Chapter 4

RENORMALIZATION OF THE VERTEX FUNCTION

4.1 INTRODUCTION

To renormalize the Z-penguin vertex, we note that the divergence of the unrenormalized vertex is cancelled by the divergence arising from the improper vertex, which involve the flavour-changing quark self-energy, when the external quark lines are on mass shell. In the next section, we define a renormalization prescription which utilizes this fact for the subtractions of the divergent term. In section 4.3, we present the calculation of the counterterm from the improper quark-Z vertex function. The explicit expression for the renormalized Z-penguin vertex function is presented in section 4.4.

4.2 THE RENORMALIZATION PRESCRIPTION

We shall use the our renormalization prescription proposed by Chia and Chong [64], which is schematically given in Fig. 4.1. The renormalized flavour-changing quark-Z vertex function is obtain by adding to the unrenormalized flavour-changing quark-Z vertex function contributions arising from the on-shell self-energy vertices. It is checked that the prescription yields the same counter-terms for gluon-penguin and photo-penguin vertices as those obtained from Ward-Takahashi identity. This prescription is much simpler than utilizing Ward-Takahashi identity.

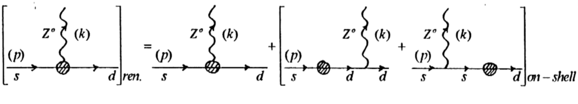

Fig.4.1: Renormalization prescription.

4.3 THE CALCULATION OF THE COUNTERTERM FROM THE IMPROPER VERTEX

In the 't Hooft-Feynman gauge, the counterterm is claculated from Fig. 4.2, which is given by

$$\Omega_\mu(p,k) = \sum_i \Omega_\mu^{(i)}(p,k) \qquad i = a,b,c,d. \tag{4.1}$$

where index i refer to the corresponding diagram in Fig. 4.2.

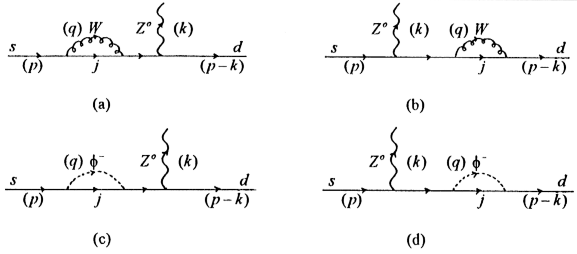

Fig.4.2: Diagrams involving $\bar{d}s$ self-energy that also contribute to the $\bar{d}sZ$ vertex.

The contribution from each of the diagrams in Fig. 4.2 are as follows:

$$\Omega_\mu^{(a)}(p,k) = -\frac{ig^3}{4\cos\theta_w}\sum_j \lambda_j \int \frac{d^4q}{(2\pi)^4}\left\{\gamma_\mu\left(\frac{2}{3}s^2 - L\right)\frac{1}{\not p - m_d}\gamma_\alpha L \times \right.$$
$$\left. \times \frac{1}{\not p - \not q - m_j}\gamma_\beta L \frac{g^{\alpha\beta}}{q^2 - M_w^2}\right\} \tag{4.2}$$

$$\Omega_\mu^{(b)}(p,k)=\frac{ig^3}{4\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\alpha L\frac{1}{\not p-\not k-\not q-m_j}\gamma_\beta L\frac{g^{\alpha\beta}}{q^2-M_w^2}\times\right.$$
$$\left.\times\frac{1}{\not p-\not k-m_s}\gamma_\mu(L-\frac{2}{3}s^2)\right\} \tag{4.3}$$

$$\Omega_\mu^{(c)}(p,k)=-\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{\gamma_\mu(L-\frac{2}{3}s^2)\frac{1}{\not p-m_d}(m_jR-m_dL)\times\right.$$
$$\left.\times\frac{1}{\not p-\not q-m_j}(m_jL-m_sR)\frac{1}{q^2-M_w^2}\right\} \tag{4.4}$$

$$\Omega_\mu^{(d)}(p,k)=-\frac{ig^3}{4M_w^2\cos\theta_w}\sum_j\lambda_j\int\frac{d^4q}{(2\pi)^4}\left\{(m_jR-m_dL)\frac{1}{\not p-\not k-\not q-m_j}\times\right.$$
$$\left.\times(m_jL-m_sR)\frac{1}{q^2-M_w^2}\frac{1}{\not p-\not k-m_s}\gamma_\mu(L-\frac{2}{3}s^2)\right\} \tag{4.5}$$

In $n=4-\varepsilon$ spacetime dimensions, Eqs. (4.2) through (4.5) become

$$\Omega_\mu^{(a)}(p,k)=-\frac{ig^3(\mu^2)^{\frac{\varepsilon}{2}}}{4\cos\theta_w(p^2-m_d^2)}\sum_j\lambda_j\int\frac{d^nq}{(2\pi)^n}U_\mu^{(a)}\left[q^2-M_w^2\right]^{-1}\left[(p-q)^2-m_j^2\right]^{-1} \tag{4.6}$$

$$\Omega_\mu^{(b)}(p,k)=\frac{ig^3(\mu^2)^{\frac{\varepsilon}{2}}}{4\cos\theta_w((p-k)^2-m_s^2)}\sum_j\lambda_j\int\frac{d^nq}{(2\pi)^n}U_\mu^{(b)}\left[q^2-M_w^2\right]^{-1}\left[(p-k-q)^2-m_j^2\right]^{-1} \tag{4.7}$$

$$\Omega_\mu^{(c)}(p,k)=-\frac{ig^3(\mu^2)^{\frac{\varepsilon}{2}}}{4M_w^2\cos\theta_w(p^2-m_d^2)}\sum_j\lambda_j\int\frac{d^nq}{(2\pi)^n}U_\mu^{(c)}\left[q^2-M_w^2\right]^{-1}\left[(p-q)^2-m_j^2\right]^{-1} \tag{4.8}$$

$$\Omega_\mu^{(d)}(p,k) = -\frac{ig^3(\mu^2)^{\frac{\varepsilon}{2}}}{4M_w^2\cos\theta_w((p-k)^2-m_s^2)}\sum_j\lambda_j\int\frac{d^nq}{(2\pi)^n}U_\mu^{(d)}\times$$
$$\times\left[q^2-M_w^2\right]^{-1}\left[(p-k-q)^2-m_j^2\right]^{-1} \tag{4.9}$$

where

$$\left.\begin{aligned}
U_\mu^{(d)} &= (m_jR-m_dL)(\not p-\not k-\not q+m_j)(m_jL-m_sR)(\not p-\not k+m_s)\gamma_\mu(L-\frac{2}{3}s^2)\\
U_\mu^{(b)} &= \gamma_\alpha L(\not p-\not k-\not q+m_j)\gamma_\beta Lg^{\alpha\beta}(\not p-\not k+m_s)\gamma_\mu(L-\frac{2}{3}s^2)\\
U_\mu^{(c)} &= \gamma_\mu(L-\frac{2}{3}s^2)(\not p+m_d)(m_jR-m_dL)(\not p-\not q+m_j)(m_jL-m_sR)\\
U_\mu^{(d)} &= (m_jR-m_dL)(\not p-\not k-\not q+m_j)(m_jL-m_sR)(\not p-\not k+m_s)\gamma_\mu(L-\frac{2}{3}s^2)
\end{aligned}\right\} \tag{4.10}$$

Again, we introduce Feynman parametrization:

$$\left.\begin{aligned}
\left[q^2-M_w^2\right]^{-1}\left[(p-q)^2-m_j^2\right]^{-1} &= \int_0^1dx\left[\tilde{q}_1^2-D_1\right]^{-2}\\
\left[q^2-M_w^2\right]^{-1}\left[(p-k-q)^2-m_j^2\right]^{-1} &= \int_0^1dx\left[\tilde{q}_2^2-D_2\right]^{-2}
\end{aligned}\right\} \tag{4.11}$$

where

$$\left.\begin{aligned}
\tilde{q}_1 &= q-(1-x)p,\\
\tilde{q}_2 &= q-(1-x)(p-k),
\end{aligned}\right\} \tag{4.12}$$

$$\left.\begin{aligned} D_1 &= xM_w^2 + (1-x)m_j^2 - x(1-x)p^2\,, \\ D_2 &= xM_w^2 + (1-x)m_j^2 - x(1-x)(p-k)^2\,. \end{aligned}\right\} \tag{4.13}$$

Substitute Eq. (4.12) into Eq. (4.10), we obtain

$$U_\mu^{(a)} = \gamma_\mu(\frac{2}{3}s^2 - L)(\not{p} + m_d)(2-n)(x\not{p} - \not{\tilde{q}}_1)L \tag{4.14}$$

$$U_\mu^{(b)} = (2-n)\left[\not{\tilde{q}}_2 - x(\not{p} - \not{k})\right]\left[(\not{p} - \not{k})(1-\frac{2}{3}s^2)R - \frac{2}{3}s^2 m_s L\right]\gamma_\mu \tag{4.15}$$

$$\begin{aligned} U_\mu^{(c)} = \gamma_\mu\Big\{ & m_j[(1-\frac{2}{3}s^2)\not{p} - \frac{2}{3}s^2 m_d]R + m_d[\frac{2}{3}s^2\not{p} - m_d(1-\frac{2}{3}s^2)]L\Big\} x \\ & x(x\not{p} - \not{\tilde{q}}_1 + m_j)(m_j L - m_s R) \end{aligned} \tag{4.16}$$

$$\begin{aligned} U_\mu^{(d)} &= (m_j R - m_d L)[x(\not{p} - \not{k}) - \not{\tilde{q}}_2 + m_j] \times \\ & \times \left\{(1-\frac{2}{3}s^2)[(\not{p} - \not{k})m_j^2 - m_s^2]R + \frac{2}{3}s^2 m_s(\not{p} - \not{k} - m_j^2)L\right\}\gamma_\mu \end{aligned} \tag{4.17}$$

Again, we drop the terms which contain odd power of $\tilde{q}$, Eqs. (4.14) through (4.17) becomes

$$U_\mu^{(a)} = (2-n)x\gamma_\mu\left[(\frac{2}{3}s^2 - 1)p^2 L + m_d\frac{2}{3}s^2 R\not{p}\right] \tag{4.18}$$

$$U_\mu^{(b)} = (2-n)x\left[\frac{2}{3}s^2 m_s(\not{p} - \not{k})L - (1-\frac{2}{3}s^2)(p-k)^2 R\right]\gamma_\mu \tag{4.19}$$

$$\begin{aligned} U_\mu^{(c)} = \gamma_\mu\Big\{ & (1-\frac{2}{3}s^2)L\left[m_j^2(xp^2 - m_d^2) + m_s(xm_d^2 - m_j^2)\not{p}\right] \\ & + \frac{2}{3}s^2 m_d R\left[m_s(m_j^2 - xp^2) + (1-x)m_j^2\not{p}\right]\Big\} \end{aligned} \tag{4.20}$$

$$U_\mu^{(d)} = (1-\frac{2}{3}s^2)\Big[m_j^2[x(p-k)^2 - m_s^2] + m_d(xm_s^2 - m_j^2)(\not p - \not k)\Big]\gamma_\mu L + \frac{2}{3}s^2 m_s\Big[m_d[m_j^2 - x(p-k)^2] + (1-x)m_j^2(\not p - \not k)\Big]\gamma_\mu R \tag{4.21}$$

After the loop momentum integration and some simple algebra, we obtain

$$\Omega_\mu^{(a)}(p,k) = \left(p^2 - m_d^2\right)^{-1} G\sum_j \lambda_j \int_0^1 \left\{x\left(\xi + 1 + \ln \hat{D}_1\right)\gamma_\mu\left[(\frac{2}{3}s^2 - 1)p^2 L + \frac{2}{3}s^2 m_d R\not p\right]\right\}dx \tag{4.22}$$

$$\Omega_\mu^{(b)}(p,k) = \left[(p-k)^2 - m_s^2\right]^{-1} G\sum_j \lambda_j \int_0^1 \left\{x\left(\xi + 1 + \ln \hat{D}_2\right)\left[(\frac{2}{3}s^2 - 1)(p-k)^2 R + \frac{2}{3}s^2 m_s(\not p - \not k)L\right]\gamma_\mu\right\}dx \tag{4.23}$$

$$\Omega_\mu^{(c)}(p,k) = \left[p^2 - m_d^2\right]^{-1} \frac{G}{M_w^2}\sum_j \lambda_j \int_0^1 \left(\xi + \ln \hat{D}_1\right)\left\{(\frac{1}{3}s^2 - \frac{1}{2})\gamma_\mu L\Big[m_j^2(xp^2 - m_d^2) + m_s(xm_d^2 - m_j^2)\not p\Big] - \frac{1}{3}s^2 m_d \gamma_\mu R\Big[m_s(m_j^2 - xp^2) + (1-x)m_j^2 \not p\Big]\right\}dx \tag{4.24}$$

$$\Omega_\mu^{(d)}(p,k) = \left[(p-k)^2 - m_s^2\right]^{-1} \frac{G}{M_w^2}\sum_j \lambda_j \int_0^1 \left(\xi + \ln \hat{D}_2\right)\left\{(\frac{1}{3}s^2 - \frac{1}{2}) \times \Big[m_j^2[x(p-k)^2 - m_s^2] + m_d(xm_s^2 - m_j^2)(\not p - \not k)\Big]\gamma_\mu L - \frac{1}{3}s^2 m_s\Big[m_d[m_j^2 - x(p-k)^2] + (1-x)m_j^2(\not p - \not k)\Big]\gamma_\mu R\right\}dx \tag{4.25}$$

where G and ξ have been defined in Eqs. (3.21) and (3.24) respectively in chapter 3. We now put the external quarks on mass shell by imposing the following on-shell conditions:

$$\left.\begin{aligned} \not{p}U(p) &= m_s U(p), & p^2 &= m_s^2 \\ \bar{U}(p-k)(\not{p}-\not{k}) &= m_d \bar{U}(p-k), & (p-k)^2 &= m_d^2 \end{aligned}\right\} \tag{4.26}$$

By substituting Eqs. (4.22) through (4.25) into Eq. (4.1) and dropping terms which are independent of m_j^2, we obtain the following expression for the counterterm derived:

$$\begin{aligned} \Omega_\mu = \frac{G}{(m_d^2 - m_s^2)} \sum_j \lambda_j \Bigg\{ &(\frac{2}{3}s^2 - 1)\Bigg[\frac{1}{4}(m_d^2 - m_s^2)\hat{m}_j^2 \xi \\ &+(1+\frac{\hat{m}_j^2}{2})\left[m_d^2 F_1(m_d^2) - m_s^2 F_1(m_s^2)\right] + \frac{1}{2}\hat{m}_s^2 m_d^2\left[F_1(m_d^2) - F_1(m_s^2)\right] \\ &+\frac{1}{2}(m_s^2 + m_d^2)\hat{m}_j^2\left[F_0(m_s^2) - F_0(m_d^2)\right]\Bigg]\gamma_\mu L \\ &+\frac{2}{3}s^2 m_s m_d\Bigg[\frac{1}{2}\left[\hat{m}_d^2 F_1(m_d^2) - \hat{m}_s^2 F_1(m_s^2)\right] \\ &+(1+\frac{\hat{m}_j^2}{2})\left[F_1(m_d^2) - F_1(m_s^2)\right] + \hat{m}_j^2\left[F_0(m_s^2) - F_0(m_d^2)\right]\Bigg]\gamma_\mu R\Bigg\} \end{aligned} \tag{4.27}$$

where $F_0(p^2) = \int_0^1 \ln\left[x + (1-x)\hat{m}_j^2 + x(x-1)\frac{p^2}{M_w^2}\right]dx$ (4.28)

and $F_1(p^2) = \int_0^1 x\ln\left[x + (1-x)\hat{m}_j^2 + x(x-1)\frac{p^2}{M_w^2}\right]dx$ (4.29)

4.4 THE RENORMALIZED Z-PENGUIN VERTEX FUNCTION

The Z-penguin vertex function is renormalized by adding the counterterm calculated in Sect. 4.3 to the unrenormalized vertex function:

$$\Gamma_{\mu,R}(p,k) = \Gamma_\mu(p,k) + \Omega_\mu \tag{4.30}$$

The divergent term in $\Gamma_\mu(p,k)$ is cancelled exactly by the divergent term in the counterterm Ω_μ. The renormalized Z-penguin vertex function is then given by

$$\begin{aligned}\Gamma_{\mu,R}(p,k) = G\sum_j \lambda_j \Big\{ & \beta_L\gamma_\mu L + \beta_R\gamma_\mu R + \frac{1}{3}\hat{m}_j^2 s^2\gamma_\mu L \\ & + \Big[E_1^L\gamma_\mu + \frac{1}{M_w^2}\left(E_2^L \not{p}\gamma_\mu \not{p} + E_3^L \not{k}\gamma_\mu \not{k} + E_4^L \not{p}\gamma_\mu \not{k} + E_5^L \not{k}\gamma_\mu \not{p} + E_6^L \not{p}\not{k}\gamma_\mu\right) \\ & + \frac{1}{M_w}\left(E_7^L \not{p}\gamma_\mu + E_8^L\gamma_\mu \not{p} + E_9^L \not{k}\gamma_\mu + E_{10}^L\gamma_\mu \not{k}\right)\Big] L \\ & + \Big[E_1^R\gamma_\mu + \frac{1}{M_w^2}\left(E_2^R \not{p}\gamma_\mu \not{p} + E_3^R \not{k}\gamma_\mu \not{k} + E_4^R \not{p}\gamma_\mu \not{k} + E_5^R \not{k}\gamma_\mu \not{p} + E_6^R \not{p}\not{k}\gamma_\mu\right) \\ & + \frac{1}{M_w}\left(E_7^R \not{p}\gamma_\mu + E_8^R\gamma_\mu \not{p} + E_9^R \not{k}\gamma_\mu + E_{10}^R\gamma_\mu \not{k}\right)\Big] R \Big\}\end{aligned} \tag{4.31}$$

where

$$\begin{aligned}\beta_L = \frac{(\frac{2}{3}s^2 - 1)}{m_d^2 - m_s^2} \Big\{ & (1 + \frac{\hat{m}_j^2}{2})\left[m_d^2 F_1(m_d^2) - m_s^2 F_1(m_s^2)\right] \\ & + \frac{1}{2}\hat{m}_s^2 m_d^2\left[F_1(m_d^2) - F_1(m_s^2)\right] + \frac{\hat{m}_j^2}{2}(m_s^2 + m_d^2)\left[F_0(m_s^2) - F_0(m_d^2)\right]\Big\}\end{aligned} \tag{4.32}$$

and

$$\begin{aligned}\beta_R = \frac{2}{3}s^2 \frac{m_s m_d}{m_d^2 - m_s^2} \Big\{ & \frac{1}{2}\left[\hat{m}_d^2 F_1(m_d^2) - \hat{m}_s^2 F_1(m_s^2)\right] \\ & + (1 + \frac{1}{2}\hat{m}_j^2)\left[F_1(m_d^2) - F_1(m_s^2)\right] + \hat{m}_j^2\left[F_0(m_s^2) - F_0(m_d^2)\right]\Big\}.\end{aligned} \tag{4.33}$$

Eq. (4.31) is the exact expression for the off-shell renormalized vertex function. The on-shell condition will be applied in the next chapter.