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CUBIC HAMILTONIAN GRAPHS
AND
GENERALIZED KNIGHT'S TOURS

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ABSTRAK

Tesis ini adalah berkaitan dengan graf hamiltonian dan ia dibahagi kepada dua bahagian.

Bahagian pertama adalah mengenai graf kubik, bipartite dan satahan dengan keterkaitan 3. Biar G mewakili graf kubik, bipartite dan satahan dengan keterkaitan 3. Dengan mengandaikan setiap G adalah graf hamiltonian, batasan bawah bagi bilangan kitar Hamilton dalam G dengan keterkaitan berkisar dua, tiga dan empat telah ditentukan dalam Bab 2. Dalam Bab 3, kami membuktikan bahawa Konjektur Barnette adalah setara dengan konjektur yang menyatakan bahawa bagi sebarang dua tepi x dan y pada muka yang sama dalam G , terdapat satu kitar Hamilton yang mengandungi x dan y , dan satu kitar Hamilton yang mengandungi x tetapi tidak mengandungi y . Seterusnya, dengan mengandaikan setiap G adalah graf Hamilton, kita menentukan graf G dengan keterkaitan berkisar tertentu yang mempunyai n kitar Hamilton bagi $n = 6$ dan $n = 12$.

Bahagian dua adalah mengenai masalah "Generalized Knight's Tour". Kita menentukan saiz papan-papan catur yang tidak membenarkan " (a, b) -knight's tour" tertutup dalam Bab 4. Dalam Bab 5, bagi setiap integer positif k , kita memperolehi nilai n yang mana papan catur dengan saiz $5k \times n$, kecuali saiz 5×18 , membenarkan " $(2, 3)$ -knight's tour" tertutup.

ABSTRACT

This thesis is divided into two parts, both related to hamiltonian graphs.

The first part deals with 3-connected cubic bipartite planar graphs. By assuming that all 3-connected cubic bipartite planar graphs are hamiltonian, lower bounds for the number of Hamilton cycles in cubic bipartite planar graphs with given cyclic connectivity are obtained in Chapter 2. In Chapter 3, we show that Barnette's Conjecture is equivalent to the conjecture which states that for any two edges x and y on the same face of a 3-connected cubic bipartite planar hamiltonian graph, there is a Hamilton cycle passing through x and y , and another one passing through x but avoiding y . As a byproduct, by assuming that every 3-connected cubic bipartite planar graph is hamiltonian, we characterize all those cubic bipartite planar graphs with given cyclic connectivity and whose number of Hamilton cycles is n for $n = 6$ and $n = 12$.

The second part deals with the Generalized Knight's Tour Problem. We show that certain rectangular chessboards do not admit a closed (a, b) -knight's tour in Chapter 4. In Chapter 5, for all positive integers k , we obtain the values of n for which the $5k \times n$ chessboard, except for the 5×18 chessboard, admits a closed $(2, 3)$ -knight's tour.

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Part I Cubic bipartite planar (CBP) graphs

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