Chapter 3
Theoretical Framework

3.1 Introduction

In a competitive exchange market, when one party has informational advantage over the other party to the transaction, the outcome may not be equilibrium. Even when equilibrium does exist it exhibits strange properties. In this circumstances, if individual willing to reveal their information, everyone in the market will be better off. Thus in asymmetry information, trade that took place is not Pareto Optimal.

This section explains the predictions of the theory of asymmetry information in insurance market:

"In competitive insurance markets with asymmetric information, high risk individuals end up purchasing larger quantities of insurance than do low risk individuals....Since the high risk consumers buy larger quantities, an insurer can break even only if marginal prices rise with quantity."    

John Cawley and Tomas Philipson. 1999

specifically, non-linearity in prices and covariance between risk and quantity. These characteristics of both insurers and insureds cause "barriers to trade" in insurance market with asymmetry information.
3.2 Model For Non-linearity In Prices

Non-linearity in prices is necessary as it ensures risk sorting, an insurer can achieve breakeven only when prices rise with quantity. The central implication is that high risk individuals will purchase larger quantity of insurance than low risk individuals. In order to avoid adverse selection in their risk portfolio, insurance companies set separating equilibrium for different category of risk. Since the high risk individual opt for larger policy, he is charged at higher price. Hence prices rise with quantity. And the low risk individuals are quantity constrained in order to make their contracts undesirable to those of high risk. This non-linearity in prices and covariance between risk and quantity will be tested in empirical section.

3.2.1 Perfect Information

Separating equilibrium, theoretically, is set to overcome the problem of pooling equilibrium which is found to be not viable. Before proceed to asymmetry information, consider a contingent market with perfect information. Suppose an individual has two contingent goods, \( W_g \) (wealth in good time) and \( W_b \) (wealth in bad time). This individual believes that good time will occur with probability \( \pi \), and therefore the expected utility is (Laffont ;1993):

\[
V(W_g, W_b) = \pi U(W_g) + (1 - \pi) U(W_b)
\]

And the budget constraint is:

\[
W = P_g W_g + P_b W_b
\]

\( P_g \)=Dollar of wealth in good time
\( P_b \)=Dollar of wealth in bad time
If the insurance market is fair then;
\[ P_s = \pi \]
\[ P_b = (1 - \pi) \]

A utility maximizing individual will opt for a situation in which \( W_s = W_b \), that is, wealth obtained is the same no matter what state occurs. Maximization of utility subject to budget constraint requires that the individual sets the marginal rate of substitution of \( W_s \) and \( W_b \) equals to the price ratio of these two goods;

\[
MRS = \frac{\partial V/\partial W_s}{\partial V/\partial W_b} = \frac{\pi U'(W_s)}{(1-\pi)U'(W_b)} = \frac{P_s}{P_b}
\]
\[ = \frac{U'(W_s)}{U'(W_b)} = 1 \]

or
\[ W_s = W_b \]

Hence this individual, faced with a fair market, will choose to insure the same level of wealth regardless of circumstances. Thus in perfect competition, high risk individuals and low risk individuals will opt for policies specifically designed for them and they opt for full coverage. Consider 2 individuals with an initial wealth at \( W_0 \) and face a possibility of loss \( L \). the high risk individual has a probability of loss of \( \pi_H \) and \( \pi_L \) for low risk individual, and \( \pi_H > \pi_L \). With fair insurance and state independence, both individuals would prefer on the certainty line; at point G for high risk type and point F for low risk type.
The above independent state situation can be explained by looking into how much coverage an individual would choose if it is perfect competition. Insurance company will offer policy on EF line (fair odds line) for low risk individuals and on EG line (fair odds line) for high risk individuals. In an exchange market consists of I identical consumers and a single good, endowment for agent i equals to \( w_0 \) with probability \( 1-\pi \) and \( w_0-L \) with probability \( \pi \). Let \( U(w) \) with \( U'(\cdot)>0 \) and \( U(\cdot)''<0 \) be the Von Neumann-Morgenstern utility function. Insurance company collects a premium \( \alpha \) and reimburses agent \( i \) for his loss \( L \) when he has an accident. The total premium collected is \( \alpha I \) and compensation is then \( I\pi L \). Zero profit constraints on the insurance company that results from perfect competition requires that \( \alpha = \pi L \). Then the income of the agent is:

\[
\begin{align*}
    w_0 - \alpha &= w_0 - \pi L & \text{If he does not have an accident} \\
    w_0 - L + L - \alpha &= w_0 - \pi L & \text{If he has an accident}
\end{align*}
\]
If the company sets \( q \) as price for per unit of compensation in the case of an accident, so that \( q = \pi \) and \( z \) as total coverage purchased. The consumers then solve the problem:

\[
\max_{x_1, x_2} \left[ (1 - \pi) U(x_1) + \pi U(x_2) \right]
\]

s.t.

\[
x_1 = w - qz \\
x_2 = w - L + z - qz
\]

or

\[
\max_z \left[ (1 - \pi) U(w - qz) + \pi U(w - L + z - qz) \right]
\]

and

\[
(1 - \pi) q U'(w - qz) = \pi (1 - q) U'(w - L + z - qz) \\
U''(w - qz) = U''(w - L + z - qz)
\]

\[
z = L
\]

Therefore, the consumer insures himself completely. If the agent faces "actuarially fair" insurance market, that is \( q = \pi \), he will always ensure that \( W_g = W_h \) (Laffont; 1993).

### 3.2.2 Imperfect Information

"...Asymmetry Information about quality of the consumers arises in insurance market when firms have difficulty judging the riskiness of those who demand insurance coverage..."

Puez and Snow (1994)

If the insurance company has imperfect information about which individuals fall into the low and high risk categories, the solution at point G and F will be unstable. The difficulty is that point F provides more wealth than point G does and will therefore be preferred by high risk types. They will have incentive to purchase insurance intended for low risk individuals, and in the absence of information about risk categories, the insurer
will have no basis for declining to offer coverage to them. With this mixed group of clients, the insurer will face a higher probability of loss than \( \pi_L \) (probability of risk for low risk type), and will lose money on each policy sold. Thus point F and G are not viable equilibrium.

Assume that the agents know their own probability of an accident but the insurer cannot distinguish between the two types. Thus given \( \pi_H \) and \( \pi_L \), and \( \pi_H > \pi_L \), company sets a price at \( q \) and each agent chooses his own level of coverage. Type H (high risk individuals) solves the following problem (Laffont; 1993):

\[
\begin{align*}
\max \left[ \pi_H U(w - L - qz + z) + (1 - \pi_H) U(w - qz) \right] \\
= \pi_H U'(w - L - qz + z)(1 - q) + (1 - \pi_H) U'(w - qz)(-q) = 0 \\
\frac{U''(w - L - qz + z)}{U'(w - qz)} = \frac{q}{(1 - q)} \frac{(1 - \pi_H)}{\pi_H}
\end{align*}
\]

and for low risk individual is:

\[
\frac{U''(w - L - qz + z)}{U'(w - qz)} = \frac{q}{(1 - q)} \frac{(1 - \pi_L)}{\pi_L}
\]

Since \( (1 - \pi_H)/\pi_H < (1 - \pi_L)/\pi_L \) it implies that \( z_H > z_L \). It explains high risk type opts for higher coverage than low risk type.

For a price corresponding to line EF (fair odds line to safe type) in figure 3.2, insurance company yields zero profit with respect to the low risk type who chooses point F, but incurring losses with respect to the high risk type. Therefore if policy is priced along the EF line, the overall average probability will be higher than \( \pi_L \), and company will make losses. Even if the company increases the price, the low risk type will buy less at
point A’ the company can make profit from them but it may not be sufficient to compensate for the losses accruing from the high risk types. At this point, high risk type will purchase at point B’, as the premium paid is lower than the risk they face.

![Figure 3.2 Premium Charged In Imperfect Information Situation](image)

And if the insurance company sets price on line EG (on the fair odds line of high risk type), the high risk type chooses point G for full coverage and company makes zero profit. But the low risk types prefer not to buy insurance as the price paid is higher than the risk they face and they remain at point E (Laffont; 1993). As the result only high risk policy is offered, and there is no market for low risk types (Arkelof; 1970).

3.2.3 Pooling Equilibrium

Therefore in imperfect information situation, we cannot find the equilibrium for both types of risk; neither on EF nor on EG curves. This gives the company one choice left: setting policy on market average odds line, that is, it will pool the customers for a common policy (Philips; 1988).
The market average fair odds are simply the odds that an insurer could offer to the average customers while breaking even on average as long as the contract is taken by a random sample of both types of customers. The premium per unit of compensation, or the market average fair premium is $p^M$ which is equal to $(n_1p + n_2p')/(n_1 + n_2)$, where $n_1$, $p$ and $n_2$, $p'$ are number of safe and risky customers and the price charged respectively.

![Diagram](image)

Figure 3.3: The market average fair odds line and Pooling Contract

The market average price, $P^M$, represents unfavourable odds to the safe customers but favourable odds to the high risk individuals. Safe types will therefore choose partial insurance and risky types will opt for more than full coverage. The contract offered at point M on EM line will not be viable; it will be driven by competition and company has to offer other contract along the EM line which optimizes safe customers utility level, this should be at point L (Hiller ;1997).
Any pooling contract offered at a point below EM line would produce supernormal profits to the insurer if it can attract both types of customers. Such a contract cannot be Equilibrium, since competition would drive the contract terms to be improved until the market average fair odds be offered.

Any contract to the right of point L along the EM line could be improved upon by another insurer offering a contract at L; since both risky and safe types would prefer the contract at L, and insurer offering such a contract would attract all customers away from insurer offering contracts to the right of L.

And any contract along the EM line to the left of L could be improved upon by an insurer offering a contract allowing location at L; because the safe customers would accept the contract at L and that contract would produce supernormal profits since it lies below the safe customer's fair odds line. Competition would drive it to point L. This contract to the left of L also becomes loss making as it attracts also risky customers as it lies above their fair odds line.

Therefore contracts offered on EM line will be driven by competition to offer at point L; which maximizes utility of the safe type customers. But if given a choice, safe types will not purchase insurance at point L, they would prefer gamble to buying insurance. This results to only the risky customers are buying the insurance. This is because at market average premium, \( P^m \), low risk types are paying more than the risk they face, in other words, expected utility is higher at point E than at any point along the EM line.
Therefore left only the risky types are prepared to accept contracts offered along the market average line. But this type of contract will not be offered since it produces losses to the insurance company, who instead will offer insurance at the premium \( P' \) for risky customers who will optimize utility at point G.

In conclusion, the safe types pay an average premium rate that is higher than they would pay in the full information world or they are excluded from the market, only the risky types purchase the insurance at an appropriate rate.

Any point along the market average line therefore cannot be Nash Equilibrium. Alternatively, if an insurer is allowed to offer contract at point Q, it will attract only those of safe types. Since point Q is below safe fair odds line, it produces supernormal profit; point Q therefore is more superior than point M and L. But point Q is also not Nash Equilibrium possible, as competition will cause insurer to offer a contract on the market average line again.

3.2.4 Separating Equilibrium

It is not possible to have single policy that is Nash Equilibrium (Cawley and Philipson ; 1999) for both high and low risk individuals, in other words both risk types of customers have to separate themselves, or self select the policies offered to them. That is there must be a separating equilibrium which commensurate with the risk each individual faces.
One contract is offered at the fair odds line $EG$, and high risk types choose full coverage at point $G$. The other contract is offered on the fair odds line for safety type, and they maximize utility at point $R$ which offers only partial coverage. Although the indifference curve $U_{H}$ intersects with fair odds line for the safety type, high risk individuals will not choose contract offered to the low risk individuals at $R$ since it offers only partial coverage. At point $G$, insurer breaks even on high risk individuals. Low risk types, on the other hand, prefers point $R$ to $G$, since $R$ lies on the safe fair odds line, also allows insurer to break even.

Notice that partial insurance offered at point $R$ is determined by the intersection of the safe fair odds line and the risky indifference curve. Any point higher than $R$ would attract high risk individuals to the contract offered to the low risk types, and any point below point $R$ would mean that it would be improved upon by offering a slightly higher coverage at a higher premium and still can attract only safe customers and profits. And competition for safe customers will drive the level of cover up to that producing at point $R$. 

26
Note that the separating equilibrium is clearly inferior to the one when there is perfect information. The policy offered to the low risk type will take a deductibles; although with a reduced premium. If the insurers can observe the risk type associated with each individual, low risk type will be better off and high risk type would not be worse off when they were offered their respective contract according to their risk level. Further more the policies at R and G are not equilibrium. (Michael Rothschild and Joseph Stiglitz 1976). Consider another firm offers contract at Y and Z; Y is making losses and Z is profitable. High risk types prefer Y to G and low risk types prefer Z to R. These contracts, when offered, do not make losses as profits from Z subsidizes losses from Y. and these contracts upset the equilibrium level at G and R.

This barrier of trade between insurers and low risk individuals is necessary for insurers to underwrite and offer contract which is only profitable. Consider in separating equilibrium, each type of individual when choosing a contract, is signalling to the insurer which type of risk they belong to. And insurer will know the risk type from their consumption. In other word, low risk type is not encouraged to purchase full coverage, as they will be paying higher premium than the risk they face.

Asymmetry information also discourages insurers to offer contract to both risk types. In the pooling equilibrium, low risk type will be dropped out as they are paying higher premium or the expected utility from the uninsured state is higher than when he is insured. On the other hand, higher insurance value or if company sets a lower premium rate, it will cause higher probability of loss in the portfolio, furthermore higher insured
value creates an incentive for the subscribers to increase risk; there is moral hazard problem.

In order to test the existence of barrier to trade due to asymmetry information, we investigate the non-linearity in prices and covariance between risk and quantity of coverage.

3.2.5 Insurance Under Asymmetric Information

Since insurers will incur loss if it charged a standard premium rate in pooling equilibrium, and less than efficiency in separating equilibrium (Rothschild and Stiglitz; 1976), we need to find a level, which is closer to perfect information equilibrium. Under assumption that high risk consumers will purchase larger than usual policy, insurer will opt for quadratic pricing strategy to solve the information problems.

Insurance under asymmetry information implies that high risk individuals end up purchasing larger quantities of insurance than do low risk individuals. Therefore insurer can break even in a competitive market only if marginal prices rise with quantity—the opposite of a bulk discounts. Thus the total price is convex with the quantity of coverage. To test the implications of asymmetry information, the covariance between contract size and risk is also examined (Cawley and Philipson ;1999).

A consumer of risk c who faces the unit price p has a demand for insurance coverage denoted D(p,c) defined as;
\[ D(p,c) = \arg\max_z U(w - L + z - qz) + (1 - c)U(w - qz) \]. This implies that risk raises demand; \( D(\geq 0) \), people of higher risk value insurance more than the lower risk. This is necessary for the separating equilibrium in which the high risk types choose a larger policy at a higher unit price. The utility function may not be state independent.

The expected profit of the insurance company is;

\[ \Pi = E[D(p,c) (p-c)] \]

The expectation is over consumer pairs \((D,c)\) of coverage and risk. The cost of production therefore is determined by buyers, the profit function can be rewritten as;

\[ \Pi = E[D(p-c)] \]

\[ = E[D] (E[p] - E[c]) + \text{cov}(D,p) - \text{cov}(D,c) \]

thus profits are decomposed into three parts. The first one is the price above claim expenditure or “loadings” which may be due to alternative cost of production or markups. Secondly the demand is positively dependent on covariance between quantity and price, but negatively on covariance between quantity and risk. In ensuring zero profit, non-linearity of prices is needed because when more riskier individuals demand larger contracts but all face the same unit price, the insurer cannot cover claim with premium. Thus the price;

\[ p(D)D = \alpha + \beta_1 D + \beta_2 D^2 \]

and the profit is then;

\[ \Pi = \alpha + E[D] (\beta_1 E[c]) + \beta_2 E[D^2] - \text{cov}(D,c) \]
In fair pricing, where if $z = L$, $\alpha = 0$ and $\beta_1 = E[c]$, then the first two terms in the equation vanish and the nonlinearity of prices ensures that $\beta_2 \geq 0$; it must cover for any positive covariance between demand and risk (Cawley and Philipson; 1999).

Thus in this section the non-linearity of prices:

$$p(D)D = \alpha + \beta_1 D + \beta_2 D^2$$

the theory of asymmetry information predicts that $\beta_2 > 0$, $\alpha$ is fixed underwriting cost and a marginal price schedule (Beliveau; 1991).

### 3.3 Covariance Between Risk And Quantity

Conventional theory under asymmetry information predicts that the amount of coverage desired by an insured will be positively correlated with the insured's probability of incurring loss:

"…...That individuals purchasing unusually large amounts of protection, relative to their observably characteristics (i.e., income, age, marital status, etc) are more likely to be of higher risk than other consumers. It is hypothesized that insurers adjust for this with larger loading factors for unusually large policies."

Barbara C. Beliveau. 1979.

Thus if consumers maximize expected utility, riskier consumers will, for any fixed price per unit of coverage, demand more coverage than less risky consumers (Lewis; 1989). In order to decide what terms the insurers should offer to let consumers buy
insurance, information about consumers’ market behavior is vital to make inference about their loss probabilities. And one way to achieve this under asymmetry information is to assume that; those with high probabilities of loss will demand more insurance than those of less accident-prone (Rothchild and Stiglitz ;1976). We examine the relationship and the likelihood of holding larger H&S insurance coverage by using logit regression.

3.4 Conclusion

Therefore quadratic pricing schedule is expected to be present for insurers to achieve breakeven point. The schedule is more efficient than price charged in pooling and separating equilibrium. Insurers who charge a standard premium rate as in pooling equilibrium will suffer losses as low risk types will not buy. This is because premium paid is higher than the level of risk they face. In separating equilibrium, low risk type is restricted from buying larger coverage. But in quadratic pricing, both categories will have a chance to buy at the same price, and are charged differently only when quantity purchased exceeds a certain level. This is because higher risk consumers are expected to purchase higher level of coverage.