CHAPTER 3

Domain Knowledge in Mathematics Vector

3.1 Mathematical Problem Solving

Problem solving is a main feature in studying mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics [Wilson,J.W. et. al]. Reitman defined a problem solving as when we have the description of something but do not yet have anything that satisfies that description. In a problem solving, there must be a goal, a blocking of that goal for the individual, and acceptance of that goal by the individual [Henderson and Pingry]. Different persons may be faced with the differenct problem solving. It depends on each individual how they face the problem and how they decide the solutions.

Mathematics teachers talk about, write about, and act upon, many different ideas under the heading of problem solving. Some have in mind primarily the selection and presentation of good problems to students. Some think of mathematics program goals in which the curriculum is structured around problem content. Others think of program goals in which the strategies and techniques of problem solving are emphasized. Some discuss mathematics problem solving in the context of a method of teaching [James W. Wilson et. al]. Thus the combination of these ideas will create a discussion of mathematics problem solving.

Problem solving in mathematics can be divided into three. Firstly, mathematics problem solving as a process. The main area, which is important in this process, is a domain specific knowledge. The effectiveness of organizing the knowledge will contribute to successful problem solving. The students with a good knowledge base were most able to use the heuristics in geometry instruction in mathematics problem solving [Kantowski]. Heuristics are kinds of information, which is available for students in making decisions during problem solving. In other words, heuristics is synonyms to strategies, techniques and rule of thumb. Kantowski used heuristic instruction to enhance the geometry problem solving performance of secondary school students.

Secondly, mathematics as an Instructional Goal. Before we go further, we should know what mathematics is. Mathematics is the process of exploration, inquiry, discovery, plausible reasoning or problem solving [James W. Wilson et. al.]. These processes are involved in algebra, geometry, number, probability, statistics or calculus. Problem solving is the heart of mathematics. So that, in order to allow students experience mathematics as problem solving, mathematics instruction should be designed.

Why problem solving is important in mathematics? This is because, first, problem solving is a major part of mathematics. Second, mathematics has many application and this application represent important problems in mathematics. Third, there is an intrinsic motivation embedded in solving mathematics problem that is can stimulate the interest and enthusiasm of the students. Finally, problem solving must be in the school mathematics curriculum to allow students to develop the art of problem solving. This art is so essential to understanding mathematics and appreciating mathematics that is must be an instructional goal [James W. Wilson et. al.].

3.2 Vector

Vectors either in physics or mathematics are used to mathematically represent objects in motion and at rest. The objects in motion and the object at rest have a mathematical relationship of direction and magnitude. For example, a train moving on a railroad has a direction and a speed of it travelling (magnitude). The direction of a vector must be in relation to a fixed coordinate system such as 1 dimension (1 D), 2-dimension (2D) and 3-dimension (3D). A vector has 2 characteristics, which are direction and magnitude.

3.2.1 Direction

Direction is motion relative to a fixed reference point. On Cartesian coordinate system, reference point is the origin (point 0,0) and can be expressed in relation to the x and y axis. For example, an object can be travelling 3 units in the x direction and –3 units in the y direction. It can be written as (3,-3). These coordinates are referred to as position coordinates and tell about the direction of travelling object. To obtain the direction of objects travelling, trigonometry functions are required to evaluate vector such as sine, cosine and tangent. Usually, the object moving can be told by moving north, south, east, west or a combination of directions from the origin. But some vectors are directed northeast (at a 45 degree angle) and some vectors are even directed northeast, yet more north than east.

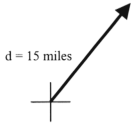
Thus, the identification of a direction of a vector is not due East, West, South or North. There are a variety of conventions for describing the direction of any vector but this study will only focus in two of these conventions.

1. The direction of a vector is often expressed as an angle of rotation of the vector about its 'tail' from east, west, north, or south. For example, a vector can be said to have a direction of 35 degrees North of West (meaning a vector pointing West has been rotated 35 degrees toward the northernly direction) or 65 degrees East of South (meaning a vector pointing South has been rotated 65 degrees towards the easterly direction).

2. The direction of a vector is often expressed as a counter-clockwise angle of rotation of the vector about its 'tail' from due East. Using this convention, a vector with a direction of 30 degrees is a vector, which has been rotated 30 degrees in a counter-clockwise direction relative to due east. A vector with a direction of 160 degrees is a vector, which has been rotated 160 degrees in a counter-clockwise direction relative to due east.

3.2.2 Magnitude

The magnitude of the vector is the motion related quantity associated to the direction of the vector [Richard et. al]. If **V** is a particular vector, then |**V**| symbolizes its magnitude. Quantities which can be specified by magnitude only are called scalar quantities to distinguish them from vectors which have both direction and magnitude. The length of the arrow depicts the magnitude of a vector in a scaled vector diagram. For example, the diagram below shows a vector with a magnitude of 15 miles with the scale used for constructing the diagram is 1 cm = 5 miles, and the vector arrow is drawn with a length of 3 cm. That is 3 cm x 5 miles/cm = 15 miles.



Scale: 1 cm = 5 miles

Figure 3.1: Example of vector magnitude

Velocity is a property used to derive each of the listed vector magnitude quantities. In a Cartesian coordinate system, the magnitude of a vector is the distance from the origin to the position of the vector point. In order to obtain the magnitude of the vector, we can use a Pythagorean Theorem i.e. $c^2 = a^2 + b^2$.

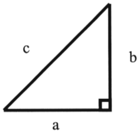


Figure 3.2 : Triangle of Pythagorean Theorem

The computation of vectors can be done either by adding, subtracting or multiplying. This can be figured out in our natural live, which happens every day without using mathematics. For example, birds will add and subtract vectors to fly south for winter and north for the summer because they have to fly with or against winds. Other example is an airplane navigating along the same principle as birds flying against winds, and boats and ships must add vectors and subtract vectors when monitoring against or with a current.

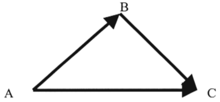
Vectors may be used to find many different types of things. It can be used to represent the velocity and acceleration of an object because all of these quantities have both a magnitude and a direction. Knowing the time over which a displacement occurred enables us to determine the velocity vector for an object and knowing the velocities at different times enable us to find the acceleration of the object. Velocity vectors and acceleration vectors are all added together in the same manner.

3.3 Vector Algebra

Vector is represented by a line with an arrowhead on it. In order to measure, we need distance and direction. Just as there is an arithmetic of numbers, there is arithmetic of vectors. There are three categories of vector arithmetic, which are as follows :

3.3.1 Vector Addition

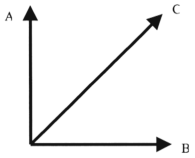
To add vectors there are two techniques available, geometric addition and algebraic addition. Both yield the same result. The choice of which technique to use in adding vectors depends on the application and is a matter of convenience. Since a vector is defined by its magnitude and direction, changing its location in our reference frame without changing its direction or magnitude leaves it as the same vector. We are free to relocate a vector anywhere in our space where we find it convenient. In geometric vector addition, to add vectors geometrically we just place the tail of one at the head to the other. The sum then is a vector from the tail of the first vector to the head of the last. The figure below shows the example of geometric addition.



$$\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AC}}$$

Figure 3.3: Example of Geometric Addition

The algebraic addition of vectors involves simply adding up the components of the vectors. Imagine a vector with its tail at the origin. The scalar components of that vector are just the coordinates of the head of the vector. The coordinates are the distances along each axis which define the position of the head of the vector. To add two vectors, add all the x components in the horizontal direction together and all the y components in the vertical direction. The algebraic addition of vectors works because the sums of the components are the components of the sum. Figure below shows the example of algebraic addition.

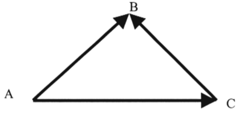


$$\mathbf{A} + \mathbf{B} = \mathbf{C}$$

Figure 3.4: Example of Algebraic Addition

3.3.2 Vector Subtraction

Vector subtraction is just like vector addition. The difference is that the sign of the entity to be subtracted will be reversed. We can say it as a negative vector. For example, consider the figure below.



$$\overrightarrow{\mathbf{AB}} - \overrightarrow{\mathbf{CB}} = \overrightarrow{\mathbf{AC}}$$

Figure 3.5: Example of Vector Subtraction

The figure shows the subtraction of vector $\overrightarrow{\mathbf{CB}}$ from vector $\overrightarrow{\mathbf{AB}}$ to produce a vector $\overrightarrow{\mathbf{AC}}$. It can be rearranged as $\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CB}} = \overrightarrow{\mathbf{AB}}$. So $\overrightarrow{\mathbf{AB}}$ must run from the tail of $\overrightarrow{\mathbf{AC}}$ to the head of $\overrightarrow{\mathbf{CB}}$.

3.3.3 Vector Multiplication

The scalar product of two vectors results in scalar quantity. It is also called the 'dot product'. Figure below shows the example of scalar product multiplication. The '.' between the vectors is the scalar product operator.

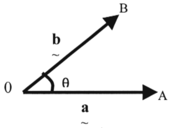


Figure 3.6: Example of Scalar Product Multiplication

The figure above shows that, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is denoted by θ. The scalar product of two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ can be expressed as **a.b** and **a.b** = |**a**| |**b**| *cos* θ.

3.4 Difficulties in Learning Vector

In studying mathematics, the problem which is usually encountered by the student, is that there is no guidance of how to solve particular problems. The traditional CAI system only provide a 'correct', 'incorrect', 'yes' or 'no' answer which didn't give any help to students. Student basically may feel lost or frustrated to solve the problem if they don't have any idea. May be some of them may give up if they have tried to answer it more than once and still get a wrong answer. So that, in order to solve this problem, VECITS is needed. VECITS guides student to solve the problem by giving guidelines and hints which is believed to help student understand and master the particular topic.

Another problem, which is usually encountered by student, is to remember the formula. In vector they have to remember and understand the rules and the concept of direction and magnitude of vector.

3.4.1 Misconception of Direction

Students are confused when the direction of vector is encountered. Even if the magnitude is correct initially, the wrong direction will result in the wrong calculation and hence the final resultant vector is wrong in both magnitude and direction besides the errors in other entities.

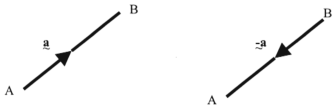


Figure 3.7 : Two different directions of vectors

For example Figure 3.7 above shows two different directions of vectors with the same length. These two opposite vectors have the same length but a different value as the arrow or direction will affect the value of a vector. Students have this misconception. This will lead them to wrong answers in solving the problem. This lack of visualization is one of the main sources of errors in the analysis of vector problems.

3.4.2 Difficulties in Solving Problem

Wrong calculation occurs when the students are trying to apply the vector algebra such as addition, subtraction and multiplication. The confusion in basic arithmetic operation i.e '+', '-' and '.' leads them to the wrong answer. They usually lack technique of the multiplying two vectors such as

$$(\mathbf{a} + \mathbf{b}).(\mathbf{a} - \mathbf{b})$$

The arithmetic operations (+, -, .) for scalar quantities and vector quantities are different. Students usually have the misconceptions by treating vector quantities as scalar quantities. They may fail to differentiate the difference between a vector and the negative vector in the opposite direction, and rules of vector addition, subtraction or scalar product are wrongly applied.

3.5 Problem Solving of the study

3.5.1 VECITS

What is VECITS

VECITS represents Vector Intelligent Tutoring System. It is a prototype system, which provide a tutorial for students in learning mathematics vector. The aim of the system is to provide a tutorial for student in studying mathematics vector and give a guideline of how to answer the question. The system is useful for student who would like to do a revision and acquire the skill of how to get correct steps in answering the question.

Characteristics of VECITS

VECITS has several characteristics, which may help student to learn and understand the topic better than traditional system, which are shown as follows.

1) Notes as Tutorial.

VECITS provides notes for student to help them make a revision or reference and understand more on the topic. The notes contain a definition of vector, rule, arithmetic and examples related to the topics. It starts by providing a basic concept of vector such as the definition of vector, and followed by the arithmetic and rule of vector. One of the note interfaces is shown below.

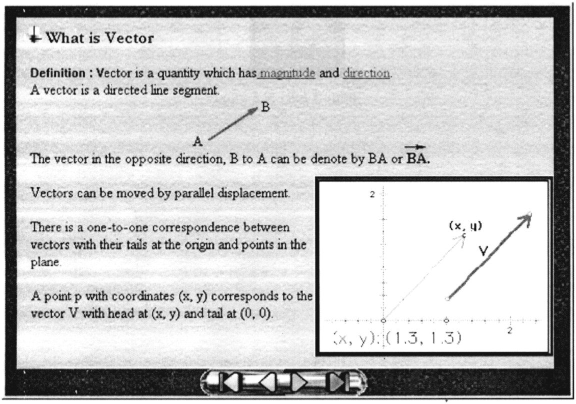


Figure 3.8 : Example of Note Interface

2) Color

Color is a wonderful thing and an important feature in multimedia development. In VECITS, color is used to differentiate between the topics and contents, hypertext, lines of arrow and the graphics itself. A different color is used to differentiate a different direction of vector. For example, a red line is used to represent a direction for **a**, green line for **b** and black for **c**.

3) Hints as a guideline

Hints is one of the important thing in ITS. It will help student to get an overview of how they can solve the problem. May be student does not have any idea of how to start answering the question. VECITS provide a guideline or hints for student in the tutorial session in order to help them understand and get an overview of how they have to start. Figure below show the interface of hints provided in VECITS.

Figure 3.9: Hint Button

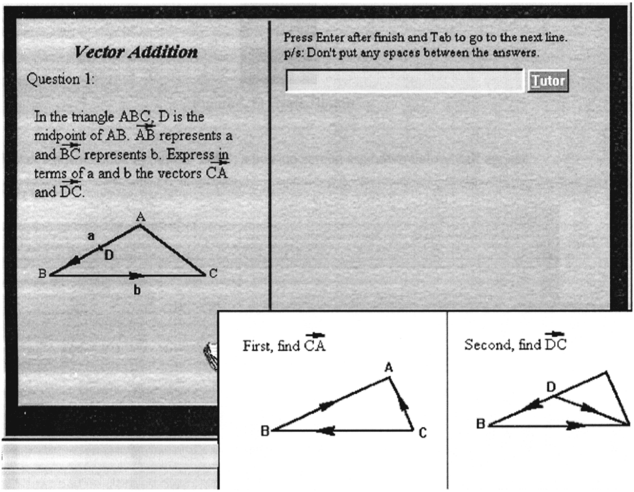


Figure 3.10: Pop-up Screen for Hint

Hint is provided with the button as shown in Figure 3.9. When students click the button, a pop up box will appear to give a guideline of what they have to do in the first step and in the second step. Second click will hide the pop up box. Figure 3.10 shows the pop-up screen for hint.

4) Note as a reference

Note button is provided for student as a reference if they forget certain notes or formulas while they are trying to solve a problem. Button note is shown as in Figure 3.11 below.

Figure 3.11: Note Button

When students click at button 'note', a pop-up screen as shown below will appear.

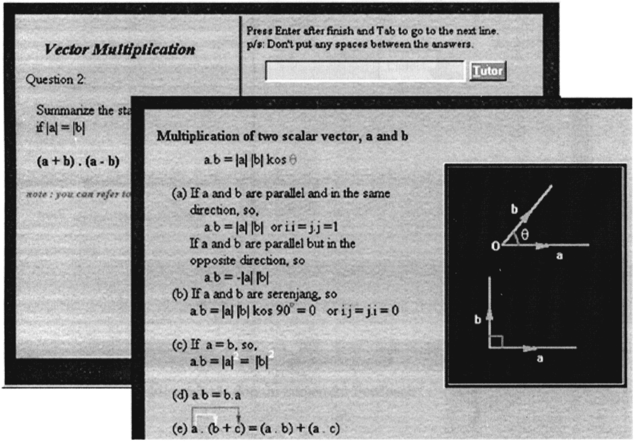


Figure 3.12: Pop-up Screen for Note Button

Pop-up screen that appears is the reference related to the question asked. Student will refer to the pop up screen when they need various aspects to refer such as formulas or rules. Second click will hide this pop-up screen.

5) Feedback

VECITS provides feedback in response to the student activity. For example if the answer is correct, a pop up message will appear to tell student that their answer is correct. Figure below shows the pop up feedback message in VECITS.

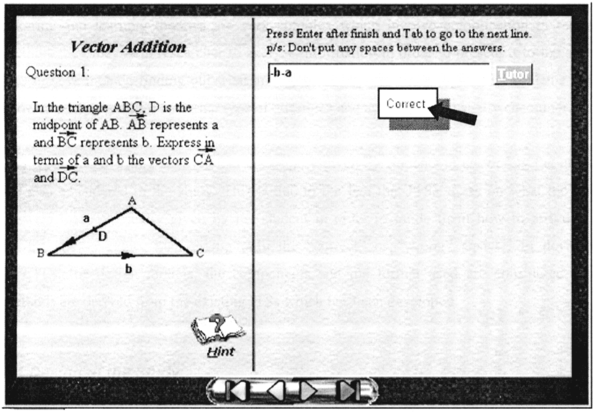


Figure 3.13: Pop-up Screen for Feedback

Arrow in figure above shows the pop-up message when the student put the correct answer. It is the same as if the student put incorrect answer. A pop-up message box will appear to tell student that their answer is incorrect together with explanation of their mistakes. It may help student to be aware of what mistakes they have done and do the necessary correction.