional strategies.

1.3 Research Questions

This study is concerned, firstly, with the understanding of algebraic notation of students in the fourth year of secondary schools in Malaysia. To achieve this, the study investigates the students' interpretations of the letters used in algebraic notation. It also determines the students' levels of understanding of algebraic notation. Secondly, the study attempts to investigate the students' cognitive levels and also determine if there is any significant relationship between the levels of understanding of algebraic notation and the cognitive levels.

This study attempts to answer the following research questions:

1. What are the students' interpretations of algebraic notation?
2. What is the distribution of the levels of understanding of algebraic notation among Form Four students as measured by the Algebra Test?

3. What is the distribution of the cognitive levels of Form Four students as measured by the Longeot Reasoning Test?

4. Are there any significant differences in the achievement of Algebra Test among students of different cognitive levels?

5. Is there any significant relationship between the students' levels of understanding of algebraic notation and their cognitive levels?

1.4 Operational Definitions

This study uses the following terms with their operational definitions:

1.4.1 Algebraic notation

This refers to the letter(s) used in an algebraic expression and algebraic equation.

1.4.2 Interpretation of the letter

This refers to the meanings given to the letter. The classifications of interpretation of letters are adapted from Kuchemann (1981a):

Letter evaluated : This category applies to responses where the letter is assigned a numerical value from the outset.

Letter not used : The letter is ignored or its existence is acknowledged without giving it a meaning.

Letter used as a concrete object : The letter is regarded as a shorthand for an object or as an object in its own right.

Letter used as a specific unknown : The letter is regarded as a specific but unknown number.

Letter used as a generalised number : The letter is seen as representing, or at least as being able to take on, several values rather than just one.

Letter used as a variable : The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.

1.4.3 Levels of Understanding

Levels of understanding refer to the achievement of the understanding of algebraic notation provided in the Algebra Test. The items in the Algebra Test are classified into four levels, Levels 1 to 4. Level 1 is the lowest level and Level 4 the highest. Attainment to each level is based on the students' performance of correctly answering more than 60 percent of items in each level. The attainment of a particular level also infers the attainment of all preceding levels.

1.4.4 Cognitive Levels

This refers to the intellectual development levels on operational stages of reasoning as described by Piaget (Inhelder & Piaget, 1958). Criteria in classifying the students' cognitive levels are based on their performance on the Longeot Reasoning Test or LRT (Ward, Nurrenbern, Lucas, & Herron, 1981). The cognitive levels used in this study are:

Concrete IIA : This refers to Piaget's early concrete level. In the present study, a student is considered to have attained Concrete IIA if his or her score on the LRT is in the range of 0 and 7.

Concrete IIB : This is Piaget's late concrete level. A student is regarded to achieve this level if his or her score on the LRT is in the range of 8 and 22.

Formal IIIA : This refers to Piaget's early formal operational level. A student is considered to have obtained Formal IIIA if his or her score on the LRT is in the range of 23 and 29.

Formal IIIB : This is Piaget's late formal operational level. A student is regarded to have achieved Formal IIIB if his or her score on the LRT is in the range of 30 to 42.

1.5 Significance of the Study

The study seeks to establish whether students who differ in their cognitive levels will also differ significantly in their understanding of algebraic notation. Such information will be useful for educators in curriculum planning to help them on selecting, sequencing and organising the content of the secondary school Mathematics curriculum. Indeed a better sequencing of the mathematics curriculum is needed to develop abstract operational ability for many students.

The findings in this study also provide a basis to recommend teaching learning strategies for the students. The secondary mathematics teachers need a basis to develop relevant and meaningful instructional materials in teaching algebra. Due to the abstract nature of algebra, the teachers need to broaden students' conceptualisation of algebra beyond memorised rules and skills assignments to a meaningful development and inclusion of varied applications as part of the learning process.

In knowing the different levels of understanding of algebraic notation, teachers will also be able to introduce and approach the topic of algebra in appropriate ways and at different depths that will be consistent with the students' individual stages of intellectual development.

Malaysia aspires to be an industrialised country by year 2020. To realise the nation's aspiration, the Seventh Malaysian Plan (1996 – 2000) has endorsed a goal of 60 percent of Forms Four and Five students in the science stream in the education sector. To encourage the secondary school students to enrol in the science stream, our Deputy Prime Minister realised that a love for mathematics must be instilled at a young age (Loh, 2000). This is due to the fact that students in the science stream of Forms Four and Five need a strong foundation, not only in science, but also in mathematics. As algebra forms the basis in mathematics learning and is also widely used in quantitative science courses, it is hoped that the findings of this study will help towards determining the root of learning difficulties involved in algebra.

1.6 Limitations of the Study

This study is only concerned with a topic in the secondary mathematics curriculum, that is, algebra. Also, only one aspect on the understanding of algebraic notation is studied here.

Many factors can affect the students' understanding of algebraic notation, such as attitude towards mathematics, family and school environment, and cognitive style, to name a few. However, in this study only their cognitive development is considered.

The sample chosen for this study consists of only 139 students selected from an urban secondary school in the Petaling Jaya District and may not be representative of the population of Form Four students in Malaysia. Thus the findings obtained in this study cannot be generalised to all the Form Four students in Malaysia.

The Longeot Reasoning Test items have multiple-choice answers and the students could guess the answers. However, the scoring of the Longeot Reasoning Test was not corrected for the students' guessing. As a result, a higher estimation of the percentage of cognitive levels of subjects might occur.