Chapter 3

THE UNRENORMALIZED Z-PENGUIN VERTEX

3.1 INTRODUCTION

In the Standard Glashow-Weinberg-Salam Theory of the electroweak interactions, the coupling of the Z boson to quarks does not change their flavours at the tree-level. However, flavour-changing quark-Z coupling arises at the one-loop or higher level as a result of quark flavour-mixing in the charged weak current. In this chapter, we present the computation of the explicit expression for the unrenormalized Z-penguin vertex function in the 't Hooft-Feynman gauge. In this gauge, the correspondence Feynman diagrams are as shown in Fig. 3.1. The renormalization of the vertex function will be discussed in the next chapter.

3.2 UNRENORMALIZED FLAVOUR-CHANGING Z-PENGUIN VERTEX FUNCTION

In the 't Hooft-Feynman gauge, besides the contribution from Fig. 3.1a and 3.1c where a W-boson is exchanged, there exist another four diagrams, Fig. 3.1b, 3.1d, 3.1e and 3.1f, which are due to the coupling of an unphysical scalar ϕ. The unrenormalized flavour-changing Z-penguin vertex function, $\Gamma_\mu(p,k)$, may then be written as

$$\Gamma_\mu(p,k) = \sum_i \Gamma_\mu^{(i)}(p,k) \qquad i = a,b,\ldots,f \tag{3.1}$$

where $\Gamma_\mu^{(i)}(p,k)$ denote the contribution from diagram i.

In the 't Hooft-Feynman gauge, the contribution to $\Gamma_\mu(p,k)$ from Fig. 3.1a through 3.1f respectively are

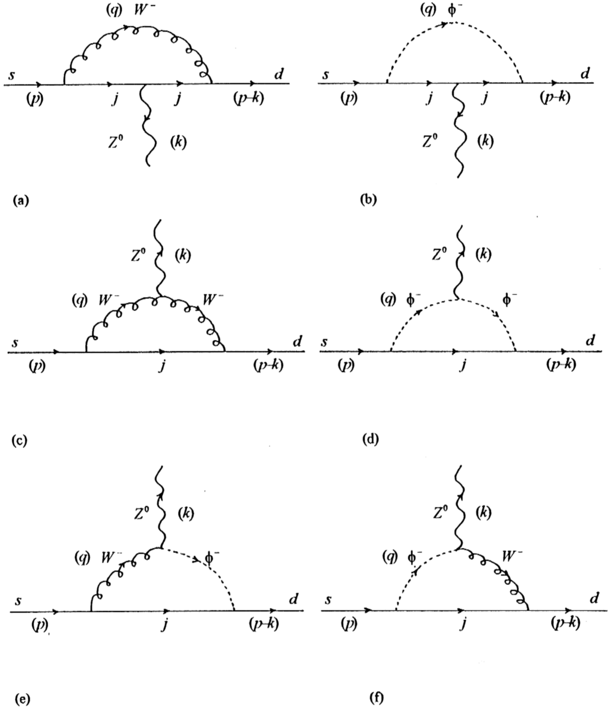

Fig. 3.1: The Feynman diagrams for the flavour-changing quark-Z vertex.

$$\Gamma_\mu^{(a)}(p,k) = -\frac{ig^3}{4\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L \frac{1}{\not p - \not q - \not k - m_j}\gamma_\mu (L - \frac{4}{3}s^2)\times \right.$$
$$\left. \times \frac{1}{\not p - \not q - m_j}\gamma_\beta L \frac{g^{\alpha\beta}}{q^2 - M_w^2}\right\} \tag{3.2}$$

$$\Gamma_\mu^{(b)}(p,k) = -\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{(m_d L - m_j R)\frac{1}{\not p - \not q - \not k - m_j}\times \right.$$
$$\left. \times \gamma_\mu (L - \frac{4}{3}s^2)\frac{1}{\not p - \not q - m_j}(m_j L - m_s R)\frac{1}{q^2 - M_w^2}\right\} \tag{3.3}$$

$$\Gamma_\mu^{(c)}(p,k) = -\frac{ig^3(1-s^2)}{2\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L \frac{1}{\not p - \not q - m_j}\gamma_\beta L \frac{g^{\alpha\nu}}{(q-k)^2 - M_w^2}\times \right.$$
$$\left. \times [-g_{\nu\lambda}(2q-k)_\mu + g_{\lambda\mu}(q+k)_\nu + g_{\mu\nu}(q-2k)_\lambda]\frac{g^{\beta\lambda}}{q^2 - M_w^2}\right\} \tag{3.4}$$

$$\Gamma_\mu^{(d)}(p,k) = \frac{ig^3(1-2s^2)}{4M_w^2\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{(m_d L - m_j R)\frac{1}{\not p - \not q - m_j}(m_j L - m_s R)\times \right.$$
$$\left. \times \frac{1}{(q-k)^2 - M_w^2}(2q-k)_\mu \frac{1}{q^2 - M_w^2}\right\} \tag{3.5}$$

$$\Gamma_\mu^{(e)}(p,k) = -\frac{ig^3 s^2}{2\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{(m_d L - m_j R)\frac{1}{\not p - \not q - m_j}\gamma_\alpha L\times \right.$$
$$\left. \times \frac{g^{\mu\beta}}{(q-k)^2 - M_w^2}\cdot\frac{g^{\alpha\beta}}{q^2 - M_w^2}\right\} \tag{3.6}$$

$$\Gamma_\mu^{(f)}(p,k) = \frac{ig^3 s^2}{2\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L \frac{1}{\not p - \not q - m_j}(m_j L - m_s R)\times \right.$$
$$\left. \times \frac{g_{\alpha\beta}}{(q-k)^2 - M_w^2}\cdot\frac{g^{\mu\beta}}{q^2 - M_w^2}\right\} \tag{3.7}$$

where m_j is the mass of the internal quark ($j = u,c,t$), $L = \frac{1}{2}(1-\gamma_5)$, $R = \frac{1}{2}(1+\gamma_5)$, $s = \sin^2\theta_w$ and $\lambda_j = U^*_{jd}U_{js}$.

Let us consider first the two diagrams in which the Z-boson is emitted from the internal quark lines. Generalizing to $n = 4-\varepsilon$ spacetime dimensions, Eqs. 3.2 and 3.3 becomes

$$\Gamma^{(a)}_\mu(p,k) = -\frac{ig^3}{4\cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}}\sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n}\eta^{(a)}_\mu F^{(Q)} \tag{3.8}$$

$$\Gamma^{(b)}_\mu(p,k) = -\frac{ig^3}{4M_w^2\cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}}\sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n}\eta^{(b)}_\mu F^{(Q)} \tag{3.9}$$

where

$$\left.\begin{aligned} \eta^{(a)}_\mu &= \gamma_\alpha L(\not{p}-\not{q}-\not{k}+m_j)\gamma_\mu(L-\tfrac{4}{3}s^2)(\not{p}-\not{q}+m_j)\gamma_\beta L g^{\alpha\beta} \\ \eta^{(b)}_\mu &= (m_d L - m_j R)(\not{p}-\not{q}-\not{k}+m_j)\gamma_\mu(L-\tfrac{4}{3}s^2)(\not{p}-\not{q}+m_j)(m_j L - m_s R) \end{aligned}\right\} \tag{3.10}$$

$$F^{(Q)} = [(p-q-k)^2 - m_j^2]^{-1}[(p-q)^2 - m_j^2]^{-1}[q^2 - M_w^2]^{-1} \tag{3.11}$$

By introducing Feynman parametrization, we get

$$F^{(Q)} = 2\int_0^1 dx \int_0^{1-x} dy[\tilde{q}^2 - D_Q]^{-3} \tag{3.12}$$

where $\tilde{q} = q-(1-x)p+yk$ $\quad$ (3.13) and $D_Q = xM_w^2 + (1-x)m_j^2 - x(1-x)p^2 - y(1-y)k^2 + 2xy(p\cdot k)$ (3.14)

Substitute Eq. (3.13) into Eq. (3.10), we obtain

$$\left.\begin{aligned}
\eta_\mu^{(a)} &= 2(\frac{4}{3}s^2 - 1)(\not{G}_1 - \not{\tilde{q}})\gamma_\mu(\not{G}_2 - \not{\tilde{q}})L + \frac{8}{3}m_j^2 s^2 \gamma_\mu L \\
\eta_\mu^{(b)} &= (m_d L - m_j R)(\not{G}_2 - \not{\tilde{q}} + m_j)\gamma_\mu(L - \frac{4}{3}s^2)(\not{G}_1 - \not{\tilde{q}} + m_j)(m_j L - m_s R)
\end{aligned}\right\} \qquad (3.15)$$

where $G_1 = xp + yk$, and $G_2 = xp + (y-1)k$. (3.16)

Odd powers of $\tilde{q}$ give zero contribution after momentum loop integration. Eq. (3.15) may then be cast into the following form:

$$\left.\begin{aligned}
\eta_\mu^{(a)} &= B_\mu^{(a)} + \frac{2-n}{n}C_\mu^{(a)}\tilde{q}^2 \\
\eta_\mu^{(b)} &= B_\mu^{(b)} + \frac{2-n}{n}C_\mu^{(b)}\tilde{q}^2
\end{aligned}\right\} \qquad (3.17)$$

where

$$\left.\begin{aligned}
B_\mu^{(a)} &= 2(\frac{4}{3}s^2 - 1)\not{G}_1\gamma_\mu\not{G}_2 + \frac{8}{3}m_j^2 s^2\gamma_\mu \\
C_\mu^{(a)} &= 2(\frac{4}{3}s^2 - 1)\gamma_\mu \\
B_\mu^{(b)} &= \left[m_s m_d(\frac{4}{3}s^2 - 1)\not{G}_2\gamma_\mu\not{G}_1 + m_s m_j^2(1 - \frac{4}{3}s^2)\gamma_\mu\not{G}_1 - \frac{4}{3}s^2 m_s m_j^2\not{G}_2\gamma_\mu\right. \\
&\quad \left. + \frac{4}{3}s^2 m_s m_d m_j^2\gamma_\mu\right] + \left[m_d m_j^2(1 - \frac{4}{3}s^2)\not{G}_2\gamma_\mu + m_j^4(\frac{4}{3}s^2 - 1)\gamma_\mu\right. \\
&\quad \left. + \frac{4}{3}s^2 m_j^2\not{G}_2\gamma_\mu\not{G}_1 - \frac{4}{3}s^2 m_d m_j^2\gamma_\mu\not{G}_2\right]L \\
C_\mu^{(b)} &= m_s m_d(\frac{4}{3}s^2 - 1)\gamma_\mu R + \frac{4}{3}s^2 m_j^2\gamma_\mu L
\end{aligned}\right\} \qquad (3.18)$$

Substituting Eqs. (3.12), (3.17) and (3.18) into Eqs. (3.8) and (3.9) and carrying out loop integration over the internal momentum with the integral formula given in Appendix, we obtain

$$\Gamma_\mu^{(a)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy \left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} \Gamma(\frac{\varepsilon}{2})\left[-\frac{\varepsilon}{2}B_\mu^{(a)}D_Q^{-1} + \frac{1}{4}C_\mu^{(a)}(\varepsilon-2)\right]L \tag{3.19}$$

$$\Gamma_\mu^{(b)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy \left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} \Gamma(\frac{\varepsilon}{2})\left[-\frac{\varepsilon}{2}B_\mu^{(b)}D_Q^{-1} + \frac{1}{4}C_\mu^{(b)}(\varepsilon-2)\right]L \tag{3.20}$$

where $G = \dfrac{g^3}{32\pi^2 \cos\theta_w}$. (3.21)

The limit $n \to 4$ is then taken to separate the divergent term :

$$\Gamma_\mu^{(a)}(p,k) = G\sum_j \lambda_j \left[\frac{1}{2}C_\mu^{(a)}\xi + \frac{1}{2}C_\mu^{(a)} + \int_0^1 dx \int_0^{1-x} dy (C_\mu^{(a)} \ln \hat{D}_Q - B_\mu^{(a)} D_Q^{-1})\right]L \tag{3.22}$$

$$\Gamma_\mu^{(b)}(p,k) = G\sum_j \lambda_j \left[\frac{1}{2}C_\mu^{(b)}\xi + \frac{1}{2}C_\mu^{(b)} + \int_0^1 dx \int_0^{1-x} dy (C_\mu^{(b)} \ln \hat{D}_Q - B_\mu^{(b)} D_Q^{-1})\right]L \tag{3.23}$$

$$\xi = \ln\left(\frac{M_w^2 e^\gamma}{4\pi\mu^2}\right) - \frac{2}{\varepsilon} \tag{3.24}$$

and $\hat{D}_Q = \dfrac{D_Q}{M_w^2}$. (3.25)

In Eq.(3.24), $\gamma = 0.5772$ is the Euler constant.

By making use of the unitarity property of the Kobayashi-Maskawa matrix, we can drop terms that are independent of m_j and obtain

$$\Gamma_\mu^{(a)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy \left\{ (\frac{4}{3}s^2 - 1)\ln \hat{D}_Q \gamma_\mu + \left[(1-\frac{4}{3}s^2)[x^2 \not{p}\gamma_\mu \not{p} + x(y-1)\not{p}\gamma_\mu \not{k} + xy\not{k}\gamma_\mu \not{p} + y(y-1)\not{k}\gamma_\mu \not{k}] + \frac{4}{3}m_j^2 s^2 \gamma_\mu \right] D_Q^{-1} \right\} L \tag{3.26}$$

$$\begin{aligned}\Gamma_\mu^{(b)}(p,k) = G\sum_j \lambda_j \Bigg\{ & \frac{1}{3}s^2 \hat{m}_j^2 \xi \gamma_\mu L + \frac{1}{3}s^2 \hat{m}_j^2 \gamma_\mu L + \int_0^1 dx \int_0^{1-x} dy \Bigg[[\frac{1}{2}\hat{m}_s \hat{m}_d (\frac{4}{3}s^2 - 1)\gamma_\mu R \\ & + \frac{2}{3}\hat{m}_j^2 \gamma_\mu L]\ln \hat{D}_Q - D_Q^{-1}\Bigg(\frac{1}{2}[x^2 \not{p}\gamma_\mu \not{p} + xy\not{p}\gamma_\mu \not{k} + x(y-1)\not{k}\gamma_\mu \not{p} + y(y-1)\not{k}\gamma_\mu \not{k}] \times \\ & \times [\hat{m}_s \hat{m}_d (\frac{4}{3}s^2 - 1)R + \frac{4}{3}\hat{m}_j^2 s^2 L] + (x\gamma_\mu \not{p} + y\gamma_\mu \not{k})[m_s \hat{m}_j^2 (\frac{1}{2} - \frac{2}{3}s^2)R - \frac{2}{3}m_d \hat{m}_j^2 s^2 L] \\ & + [x\not{p}\gamma_\mu + (y-1)\not{k}\gamma_\mu][m_d \hat{m}_j^2 (\frac{1}{2} - \frac{2}{3}s^2)L - \frac{2}{3}s^2 m_s \hat{m}_j^2 R] + \frac{2}{3}s^2 m_s m_d \hat{m}_j^2 \gamma_\mu R \\ & - \hat{m}_j^2 m_j^2 (\frac{1}{2} - \frac{2}{3}s^2)\gamma_\mu L \Bigg) \Bigg] \Bigg\}\end{aligned} \tag{3.27}$$

where $\hat{m}_s = \dfrac{m_s}{M_w}, \hat{m}_d = \dfrac{m_d}{M_w}, \hat{m}_j = \dfrac{m_j}{M_w}$. (3.28)

Next we look at the remaining diagrams of Fig. 3.1. Generalizing to $n = 4 - \varepsilon$ spacetime dimensions, Eqs. (3.4) through (3.7) become

$$\Gamma_\mu^{(c)}(p,k) = -\frac{ig^3(1-s^2)}{2\cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}} \sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n} \eta_\mu^{(c)} F^{(w)} \tag{3.29}$$

$$\Gamma_\mu^{(d)}(p,k) = \frac{ig^3(1-2s^2)}{4M_w^2 \cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}} \sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n} \eta_\mu^{(d)} F^{(w)} \tag{3.30}$$

$$\Gamma_\mu^{(e)}(p,k) = -\frac{ig^3 s^2}{2\cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}} \sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n} \eta_\mu^{(e)} F^{(w)} \tag{3.31}$$

$$\Gamma_{\mu}^{(f)}(p,k) = \frac{ig^3 s^2}{2\cos\theta_w}(\mu^2)^{\frac{\varepsilon}{2}} \sum_j \lambda_j \int \frac{d^n q}{(2\pi)^n} \eta_{\mu}^{(f)} F^{(w)} \tag{3.32}$$

where

$$\left.\begin{aligned}
\eta_{\mu}^{(c)} &= \gamma_{\nu} L(\not{p} - \not{q} + m_j)\gamma_{\lambda}[-g_{\nu\lambda}(2q-k)_{\mu} + g_{\lambda\mu}(q+k)_{\nu} + g_{\mu\nu}(q-2k)_{\lambda}] \\
\eta_{\mu}^{(d)} &= (m_d L - m_j R)(\not{p} - \not{q} + m_j)(m_j L - m_s R)(2q-k)_{\mu} \\
\eta_{\mu}^{(e)} &= (m_d L - m_j R)(\not{p} - \not{q} + m_j)\gamma_{\mu} L \\
\eta_{\mu}^{(f)} &= \gamma_{\mu} L(\not{p} - \not{q} + m_j)(m_j L - m_s R)
\end{aligned}\right\} \tag{3.33}$$

$$\text{and} \quad F^{(w)} = [(p-q)^2 - m_j^2]^{-1}[(q-k)^2 - M_w^2]^{-1}[q^2 - M_w^2]^{-1}. \tag{3.34}$$

Introducing Feynman parametrization, Eq. (3.34) becomes

$$F^{(w)} = 2\int_0^1 dx \int_0^{1-x} dy[\tilde{q}^2 - D_w]^{-3} \tag{3.35}$$

$$\text{where} \quad \tilde{q} = q - (xp + yk) \tag{3.36}$$

$$\text{and} \quad D_w = (1-x)M_w^2 + xm_j^2 - x(1-x)p^2 - y(1-y)k^2 + 2xy(p\cdot k) . \tag{3.37}$$

Substituting Eq. (3.36) into Eq. (3.33), we obtain

$$\left.\begin{aligned}
\eta_{\mu}^{(c)} &= \Big[(\not{Q}_1 - \not{\tilde{q}})\gamma_{\mu}(2\not{\tilde{q}} + \not{Q}_2) + (\not{Q}_1 - \not{\tilde{q}})(2\not{\tilde{q}} + \not{Q}_2)\gamma_{\mu} + (\not{\tilde{q}} + \not{Q}_3)(\not{Q}_1 - \not{\tilde{q}})\gamma_{\mu} \\
&\quad + \gamma_{\mu}(\not{Q}_1 - \not{\tilde{q}})(\not{\tilde{q}} + \not{Q}_4)\Big]L \\
\eta_{\mu}^{(d)} &= \Big[(m_s m_d L + m_j^2 R)(\not{\tilde{q}} - \not{Q}_1)(2\tilde{q} + Q_2)_{\mu} + (m_d m_j^2 L + m_s m_j^2 R)\Big](2\tilde{q} + Q_2)_{\mu} \\
\eta_{\mu}^{(e)} &= m_d(\not{Q}_1 - \not{\tilde{q}})\gamma_{\mu} R - m_j^2 \gamma_{\mu} L \\
\eta_{\mu}^{(f)} &= R\gamma_{\mu}\Big[m_j^2 - m_s(\not{Q}_1 - \not{\tilde{q}})\Big]
\end{aligned}\right\} \tag{3.38}$$

where

$$\left.\begin{aligned} Q_1 &= (1-x)p - yk\,, \\ Q_2 &= 2xp + (2y-1)k\,, \\ Q_3 &= xp + (1+y)k\,, \\ Q_4 &= xp + (y-2)k\,. \end{aligned}\right\} \tag{3.39}$$

Again we omit terms that contain odd powers of $\tilde{q}$. Eq. (3.38) can be rewritten into the following form:

$$\left.\begin{aligned} \eta_\mu^{(c)} &= B_\mu^{(c)} L - \left[\frac{2}{n}(2-n) + 4\right]\gamma_\mu L \tilde{q}^2 \\ \eta_\mu^{(d)} &= B_\mu^{(d)} - \frac{2}{n} C_\mu^{(d)} \tilde{q}^2 \\ \eta_\mu^{(e)} &= m_d Q_1 \gamma_\mu R - m_j^2 \gamma_\mu L \\ \eta_\mu^{(f)} &= m_j^2 \gamma_\mu L - m_s \gamma_\mu Q_1 R \end{aligned}\right\} \tag{3.40}$$

where

$$\left.\begin{aligned} B_\mu^{(c)} &= Q_1 \gamma_\mu Q_2 + Q_1 Q_2 \gamma_\mu + Q_3 Q_1 \gamma_\mu + \gamma_\mu Q_1 Q_4 \\ B_\mu^{(d)} &= -\frac{1}{2}(m_s m_d L + m_j^2 R)(Q_1 \gamma_\mu Q_2 + Q_1 Q_2 \gamma_\mu) \\ &\quad + \frac{1}{2} m_j^2 (m_d L + m_s R)(\gamma_\mu Q_2 + Q_2 \gamma_\mu) \\ C_\mu^{(d)} &= \gamma_\mu (m_s m_d R + m_j^2 L) \end{aligned}\right\} \tag{3.41}$$

After substituting Eqs.(3.35), (3.40) and (3.41) to Eqs. (3.29) through (3.32), we perform momentum loop integration and obtain

$$\Gamma_\mu^{(c)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy (1-s^2)\left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} \Gamma(\frac{\varepsilon}{2})\left[-\frac{\varepsilon}{2} B_\mu^{(c)} D_w^{-1} + (\varepsilon - 6)\gamma_\mu\right] L \quad (3.42)$$

$$\Gamma_\mu^{(d)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy \frac{1-2s^2}{2M_w^2}\left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} \Gamma(\frac{\varepsilon}{2})\left[-\frac{\varepsilon}{2} B_\mu^{(d)} D_w^{-1} + C_\mu^{(d)}\right] L \quad (3.43)$$

$$\Gamma_\mu^{(e)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy s^2\left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} (\frac{\varepsilon}{2})\Gamma(\frac{\varepsilon}{2})\left[m_j^2 \gamma_\mu L - m_d \mathcal{Q}_1 \gamma_\mu R\right] D_w^{-1} \quad (3.44)$$

$$\Gamma_\mu^{(f)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy s^2\left(\frac{D_Q}{4\pi\mu^2}\right)^{-\frac{\varepsilon}{2}} (\frac{\varepsilon}{2})\Gamma(\frac{\varepsilon}{2})\left[m_j^2 \gamma_\mu L - m_s \gamma_\mu \mathcal{Q}_1 R\right] D_w^{-1}. \quad (3.45)$$

After taking the limit $n \to 4$, Eqs. (3.42) through (3.45) become

$$\Gamma_\mu^{(c)}(p,k) = G\sum_j \lambda_j (1-s^2)\left[3(\xi + \frac{1}{3})\gamma_\mu + \int_0^1 dx \int_0^{1-x} dy (6\gamma_\mu \ln \hat{D}_w - B_\mu^{(c)} D_w^{-1})\right] L \quad (3.46)$$

$$\Gamma_\mu^{(d)}(p,k) = G\sum_j \lambda_j \frac{(1-2s^2)}{2M_w^2}\left[\frac{1}{4}\xi C_\mu^{(d)} + \int_0^1 dx \int_0^{1-x} dy (C_\mu^{(d)} \ln \hat{D}_w + B_\mu^{(d)} D_w^{-1})\right] \quad (3.47)$$

$$\Gamma_\mu^{(e)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy s^2 \left(m_j^2 \gamma_\mu L - m_d[(1-x)\not{p} - y\not{k}]\gamma_\mu R\right) D_w^{-1} \quad (3.48)$$

$$\Gamma_\mu^{(f)}(p,k) = G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy s^2 \left(m_j^2 \gamma_\mu L - m_s \gamma_\mu[(1-x)\not{p} - y\not{k}] R\right) D_w^{-1} \quad (3.49)$$

where $\hat{D}_w = \dfrac{D_w}{M_w^2}$. (3.50)

In Eqs. (3.46) and (3.47), there are terms which are independent of m_j and can be dropped as a result of the unitarity property of the KM matrix. We then obtain

$$
\begin{aligned}
\Gamma_\mu^{(c)}(p,k) = {} & G\sum_j \lambda_j \int_0^1 dx \int_0^{1-x} dy (1-s^2)\Big\{6\gamma_\mu \ln \hat{D}_w - \big[2x(1-x)\not{p}\gamma_\mu \not{p} + (1-x)(2y-1)\not{p}\gamma_\mu \not{k} \\
& - 2xy\not{k}\gamma_\mu \not{p} + y(1-2y)\not{k}\gamma_\mu \not{k} + 2[2x(1-x)p^2 + y(1-2y)k^2]\gamma_\mu\big] D_w^{-1}\Big\}
\end{aligned} \tag{3.51}
$$

$$
\begin{aligned}
\Gamma_\mu^{(d)}(p,k) = {} & \frac{1}{2}(1-2s^2)G\sum_j \lambda_j \Big\{\frac{1}{2}\hat{m}_j^2 \xi \gamma_\mu L + \int_0^1 dx \int_0^{1-x} dy \Big[(\hat{m}_s \hat{m}_d L + m_j^2 R)x \\
& x[2x(x-1)\not{p}\gamma_\mu \not{p} + (1-x)(1-2y)\not{p}\gamma_\mu \not{k} + 2xy\not{k}\gamma_\mu \not{p} + y(2y-1)\not{k}\gamma_\mu \not{k} \\
& + 2x(x-1)p^2\gamma_\mu + y(2y-1)k^2\gamma_\mu] + \hat{m}_j^2(m_d L + m_s R)[2x(\gamma_\mu \not{p} + \not{p}\gamma_\mu) \\
& + (2y-1)(\gamma_\mu \not{k} + \not{k}\gamma_\mu)]\Big] D_w^{-1} + 2\int_0^1 dx \int_0^{1-x} dy (\hat{m}_s \hat{m}_d + \hat{m}_j^2 R)\ln \hat{D}_w \gamma_\mu L\Big\}
\end{aligned} \tag{3.52}
$$

Substitute Eqs. (3.26), (3.27), (3.48), (3.49), (3.51) and (3.52) into Eq. (3.1), we obtain the following expression for the unrenormalized flavour-changing quark-Z vertex function

$$
\begin{aligned}
\Gamma_\mu(p,k) = {} & G\sum_j \lambda_j \Big\{-\frac{1}{4}\Big(\frac{2}{3}s^2 - 1\Big)\xi \hat{m}_j^2 \gamma_\mu L + \frac{1}{3}\hat{m}_j^2 s^2 \gamma_\mu L \\
& + \Big[E_1^L \gamma_\mu + \frac{1}{M_w^2}\Big(E_2^L \not{p}\gamma_\mu \not{p} + E_3^L \not{k}\gamma_\mu \not{k} + E_4^L \not{p}\gamma_\mu \not{k} + E_5^L \not{k}\gamma_\mu \not{p} + E_6^L \not{p}\not{k}\gamma_\mu\Big) \\
& + \frac{1}{M_w}\Big(E_7^L \not{p}\gamma_\mu + E_8^L \gamma_\mu \not{p} + E_9^L \not{k}\gamma_\mu + E_{10}^L \gamma_\mu \not{k}\Big)\Big]L \\
& + \Big[E_1^R \gamma_\mu + \frac{1}{M_w^2}\Big(E_2^R \not{p}\gamma_\mu \not{p} + E_3^R \not{k}\gamma_\mu \not{k} + E_4^R \not{p}\gamma_\mu \not{k} + E_5^R \not{k}\gamma_\mu \not{p} + E_6^R \not{p}\not{k}\gamma_\mu\Big) \\
& + \frac{1}{M_w}\Big(E_7^R \not{p}\gamma_\mu + E_8^R \gamma_\mu \not{p} + E_9^R \not{k}\gamma_\mu + E_{10}^R \gamma_\mu \not{k}\Big)\Big]R\Big\},
\end{aligned} \tag{3.53}
$$

where E_i^L and E_i^R are given by

$$
\left.
\begin{aligned}
E_i^L &= \int_0^1 dx \int_0^{1-x} dy\, H_i^L \\
E_i^R &= \int_0^1 dx \int_0^{1-x} dy\, H_i^R \qquad i = 1,2,\ldots,10
\end{aligned}
\right\} \tag{3.54}
$$

The functions H_i^L , H_i^R are explicitly given as following:

$$
\begin{aligned}
H_1^L = & \left\{ \left(2(1-s^2) + \frac{1}{4}(1-2s^2)\hat{m}_j^2 \right)\left(2x(x-1)p^2 + y(2y-1)k^2 \right) \right. \\
& \left. + [xy(1-2s^2)\hat{m}_j^2 - 2(x+2y-4xy-1)(1-s^2)]k \cdot p + 2\hat{m}_j^2 \right\} \hat{D}_w^{-1} \\
& + \left[6(1-s^2) + \frac{1}{2}(1-2s^2)\hat{m}_j^2 \right] \ln \hat{D}_w - \hat{m}_j^2 \left[\frac{4}{3}s^2 + \frac{1}{2}\left(\frac{4}{3}s^2 - 1 \right)\hat{m}_j^2 \right] \hat{D}_Q^{-1} \\
& + \left(\frac{4}{3}s^2 - 1 + \frac{2}{3}s^2\hat{m}_j^2 \right) \ln \hat{D}_Q^{-1}
\end{aligned}
\tag{3.55}
$$

$$
H_2^L = 2x(x-1)\left[\frac{1}{4}(1-2s^2)\hat{m}_j^2 + 1 - s^2 \right]\hat{D}_w^{-1} - x^2\left[\frac{4}{3}s^2 - 1 + \frac{2}{3}s^2\hat{m}_j^2 \right]\hat{D}_Q^{-1} \tag{3.56}
$$

$$
H_3^L = y(2y-1)\left[\frac{1}{4}(1-2s^2)\hat{m}_j^2 + 1 - s^2 \right]\hat{D}_w^{-1} + y(y-1)\left[\frac{4}{3}s^2 - 1 + \frac{2}{3}s^2\hat{m}_j^2 \right]\hat{D}_Q^{-1} \tag{3.57}
$$

$$
\begin{aligned}
H_4^L = & \left[\frac{1}{4}(1-2y)(1-x)(1-2s^2)\hat{m}_j^2 + (x+2xy-y-1)(1-s^2) \right]\hat{D}_w^{-1} \\
& + x\left[(1-y)\left(\frac{4}{3}s^2 - 1 \right) - \frac{2}{3}ys^2\hat{m}_j^2 \right]\hat{D}_Q^{-1}
\end{aligned}
\tag{3.58}
$$

$$
\begin{aligned}
H_5^L = & \left[\frac{1}{2}xy(1-2s^2)\hat{m}_j^2 + (2xy-y+2-2x)(1-s^2) \right]\hat{D}_w^{-1} \\
& + x\left[y\left(1 - \frac{4}{3}s^2 \right) + \frac{2}{3}(1-y)s^2\hat{m}_j^2 \right]\hat{D}_Q^{-1}
\end{aligned}
\tag{3.59}
$$

$$
H_6^L = \frac{1}{4}(1-x-2y)(1-2s^2)\hat{m}_j^2\hat{D}_w^{-1} \tag{3.60}
$$

$$H_7^L = \hat{m}_d\left[(x-1)s^2 + \frac{1}{2}x(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} - \frac{x}{2}\left(1-\frac{4}{3}s^2\right)\hat{m}_d\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.61}$$

$$H_8^L = \frac{x}{2}\hat{m}_d\hat{m}_j^2\left[(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}s^2\hat{D}_Q^{-1}\right] \tag{3.62}$$

$$H_9^L = \hat{m}_d\left[ys^2 + \frac{1}{4}(2y-1)(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{1}{2}(1-y)\left(1-\frac{4}{3}s^2\right)\hat{m}_d\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.63}$$

$$H_{10}^L = \frac{1}{4}\hat{m}_d\hat{m}_j^2\left[(2y-1)(1-2s^2)\hat{D}_w^{-1} + \frac{8}{3}ys^2\hat{D}_Q^{-1}\right] \tag{3.64}$$

$$\begin{aligned} H_1^R = {} & \frac{1}{2}(1-2s^2)\hat{m}_s\hat{m}_d\left\{\left[x(x-1)p^2 + y(y-\frac{1}{2})k^2 + 2xyk\cdot p\right]D_w^{-1} + \ln\hat{D}_w^{-1}\right\} \\ & - \hat{m}_s\hat{m}_d\left[\frac{2}{3}s^2\hat{m}_j^2\hat{D}_Q^{-1} + \left(\frac{1}{2}-\frac{2}{3}s^2\right)\ln\hat{D}_Q\right] \end{aligned} \tag{3.65}$$

$$H_2^R = \frac{x}{2}\hat{m}_s\hat{m}_d\left[(x-1)(1-2s^2)\hat{D}_w^{-1} + x\left(1-\frac{4}{3}s^2\right)\hat{D}_Q^{-1}\right] \tag{3.66}$$

$$H_3^R = \frac{y}{2}\hat{m}_s\hat{m}_d\left[\left(y-\frac{1}{2}\right)(1-2s^2)\hat{D}_w^{-1} + (1-y)\left(\frac{4}{3}s^2-1\right)\hat{D}_Q^{-1}\right] \tag{3.67}$$

$$H_4^R = \frac{1}{2}\hat{m}_s\hat{m}_d\left[(y-\frac{1}{2})(x-1)(1-2s^2)\hat{D}_w^{-1} + xy(1-\frac{4}{3}s^2)\hat{D}_Q^{-1}\right] \tag{3.68}$$

$$H_5^R = \frac{x}{2}\hat{m}_s\hat{m}_d\left[y(1-2s^2)\hat{D}_w^{-1} + (1-y)(\frac{4}{3}s^2 - 1)\hat{D}_Q^{-1}\right] \tag{3.69}$$

$$H_6^R = \frac{1}{4}(1-x-2y)(1-2s^2)\hat{m}_s\hat{m}_d\hat{D}_w^{-1} \tag{3.70}$$

$$H_7^R = \frac{x}{2}\hat{m}_s\hat{m}_j^2\left[(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}s^2\hat{D}_Q^{-1}\right] \tag{3.71}$$

$$H_8^R = \hat{m}_s\left[(x-1)s^2 + \frac{x}{2}(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{x}{2}(\frac{4}{3}s^2 - 1)\hat{m}_s\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.72}$$

$$H_9^R = \frac{1}{2}\hat{m}_s\hat{m}_j^2\left[(y-\frac{1}{2})(1-2s^2)\hat{D}_w^{-1} + \frac{4}{3}(y-1)s^2\hat{D}_Q^{-1}\right] \tag{3.73}$$

$$H_{10}^R = \hat{m}_s\left[ys^2 + \frac{1}{4}(2y-1)(1-2s^2)\hat{m}_j^2\right]\hat{D}_w^{-1} + \frac{y}{2}(\frac{4}{3}s^2 - 1)\hat{m}_s\hat{m}_j^2\hat{D}_Q^{-1} \tag{3.74}$$

Expression (3.53) for the flavour-changing quark-Z vertex is divergent because of the factor ξ which tends to infinity as $n \to 4$. Renormalization of the vertex function is needed, which is given in the next chapter.