CHAPTER IV

RANDOM VARIABLES

We can assign a number to every elementary event from a sample space of elementary events. In the coin tossing example, we can assign the number 1 to the appearance of heads and 0 to the appearance of tails. Then the probability of obtaining one as a result of an experiment will be the same as the probability of obtaining a head and the probability of obtaining 0 will be the same as the probability of obtaining a tail.

$$P(X) = P(\text{Tails}) = 1/2$$

In the throwing of a die if we assign the number 1 to the odd numbers of die and the number 0 to the even numbers of the die. We have again the probability of obtaining an even face of the die.

$$P(X = 0) = P(\text{an even face}) = \tfrac{1}{2}.$$

Here we see that the outcomes of the underlying experiment are not considered. The attention is focused on the values 0 and 1, as defined in each experiment, assumes

In general, let ei denote an elementary event of the sample space of elementary events. On the , we define a single value function X(e) such that roughly speaking, the probability that this function will assume certain values is defined.

Let us illustrate this with an example. If 3 coins are tossed consider the number of heads that appear. This can be found by looking at the elementary events, and their associated probabilities.

Elementary Event	No. of Heads	Probability
HHH	3	1/8
HHT	2	1/8
		1/8
HTH	2	1/8
THH	2	1/8
HTT	1	1/8
THT	1	1/8
TTH	1	1/8
TTT	0	

If we define X as the number of heads then we have a function of X for which the following probabilities are true.

No. of heads:	0	1	2	3
Probability :	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

This set of ordered pairs each of the form, (number of heads, probability of that number); in the example above $(0; \frac{1}{8})$ $(1; \frac{3}{8})$ $(2; \frac{3}{8})$ $(3; \frac{1}{8})$ is the probability function of X.

If we used X to denote a random variable and a small x for one of its values and f(x) for the probability that the random variable X takes on value x then

$$f(x) = P(X = x)$$

This is the example we have

$$f(0) = P(X = 0) = \tfrac{1}{8}$$
$$f(1) = P(X = 1) = \tfrac{3}{8}$$
$$f(2) = P(X = 2) = \tfrac{3}{8}$$
$$f(3) = P(X = 3) = \tfrac{1}{8}$$

Here we see that the value of a random variable X is determined by the outcomes of an experiment and we define a random variable as a numerical-valued function on the outcomes of chance experiment.

Distribution Function

In the example given above, a probability model was given by a priori assignment of probabilities. These probabilities are automatically transferred to the numbers or sets of numbers which are the values of a given random variable defined on the experiment. However in constructing a probability model without a priori assistance it is convenient to assign probabilities directly to the values or sets of values of the random quantity of interest. This is done by specifying the distribution function of the random variable.

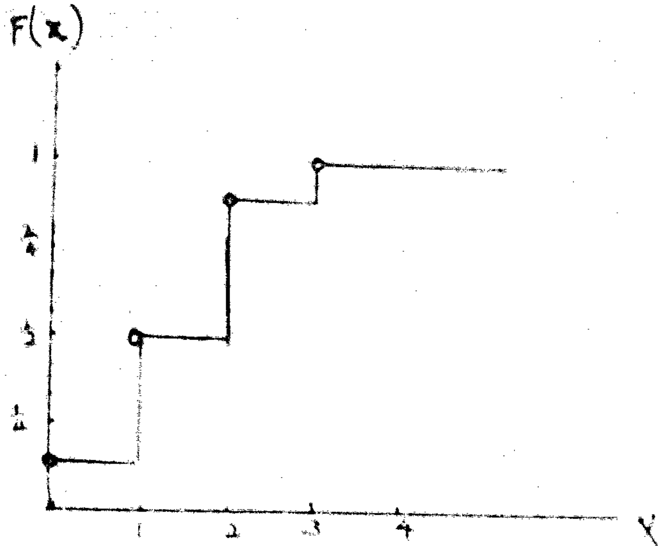
The distribution function is given by $F(x) = P(X \le x)$ where x is an arbitrary interval boundary. In the example of a tossing of 3 coins the distribution function is given as

$$F(x) = P(X \le x) = \begin{cases} 0 \text{ if } x < 0 \\ \frac{1}{8} \text{ if } 0 \le x < 1 \\ \frac{4}{8} \text{ if } 1 \le x < 2 \\ 7/8 \text{ if } 2 \le x < 3 \\ 1 \text{ if } x > 3 \end{cases}$$

The graph of the distribution function is thus, and it gives a step distribution.

DIAGRAM 9

PROBABILITY DISTRIBUTION



The probability function of a random variable X is determined by its distribution function. When setting up a probability model by specifying a probability distribution function it is not always clear from the text what to use but often one must make an educated guess.

In any event the function used as $F(x)$ cannot be completely arbitrary; it must satisfy certain conditions as a consequence of having a probability.

Since probability has been defined as a number between 0 and 1 we must require that

(a) $\quad 0 \leq F(x) \leq 1$

and since we should not want the probability of an interval to be negative we must have

(b) $\quad F(b) \geq F(a)$ whenever $a \leq b$.

This means that $F(x)$ must be a non-decreasing function, i.e. non-decreasing as we move from left to right.

Further we should have

(c) $\quad F(+\infty) = P(X \leq +\infty) = 1$

$$F(-\infty) = P(X \leq -\infty) = 0$$

One would also want for positive increments h

(d) $$\lim_{h \to 0} P(x < X \leq x + h) = \lim_{h \to 0} [F(x + h) - F(x)] = 0$$

This condition is described by saying F(x) is continuous from the right.