CHAPTER 4

DATA ANALYSIS AND DISCUSSION

4.1 Introduction

The purpose of this study is to investigate the interpretations of letters used in algebraic notation of Form Four students. It also determines the students' levels of understanding of algebraic notations and their cognitive levels. In addition, the study also seeks to determine any significant relationship between the levels of understanding of algebraic notation and the cognitive levels of the students.

The results of the analysis of data and the interpretations made based on them are organised into five major categories according to the research questions. They are:

1. Interpretation of letters.
2. Levels of understanding of algebraic notation.
3. Cognitive levels.
4. Achievement of Algebra Test and cognitive levels.
5. Relationship between cognitive levels and levels of understanding of algebraic notation.

4.2 Interpretation of Letters

The responses of the subjects in the Algebra Test were analysed individually for the interpretation of letters used in algebraic expressions and algebraic equations, with reference to those identified by Kuchemann (1981a). In addition, misinterpretations and errors made were also identified. The presentation of the analysis will be under:

1. Meanings attached to letters
2. Difficulties in interpretation of letters.

4.2.1 Meanings Attached to Letters

The items in the Algebra Test can be solved successfully by using six interpretations of letters as advocated by Kuchemann (1981a). The distribution of the different interpretations of letters of the students is as shown in Table 4.1.

Table 4.1
Distribution of Students According to Their Successful Interpretations of Letters

Interpretation of letters	Students	
	n.	%
Letter evaluated	133	95.7
Letter not used	129	92.8
Letter used as a concrete object	118	84.9
Letter used as a specific unknown	88	63.3
Letter used as a generalised number	41	29.5
Letter used as a variable	25	18.0

Students who could successfully use the interpretation of letter evaluated constituted the highest percentage (95.7%). This was followed by 92.8% of the students using the interpretation of letter not used and 84.9% of the students using the interpretation of letter as a concrete object. On the other hand, 63.3% of the students were successful in using letter as a specific unknown. In contrast, less than 30% of the students could successfully interpret letter as a generalised number and letter as a variable.

These findings are consistent with that of Kuchemann who reported that the first three interpretations of letters are considered as the lower level of interpretations of letters and the latter three as the higher level of interpretations of letters.

4.2.2 Difficulties in Interpretation of Letters

The responses of the students were analysed for errors made by the students. The types of errors were identified with reference to past researchers' classifications. However, any other error types made by the students were also noted. Table 4.2 presents the distribution of nine types of errors made by the students.

Among the errors, conjoining of numerical and algebraic elements was the most prevalent, with 50.4% of the students making this error. This was followed by the error of numerical substitution (38.1%). On the other hand, only one student made the error of wrong concatenation. It should be noted that the error of writing down of product instead of sum of letters is peculiar to the study and is not noted in the past studies. It was found that 29.5% of the students made this error.

Table 4.2
Distribution of Types of Errors

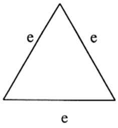
Error	No. of students making the error (N = 139)	Percentage of students
Numerical substitution	53	38.1
Ignoring the letter	10	7.2
Letter treated as a concrete object	41	29.5
Conjoining of numerical and algebraic elements	70	50.4
Wrong concatenation	1	0.7
Writing down of product instead of sum of letters	41	29.5
Changing algebraic expressions to algebraic equations	27	19.4
Omission of brackets	18	12.9
Treating *t* as 0*t*	35	25.2

4.2.2.1 Numerical Substitution

The error referred to students assigning a value to the letters to arrive at numerical answers rather than leaving the answers in terms of a variable. The value assigned could be "1" or any other values (for instance, "3" in Item 9a). Some items in which students made this error are as shown in Table 4.3.

For item 9d, a total of 33.1% of the students gave a value to n before multiplying it by two. Furthermore, since the figure was drawn with 16 sides, it was common for the students to multiply 2 by 16 to arrive at the answer 32. This was shown by 18.7% of the students.

Table 4.3
Items and Error of Numerical Substitution

Item	Question	Students' responses	Percentage of students
	Tuliskan perimeter bagi setiap bentuk de bawah.		
9a		$3 + 3 + 3 = 9$ or $1 + 1 + 1 = 3$	2.9
9d		$2 \times 16 = 32$	18.7
		other values	14.4
	Bentuk mempunyai n sisi. Panjang setiap sisi ialah 2 unit.		

4.2.2.2 Ignoring the Letter

The error referred to students ignoring or disregarding the existence of the letter and just operating on the numerical values. An example of this error was found in Item 6. For this item, the students were asked to multiply (n + 5) by 4. It was found that 6.5% of the students just multiplied the 5 by 4 to arrive at the answer of 20, ignoring the letter *n* altogether.

4.2.2.3 Letter Treated as a Concrete Object

The error referred to the letter being treated as a concrete object. The meaning of the letter was reduced from something abstract to something more concrete. However, such reduction in the meaning was found to occur frequently despite it being not appropriate. For instance, the intended meaning of the letter required a higher level of interpretation, such as interpreting the letter as a specific unknown but the letter was reduced to concrete object. Such misuse of letters as concrete objects was frequently found in problems that involved quantities that had to be translated into mathematical language (Item 13, p. 90) or when a mathematical statement had to be interpreted (Item 18, p. 90).

In Item 13, 18.0% of the students gave the answer as "p + n = RM8", which might be taken as an abbreviation of "pencils plus pens cost 8 ringgit" rather than a relationship between the numbers *p* and *n*. For Item 18, 18.0% of the students considered the expression 4k + 3r as 4 pieces of cakes and 3 pieces of buns bought by Ali instead of the total amount Ali had to pay. These two examples showed the tendency of students to treat the letter as a concrete object, thereby causing errors in interpretation.

4.2.2.4 Conjoining of Numerical and Algebraic Elements

This error was made when the meaning of the letter was ignored and the problems treated as mere manipulation of symbols and rules (Booth, 1984). It was done by

a) adding up all the numbers, and then putting down each letter that occurs only once. For example, in Item 5, 32.4% of the students wrote 4 + 3n = 7n and in Item 10b, 34.5% of them wrote 2a + 5b = 7ab.

b) adding up all the numbers, and then putting the letter for each time it occurs in the expression. For example, in Item 10c, 0.7% of the students wrote 2a + 5b + a = 7aba. The student also regarded the final *a* in the expression as *0a*.

c) adding up all the numbers, and then putting down the letter that occurs most frequently (for instance, 3.6% of the students wrote 2a + 5b + a = 7a and regarded *a* as *0a*) or which has the bigger coefficient and 2.2% of the students wrote 2a + 5b + a = 7b, regarding the final *a* as *0a*.

4.2.2.5 Wrong Concatenation

Concatenation in arithmetic denotes addition (45 = 40 + 5), whereas in algebra, it denotes multiplication, for instance, 3a = 3 x a (cited by Chalouh & Herscovics, 1988). Due to the confusion in the understanding of concatenation in arithmetic, one student (0.7%) mistook the term *3n* in Item 8 as a placeholder instead of as a product. Hence the student evaluated *3n* as 34, where n = 4.

4.2.2.6 Writing Down of Product Instead of Sum of Letters

It would seem that the students mixed up the multiplication and addition of the letters. The students wrote e^3 for e + e + e in Item 9a and h^4 + t for h + h + h + h + t in Item 9b. It was found that 10.1% made the former mistake and 5.0%, the latter. This suggests that there was confusion between the two operations. As the operation of addition is normally taught first before multiplication, the learning of the latter may have interfered with the former. This error is peculiar to the present study as no such error was reported in earlier studies. A total of 29.5% of the students made this error.

4.2.2.7 Changing of Algebraic Expressions to Algebraic Equations

It was found that students changed algebraic expressions to algebraic equations by adding "= 0" and then evaluating the unknown. For example, in Item 5, after adding 4 to 3n to arrive at the answer of 3n + 4, the students would then evaluate n. This was done by writing 3n + 4 = 0, then 3n = -4, and arriving at n = -4/3. The need to transform algebraic expressions into equations and then evaluating the letter was expressed by 19.4% of the students. This finding is in agreement with the findings of Chalouh and Herscovics (1988) as well as that of Kieran (1981).

4.2.2.8 Omission of Brackets

This error was found in 12.9% of the students' responses. Two items in the Algebra Test in which this error occurred were Items 4c and 6. In Item 4c, the students wrote e + 2 x 5 = e + 10 instead of (e + 2) x 5 = 5e + 10. In Item 6, even though a bracket was given to (n + 5) and the students are asked to multiply it by 4, this error still persisted. Kieran (1992) reported that children did not use parentheses for algebraic expressions that required parentheses. Instead they performed the computation according to the order of the written sequence of operations. Therefore, in Item 6, the students wrote n + 5 x 4 and gave the answer as n + 20. This error of omission of brackets is consistent with the studies of Booth (1984) as well as that of Linchevski and Linveh (1999).

4.2.2.9 Treating *t* As 0*t*

In this error, "t" was not regarded as "1t" but rather as "0t". This error was prevalent in conjoin errors or in errors where students wrote down the product instead of the sum of letters. For example, in Item 10c, when the students were asked to simplify 2a + 5b + a, 25.2% of them gave the answers as 7a, 7b, 7ab, 7aba, $7a^2$, $7a^2b$, $2a^2 + 5b$ or $2a^25b$. In this case, the students regarded the final "a" in the expression as "0a". This finding is consistent with the study of Yee and Soon (1996).

4.3 Levels of Understanding of Algebraic Notation

The distribution of the students according to their levels of understanding of algebraic notation is presented in Table 4.4. Any student in a particular level was considered to have achieved the preceding levels.

Table 4.4
Distribution of Levels of Understanding of Algebraic Notation of Students in Algebra Test

Level	*n*	Percentage
0	29	20.9
1	31	22.3
2	25	18.0
3	35	25.2
4	19	13.6
Total	139	100.0

The data indicate that 20.9% of the 139 students could not handle the items of Level 1 which were purely numerical, had simple structures and required the lower level of interpretation of letters. Only 13.6% of the students had achieved the highest algebraic hierarchical order of Level 4.

According to Kuchemann (1981a), letters in the items at Levels 1 and 2 could be solved using the lower level of interpretation of letters. On the other hand, the letters in the items at Levels 3 and 4 had to be treated at least as specific unknowns and in some cases, as generalised numbers or variables. The results show that a total of 61.2% of the students was operating at the lower level of interpretations (Levels 0, 1, and 2). Only 38.8% were responding at the higher level of interpretations (Levels 3 and 4). This finding is consistent with that of Kuchemann's, whose study revealed that a total of 58.0% of 15-year-olds were at Levels 0, 1, and 2.

4.4 Cognitive Levels

The 28-item Longeot Reasoning Test was used to measure the cognitive levels of the students. The test was scored according to the scale modified by Ward et al. (1981). Based on the total test scores, the students were then categorised into different cognitive levels following the classification scheme advocated by Ward et al. as shown in Table 3.8 in section 3.3.2.1. The result of the distribution of the cognitive levels of the students is presented in Table 4.5.

Table 4.5
Distribution of Cognitive Levels of Students

Cognitive Level	*n*	Percentage
Concrete IIA	17	12.2
Concrete IIB	83	59.7
Formal IIIA	19	13.7
Formal IIIB	20	14.4
Total	139	100.0

The results indicate that 13.7% of the 139 students were at Formal IIIA and 14.4% at Formal IIIB. For the Concrete IIA and Concrete IIB, 12.2% and 59.7% were found at these two levels respectively. These results imply that a majority of the students (a total of 71.9%) were functioning at the concrete operational levels compared to the formal operational levels (a total of 28.1%). These findings on the cognitive level of development of the students in this study were not in total agreement with the claim of Piaget. Not all the students in this study, whose ages ranged from 15 to 16 years, were formal operational students. A total of 73.4% students were either in Concrete IIB or in Formal IIIA.

However, these findings are consistent with those of other researchers in and outside Malaysia. The study of cognitive attainment of Form Four students in Malaysia by Lew (1987) and Palanisamy (1986) revealed that a majority of the students were operating at late concrete and early formal levels. The students at these two cognitive levels constituted 80.3% and 68.5% of the total sample in Lew's and Palanisamy's study respectively. These findings were also in agreement with the findings of Ng's (1991) study on cognitive development of Form Five science students. His finding indicated that a majority of the Form Five science students (80.6%) were also operating at the late concrete and early formal levels. Similarly, Cheah (1984) reported that 87.5% of the 271 Forms Four and Five students were functioning at the late concrete and early formal levels.

A similar pattern in the cognitive levels was found by researchers in the West (Karplus & Karplus, 1970; Lawson, 1978; Lawson & Blake, 1976; Lawson & Nordland, 1976; Shemesh, Eckstein, & Lazarowitz, 1992; Valanides, 1996). They reported that a majority of their subjects were either at the concrete or transitional stage of cognitive development and less than half of the students had reached the formal stage of development. In fact Shemesh, Eckstein, and Lazarowitz's (1992) study showed that young adults (18-year-olds) were still in the process of developing their cognitive levels.

4.5 Achievement of Algebra Test and Cognitive Levels

The differences in achievement of Algebra Test and the cognitive levels of the students were investigated using the following statistical techniques:

1. *t*-test comparison of the mean scores in Algebra Test of students in concrete (Concrete IIA and IIB) and formal levels (Formal IIIA and IIIB).
2. One-way analysis of variance to compare the performance in Algebra Test of students in Concrete IIA, Concrete IIB, Formal IIIA, and Formal IIIB levels of cognitive development. Significant result of the one-way analysis is subjected to multiple comparison test to determine the significant differences among the four cognitive operational levels. The Scheffe Multiple Comparison Test is used to test for these differences.

4.5.1 Comparison of Students at Concrete and Formal Levels

Table 4.6 presents the *t*-test analysis to compare the achievement of Algebra Test of students in the concrete and formal levels.

Table 4.6
t-test Comparison Between Students at Concrete and Formal Levels in Algebra Test

	Concrete level	Formal level	*t*-test	
			t	p
mean	10.70	23.79	-13.722	sig.
std. deviation	7.29	3.84		

Students at the concrete level had a mean score of 10.70 and a standard deviation of 7.29, while the mean score and standard deviation for those at the formal levels were 23.79 and 3.84 respectively. The value of t was significant at $p < .05$. The results indicate that students operating at formal level of development achieved better in the algebra test compared to those at concrete level of development. One probable reason for this difference in achievement is that the learning of algebra involves higher abstract thinking (Fong & Chong, 1995).

The results are consistent with the findings of Palanisamy (1986) who reported significant differences between cognitive development and achievement in both the fraction test as well as the ratio and proportion test among Malaysian Forms Two and Four students. His findings showed that students at the formal level of cognitive development achieved better in the mathematics tests compared with those at the concrete levels.

4.5.2 Comparisons of Cognitive Levels on Achievement in Algebra Test

Table 4.7 presents the analysis of variance for Algebra Test scores as the dependent variable and cognitive levels as the independent variable. The one-way analysis of variance gave a F-ratio of 51.812 which was significant at the $p < .05$ level. The results of the analysis indicate that there was a significant difference in the algebra achievement of students in different cognitive operational levels.

Table 4.7
One-way Analysis of Variance for Algebra Test by Cognitive Levels

Source	Sum of squares	df	Mean square	*F*	Sig.
Between Groups	5690.344	3	1896.781	51.812	.001
Within Groups	4942.203	135	36.609		
Total	10632.547	138			

Multiple Range Test
Scheffe Procedure

Cognitive Levels	Concrete IIA	Concrete IIB	Formal IIIA	Formal IIIB
Concrete IIA		*	*	*
Concrete IIB	*		*	*
Formal IIIA	*	*		
Formal IIIB	*	*		

(*) Denotes pairs of groups significantly different at the .05 level

The Scheffe Multiple Comparison Procedure indicates that the mean algebra achievement of the students in the levels Formal IIA and Formal IIIB were significantly higher than the mean algebra achievement of the students in levels Concrete IIA and Concrete IIB. Furthermore, the mean algebra achievement of the students in level Concrete IIB was also higher than that of those in level Concrete IIA. However, there was no difference in algebra achievement between the students in levels Formal IIIA and Formal IIIB.

The results suggest that the students who were at formal operational level performed better in algebra compared to students who were at the concrete operational levels. Similarly, the students in Concrete IIA performed better than those in Concrete IIB.

4.6 Relationship Between Cognitive Levels and Levels of Understanding of Algebraic Notation

A cross-tabulation of the levels of understanding of algebraic notation with different cognitive levels was carried out. To ascertain the relationship between cognitive levels and levels of understanding of algebraic notation, a chi-square analysis was used.

4.6.1 Levels of Understanding of Algebraic Notation by Cognitive Levels

Table 4.8 presents the distribution of levels of understanding of algebraic notation and cognitive levels of the students. The results show that none of the students in Concrete IIA level progressed beyond level 2 of understanding of algebraic notation. Of the 83 students in the Concrete IIB operational level, only 16.9% (14) and 4.8% (4) of them were in Levels 3 and 4 of understanding of algebraic notation respectively. This implies that a majority of the students (a total of 78.3%) in Concrete IIB cognitive levels were operating at the lower levels of understanding of algebraic notation (Levels 0, 1, and 2).

Among the 19 students in Formal IIIA, 57.9% of them (11) were found at the Level 3 of understanding of algebraic notation. All the students in the Formal IIIB levels were also in the higher levels of understanding (Levels 3 and 4). The results indicate a progressive increase in the students' levels of understanding of algebraic notation with that of cognitive development. The results imply that students at higher cognitive levels achieved higher levels of understanding of algebraic notation more frequently than the students at lower cognitive levels. The findings support the results of Palanisamy (1986) who reported that Form Four students in Concrete IIA achieved mainly Levels 1 and 2 understanding in both the fraction and the ratio and proportion tests and that the majority of the formal level students attained Levels 3 and 4 of understanding.

Table 4.8
Distribution of Levels of Understanding of Algebraic Notation and Cognitive Levels of Students

Algebra Levels	Concrete				Formal				Total	
	IIA		IIB		IIIA		IIIB			
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
0	12	70.6	17	20.5					29	20.3
1	4	23.5	27	32.5					31	22.3
2	1	5.9	21	25.3	3	15.8			25	18.0
3			14	16.9	11	57.9	10	50.0	35	25.2
4			4	4.8	5	26.3	10	50.0	19	13.7
Total	17	100	83	100	19	100	20	100	139	100

4.6.2 Comparisons of Levels of Understanding of Algebraic Notation Between Students at Concrete and Formal Levels

As shown in Table 4.8, there were many blank cells or cells which had values less than 5. Therefore, the Concrete IIA and Concrete IIB levels were condensed to form the Concrete level and Formal IIIA and Formal IIIB, to form the Formal level.

According to Kuchemann (1981a), Levels 0, 1, and 2 of understanding of algebraic notation were considered as the lower levels of understanding compared to

Levels 3 and 4, which were considered as the higher levels of understanding. Hence Levels 0, 1, and 2 were condensed to form the lower level of understanding of algebraic notation and Levels 3 and 4, the higher level of understanding. The data are reduced to that displayed in Table 4.9. A chi-square analysis was then carried out to determine if there was any significant relationship between the levels of understanding of algebraic notation and cognitive levels.

Table 4.9
Cross-tabulation of Levels of Understanding of Algebraic Notation with Cognitive Levels of Students

	Concrete		Formal		Total	
	n	%	*n*	%	*n*	%
Lower levels	82	82.0	3	7.7	85	61.2
Higher levels	18	18.0	36	92.3	54	38.8
Total	100	100.0	39	100.0	139	100.0

Chi-square = 65.213, df = 1, p = .001

It should be noted that one of the cells had a value of 3. Hence the data are interpreted with caution. Among the100 students at concrete level, 82.0% (82) were at the lower levels of understanding whereas a higher proportion (92.3 %) of those at formal level were at higher levels of understanding. Of the 85 students in the lower algebra levels, only 3 of them were in the formal level. The majority of those in the formal level (36 out of 54) were found at higher algebra levels. The chi-square analysis revealed a chi-square value of 65.21 which was significant at $p < .05$. The results show that a significant relationship existed between cognitive levels and levels of understanding of algebraic notation.