CHAPTER 5
SUMMARY AND CONCLUSIONS

5.1 Introduction

The present study was designed to achieve a two-fold purpose. It aims to investigate the students’ interpretations of the letters used in algebraic notation of Form Four students and their levels of understanding of algebraic notation. It also seeks to ascertain if there is any significant relationship between the understanding of algebraic notation with the students’ cognitive development.

Five research questions were formulated. They were:

1. What are the students’ interpretations of algebraic notation?
2. What is the distribution of the levels of understanding of algebraic notation among Form Four students as measured by the Algebra Test?
3. What is the distribution of the cognitive levels of Form Four students as measured by the Longeot Reasoning Test?
4. Are there any significant differences in the achievement of Algebra Test among students of different cognitive levels?
5. Is there any significant relationship between the students’ levels of understanding of algebraic notation and their cognitive levels?

The study utilises both the qualitative and quantitative approaches to answer the research questions. Subjects of the study comprised 69 boys and 70 girls of Form Four students from a national secondary school in the Petaling Jaya District.
They were selected based on their Mathematics performance in the Lower Secondary School Assessment (PMR).

Two instruments were used in this study. They were the Algebra Test and the Longeot Reasoning Test. The Algebra Test was adapted from the 'Concepts in Secondary Mathematics and Science' [CSMS] algebra research project. It contained 30 items, which required written responses from the students. The 28-item Bahasa Malaysia version of the Longeot Reasoning Test (Lew, 1987) was adapted to measure the students' levels of cognitive development.

From the responses to the Algebra Test, the students were categorised into different levels of understanding of algebraic notation. The responses were also analysed for the successful using of six categories of interpretation of the letter identified by Kuchemann (1981a). Errors made by the students in their interpretation of letters were also identified with reference to past researchers' classifications.

From the responses of the Longeot Reasoning Test, the students were categorised into four different cognitive levels. Statistical analysis employed to analyse the data gathered in this study include the computation of frequencies and percentages, cross-tabulation and chi-square analysis, t-test, and one-way analysis of variance.

In addition to the summary of the findings of the study, the implications of the findings as well as suggestions for improvement in classroom teaching and recommendations for further research are presented.
5.2 Summary of Findings

The findings obtained in this study are reported according to the research questions.

1. The analysis of the responses of the students to the Algebra Test revealed that more than 80% of the students were successful in using the lower level of interpretations of the letters, that is, letter evaluated, letter not used, and letter used as a concrete object. However, 63.3% of them were successful to interpret the letter as a specific unknown. Less than 30% of the students could only successfully use another two higher levels of interpretations of letter as a generalised number and as a variable.

Nine errors made by the students in their interpretations of the letters were:

(a) numerical substitution; (b) ignoring the letters; (c) letter treated as a concrete object; (d) conjoining of numerical and algebraic elements; (e) wrong concatenation; (f) writing down of product instead of sum of letters; (g) changing of algebraic expressions to algebraic equations; (h) omission of brackets; and (i) treating $t$ as 0.

Of these errors making "conjoining the numerical and algebraic elements" constituted the highest proportion (50.4%), while making "wrong concatenation", the lowest (0.7%). The error of writing down of product instead of sum of letters is peculiar to the present study as it is not noted in past studies. It was found that 29.5% of the students made this error.
2. Analysis and classification of the levels of understanding of algebraic notation showed that a total of 61.2% of the students were in the lower levels of understanding of algebraic notation (Levels 0, 1, and 2) as compared to 38.8% in the higher levels (Levels 3 and 4). Only 13.6% were found in the highest hierarchical Level 4 while 25.5% were in Level 3. The data also indicated that 20.9% of the students could not handle the Level 1 items. For Levels 1 and 2, the percentages of students reaching these levels were 27.3% and 18.0% respectively.

3. The analysis of the cognitive development of the students revealed that a total of 71.9% of the students was in concrete operational level, compared to 28.1% in formal operational level. A majority of the students were in Concrete IIIB level (59.7%) and 12.2% were in Concrete IIA. The percentage of students in Formal IIIA and Formal IIIB were 13.7 and 14.4 percent respectively.

4. Findings from the \( t \)-test comparison of mean scores in the Algebra Test between the students in the concrete and formal levels revealed that there was a significant difference in the performance of students operating at concrete and formal levels of cognitive development. The results indicated that formal operational students performed better in the Algebra Test compared to the concrete operational students.

The one-way analysis of variance for the Algebra Test scores as the dependent variable and the four cognitive levels as the independent variable indicated that there were significant differences in the Algebra achievement of the
students with differing cognitive operational levels. The results of Scheffe Multiple Comparison Procedure showed that there were substantial differences in the mean Algebra Test scores between students in these levels: Concrete IIA and Concrete IIB levels; Concrete IIA and Formal IIIA levels; Concrete IIA and Formal IIIB levels; Concrete IIB and Formal IIIA levels; and Concrete IIB and Formal IIIB levels. However, no significant difference in the achievement of the Algebra Test was found between students in Formal IIIA and Formal IIIB levels.

5. The analysis of the levels of understanding of algebraic notation by cognitive levels revealed that all the students at Concrete IIA level did not progress beyond Level 2 of understanding of algebraic notation. A majority of the students in the Concrete IIB level (78.3%) were found in Levels 0, 1, and 2 of understanding of algebraic notation. On the other hand, a majority of the Formal IIIA students (84.2%) and all the students in the Formal IIIB level were found in Levels 3 and 4 of understanding of algebraic notation.

The results of the chi-square analysis indicated that a relationship existed between cognitive levels and the levels of understanding of algebraic notation. A majority of the students in the formal levels (92.3%) were also at the higher levels of understanding of algebraic notation (Levels 3 and 4) whereas a majority of the students in the concrete level (80.0%) were at the lower levels of understanding of algebraic notation (Levels 0, 1, and 2). The results implied that to achieve higher levels of understanding of algebraic notation, a student would have to display higher levels of cognitive functioning.
5.3 Implications and Suggestions

This study is the first to be conducted in Malaysia on the students’ level of understanding of algebraic notation based on CSMS test. Furthermore, the meanings attached to the letters used in algebraic expressions and equations are very basic in algebra. So, although this study involved a small sample of Form Four students selected from only one secondary school in an urban city, the findings do suggest several noteworthy implications. These implications warrant the attention of educators in curriculum planning, the mathematics teachers in the school concerned as well as lower secondary school mathematics teachers. This is important because the lower secondary school mathematics teachers, especially those teaching in Forms Two and Three classes, are those who laid the foundation for the topic of algebra when it was taught formally at these levels.

This study shows that a high proportion (61.2%) of the students uses the lower interpretations of the letters, that is, letter evaluated, letter ignored and letter used as an object. This means that these students would not be able to succeed in the higher-level questions in which the letter represents a range of unspecified values and for which a systematic relationship existed between two such sets of values. Operating at the lower levels of interpretation of letter shows that the students had not acquired a structural conception of variables in algebraic expressions. This implies that greater effort and time are needed in classroom instruction towards creating a solid base in developing the structural conceptions of algebra.

In view that students are predisposed to the lower levels of interpretation of letters and also to the idea of letter as specific unknown values, it may be useful for
teachers to adopt the interpretation of letters as generalised number from the time that letters are first introduced. Booth (1989) suggested that treating $2a + 5b$ as 2 apples plus 5 bananas might not be helpful. This is because "not only does it encourage an erroneous view of the meaning of letters, but also it can be used by students to justify this simplification of $7ab$" (p. 26). Hence, teachers may be alerted on the pitfalls of such approach in algebra instruction to avoid conflict with the eventual aim of using letters to represent number of objects and not to consider letters as objects.

The study identifies many errors made by the students. Researchers had shown that some of these errors had their roots in the previous learning of arithmetic (Bodin & Capponi, 1996; Herscovics & Linchevski, 1994; Kieran, 1989). Therefore, teachers may be made aware of the subtle interference of arithmetic in the learning of algebra, perhaps through in-service courses and seminars. Such information could be used by the mathematics teachers to plan a clear presentation of algebraic concepts to the students. It is useful to students that teachers make explicit distinction between arithmetic and algebraic usage of variables.

The results of the study indicate the students' tendency to view a conjoin term (for instance, $3n$) to represent the sum rather than the product in algebra. This indicate that in introducing such terms in algebra, the teacher are encouraged to write the product in full ($n \times 3$ or $3 \times n$) for a substantial period of the students' early work in algebra.

The study has identified conjoined answers as one of the errors made by students. However, Tirosh, Even and Robinson (1995) had shown that there are
some teachers who are not aware of the students’ tendency to conjoin or “finish” open expressions. The knowledge of this error and other errors made by the students together with the possible reasons for them would be helpful to teachers in planning their teaching strategies.

The results show that there is a wide difference in the levels of cognitive development among secondary school students, ranging from early concrete to late formal operations. In addition, the achievement of the Algebra Test and the levels of understanding of algebraic notation of the students are related to their cognitive levels. Students who are at a concrete operational level might encounter difficulty in understanding concepts that require formal operations. Therefore, the curriculum developer would have to take this into consideration when selecting and sequencing the contents of the secondary school mathematics curriculum.

Teachers are encouraged to treat their students as individuals in the mathematics class. The students can best benefit from mathematics instruction when they are provided with differential learning experiences suitable to their level of cognitive development. The teacher could attempt to introduce lessons to students in different ways and at different depths that are appropriate to their levels of cognitive development. In challenging the students, it is also important to align the challenges with their cognitive development. Only such alignment would promote confidence and a positive attitude among students towards mathematics and ultimately result in an overall improvement of mathematics achievement.
5.4 Recommendations for Further Research

The sample of this study is restricted to Form Four students in one particular urban school. This poses a limitation to the study. Therefore, studies on samples drawn from other urban schools as well as rural schools for comparison are recommended. Furthermore, a larger sample will make the findings more conclusive.

In the Malaysian secondary school mathematics curriculum, algebra is formally taught in Form Two. Similar studies conducted at the end of Form One or in the beginning of Form Two is recommended to determine the students’ understanding of algebraic notation gleaned informally in their primary school years. Such information would be useful for the mathematics teachers teaching this topic in Form Two.

Longitudinal studies on students from Forms One to Five on the understanding of algebraic notation are also recommended. Such studies will provide information on the hierarchical levels of understanding of the students as they progress through their secondary mathematics.

The independent variable in this study is the cognitive development of the students. It is a factor affecting the performance of the students in the Algebra Test as well as their levels of understanding of algebraic notation. However, due to the scope of the study, the data provide not the reasons for the errors made by students nor the reasons for the misinterpretation of the notation. Therefore, it is recommended that further studies could be planned to explore the sources of the misinterpretations.
This study confines to only one aspect of algebra, that is, students’ interpretation of letters. In-depth studies on other algebraic concepts are also recommended.

5.5 Conclusion

This exploratory study has attempted to analyse the interpretations of letters used in algebraic expressions and equations by Form Four students and also identified some errors made by the students. In addition, the study has also determined the students’ distribution of the levels of understanding of algebraic notation, their cognitive levels, and the relationship between them. The findings of the study reveal that a majority of the students are at the lower levels of understanding of algebraic notation. Less than one-third of them could successfully use higher levels of interpretations of letters. These imply that a majority of them would not be able to meet with the demands of higher mathematics. Although the sample of this study may not be a general representative of students, it is believed that the findings would be useful to both the mathematics teachers and curriculum planners. It is hoped that the study will inspire other researchers in Malaysia to consider the topic of algebraic notation and to further extend this study. It is also the hope of the researcher that the study may contribute toward the building of a strong foundation in algebra, the cornerstone of secondary school mathematics.