CHAPTER I

THEORY OF PROBABILITY

Set Theory

The fundamental work done on the measure theoretical approach to probability theory is by A.N. Kolgomorov, a famous Russian mathematicia whose work has been translated into English. It is not an overstatement to say that for the past thirty-three years most of the research work in probability has been influenced by this approach and that the Axiomatic theory advanced by Kolmogorov is considered by workers in Probability and Statistics to be the correct one.

The model developed by Kolmogorov can be briefly described as follows:

In every situation, (i.e. an experiment, observation, etc.) in which random factors enter, there is an associated probability space or trible (Ω , β , P) where Ω is an abstract space (the space of elementary events) β is a set of subsets of Ω (sets of events) and P(E) is a measure (the probability of event (E) defined) for $E \in \beta$ and satisfying the condition $P(\Omega) = 1$.

The mathematical model is best described in terms of set theory and to get a fore-knowledge of probability we can start with a non-axiomatic treatment of set theory.

A set is a collection of objects[1]. For example, the set of the former presidents of the United States or the set of integers from 1-10 or the set of solutions of the equation $X^2+3X-2 = 0$. Each of these is a finite set. The set of all real numbers, set of all points on a given straight line, set of even integers are examples of infinite sets. If A is a set and b an object in the collection of objects which is A we say b is a member of A or an element of A[2], e.g. 4 is a member (or element) of the set of even integers. This fundamental relationship of membership or belonging between objects and sets can be denoted by symbols.

e.g. $b \in A$ - b is a member of set A.

---

[1] Op. Cit., J.R. Mocord & R.M. Moroney: Introduction to probability theory, page 15.

[2] Abid., Page 15.

If b is not a member of A we write $b \notin A$.
e.g. If N is a set of positive integers
then $15 \in N$ and $\pi \notin N$.

If A and B are sets and every member of B is a member of A, we say that B is a subset of A and write $B \subset A$[1] which can read B is included in A

e.g. if $S = \{a, b, c\}$ and

$T = \{a, b, c, d, e\}$

then $S \subset T$ since every member of S is a member of T.

The contrary statement that there are members of B which are not members of A is abbreviated $B \not\subset A$.

2 sets are equal if and only if they have the same members i.e. if every member of set A is a member of set B and if every member set B is also a member of setA,

Then $A = B$

To prove $A = B$ we have to show that

$x \in A$ then $x \in B$

and if $x \in B$ then $x \in A$

To prove $A \not\subset B$ we must show that x which is a member of one of the sets but is not a member of the other.

---

[1]Op. Cit., J.R. Mccord & R.M. Moroney: Introduction to probability theory, page 15.