Chapter 7

CONCLUSION

In this work we have studied the behaviour of the flavour-changing quark-Z vertex arising from the penguin diagram, which involves the change of the external quark flavour, say from $s$ to $d$, through the internal conversion into a quark of flavour $j$ ($j$ can be of flavour $u$, $c$ or $t$ in this case) with which a $W$ boson is exchanged and accompanied by the emission of a virtual $Z$ boson. The calculation of the flavour-changing quark-Z vertex function is performed in the 't Hooft-Feynman gauge.

The divergent term in the vertex function is isolated by using dimensional regularization. Renormalization is effected by a simple renormalization prescription, in which the counterterm is obtained from the improper on-shell vertex function (one-particle reducible diagram).

The $\bar{d}sZ$ vertex function is obtained without making any approximations and is expressed in terms of double integrals. The vertex function is found to contain 6 form factors $A_j$, $B_j$ and $C_j$ ($j = u, c, t$). Our expression is thus more general than earlier work which involves only the $C_j$ form factor. The double integrals have to be evaluated numerically in general. We have made the simplifying assumption that the masses of the external quarks and the momentum of the emitted $Z$ boson are small compared to the mass of the $W$ boson, so that the double integrals may be evaluated analytically.

We find that $C_j$ grows quadratically with $m_j$, the mass of the internal quark $j$, and therefore is the dominant form factor for large $m_j$. The other form factors are much smaller; $A_j$ grows only logarithmically with $m_j$, while $B_j$ approaches a constant value as
$m_j$ increases. For small value of $m_j$, however, we find that $A_j$ is more dominant than the other two.

We have applied our result for the vertex in three different processes; $s \to dd\bar{d}$, $b \to ss\bar{s}$ and $b \to ds\bar{s}$. In the applications, we have dropped the term containing $B_j$ because it is very small compared to the terms containing $A_j$ or $C_j$. After analysis, the contribution of $A_j$ term in $s \to dd\bar{d}$ is extremely small whereas its contribution in $b \to ss\bar{s}$ and $b \to ds\bar{s}$ are up to 0.31% and 0.29% respectively. The branching ratio for $s \to dd\bar{d}$ is obtained, i.e. $2.105 \times 10^{-10}$. The branching ratios for $b \to ss\bar{s}$ and $b \to ds\bar{s}$ are found to depend linearly on $\text{Re}(\lambda_s \lambda^*_c)$ as shown in Figs. 6.3 and 6.4 respectively. The results show that the branching ratios of these 3 processes are very small, which are expected for processes in one-loop level.

Our work may be extended by applying the vertex function without making any approximation. Another extension is to study the contribution of the Z-penguin vertex function to CP violation. When $k^2$ is larger than a certain threshold value, absorptive part may appear in the vertex function. This absorptive part gives rise to nonzero decay rate asymmetry parameter, which may then serve as a measurement of its contribution to direct CP violation. Furthermore, the Z-penguin vertex may be applied to study the flavour-changing decays of the Z-boson. In such a case, $k^2$ shall not be omitted because $k^2 = M_f^2$, which is larger than $M_w^2$. 