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EXACT CALCULATION  
OF THE  
FLAVOUR-CHANGING QUARK-Z VERTEX

BY  
CHONG NYOK KIAN  
DEPARTMENT OF PHYSICS  
FACULTY OF SCIENCE  
UNIVERSITY MALAYA.

Perpustakaan Universiti Malaya



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HAMSI AH BT. MOHAMMAD ZAHARI

UPR UNIT FOTODUPLIKASI  
PERPUSTAKAAN UTAMA  
UNIVERSITI MALAYA

## ABSTRACT

In the standard model of particle theory, flavour mixing in the quark sector has given rise to induced neutral flavour-changing processes. The flavour-changing  $Zq_i\bar{q}_j$  vertex has attracted considerable attention, first in connection with the rare decay of  $K$  meson, and later in relation to the flavour-changing decays of  $Z$  boson. The earlier calculation of the  $Zq_i\bar{q}_j$  vertex was performed with the assumption that the  $W$  mass is much larger than the quark masses. Later work relaxed this assumption. But the structure of the effective vertex obtained was not general enough. Only one dominant form factor was computed while the others were neglected.

In this thesis we present an investigation of the behaviour of the flavour-changing quark- $Z$  vertex and the contribution of its form factors. An exact expression for the flavour-changing quark- $Z$  vertex function in the 't Hooft-Feynman gauge is obtained. A single renormalization prescription is proposed, and the counterterm for renormalization is obtained in a straightforward way. The general expression for the on-shell vertex function is obtained, which is expressed in terms of double integrals. For practical application, we have obtained a simplified expression for the vertex function by neglecting the external quark masses and the four-momentum of virtual  $Z$  boson compared to the mass of  $W^\pm$  boson. The vertex function is found to depend on six form factors,  $A_j$ ,  $B_j$  and  $C_j$  ( $j = u, c, t$ ). Our result is more general than earlier calculation which gives only the  $C_j$  form factors. The behaviour of the form factors in the vertex function with respect to internal quark masses were investigated. Our result is then applied to calculate the branching ratio of three decay processes;  $s \rightarrow d\bar{d}\bar{d}$ ,  $b \rightarrow s\bar{s}\bar{s}$  and  $b \rightarrow d\bar{s}\bar{s}$ .

## ABSTRAK

Di dalam model piawai untuk teori zarah keunsuran, percampuran citarasa dalam sektor kuark telah menimbulkan proses-proses citarasa-tertukar neutral yang teraruh. 'Vertex'  $Zq, \bar{q}_j$  citarasa-tertukar telah menarik perhatian para penyelidik, pada awalnya kepada reputan nadir meson  $K$ , kemudian kepada reputan boson  $Z$  citarasa-tertukar. Penghitungan 'vertex'  $Zq, \bar{q}_j$  yang lepas dilakukan dengan anggapan bahawa jisim  $W$  adalah jauh lebih besar daripada jisim kuark-kuark. Anggapan ini tidak dipakai dalam kerja yang kemudiannya tetapi 'vertex' yang diperolehi masih tidak cukup umum. Hanya satu faktor-bentuk yang dominan dihitung manakala yang lain diabaikan.

Di dalam tesis ini kami membentangkan satu pengkajian terhadap kelakuan 'vertex' kuark- $Z$  citarasa-tertukar dan sumbangan-sumbangan faktor-bentuknya. Satu pernyataan tepat bagi fungsi 'vertex' kuark- $Z$  citarasa-tertukar dalam ge<sup>t</sup> Hooft-Feynman diperolehi. Satu preskripsi renormalisasi dicadangkan, dan 'counterterm' bagi renormalisasi diperolehi secara terus. Pernyataan umum bagi fungsi 'vertex' jisim-pada-petala diperolehi dan dinyatakan dalam sebutan-sebutan kamilan dubel. Untuk penggunaan praktikal, kami memperoleh satu pernyataan teringkas bagi fungsi 'vertex' dengan mengabaikan jisim kuark-kuark luaran dan empat-momentum boson  $Z$  khayalan berbanding dengan jisim boson  $W^\pm$ . Fungsi 'vertex' itu didapati bergantung kepada enam faktor-bentuk,  $A_j, B_j$  dan  $C_j$  ( $j = u, c, t$ ). Keputusan kami adalah lebih umum berbanding dengan penghitungan yang lepas dimana hanya faktor-bentuk  $C_j$  dikekalkan. Kelakuan faktor-bentuk dalam fungsi 'vertex' terhadap jisim kuark dalaman dikaji. Keputusan kami kemudian dipergunakan untuk menghitung nisbah cabang bagi tiga proses reputan,  $s \rightarrow du\bar{u}$ ,  $b \rightarrow ss\bar{s}$  dan  $b \rightarrow ds\bar{s}$ .

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*TO*  
*MY PARENTS, BROTHERS,*  
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