Chapter 5

THE ON-SHELL FLAVOUR-CHANGING QUARK-Z VERTEX

5.1 INTRODUCTION

For practical applications, we need to consider the case when the external s and d quarks are on-mass shell. Therefore, in sect. 5.2, we obtain expression for the on-shell flavour-changing quark-Z vertex function, $\Gamma_{\mu,R}(on-shell)$. We then simplifies our calculations by assuming that the momentum of the emitted Z boson as well as the mass of the s and d quarks are very small compared to the mass of the W boson. In sect. 5.3, our results are tabulated and plotted as functions of the internal quark mass.

5.2 THE ON-SHELL VERTEX FUNCTION

We obtain the on-shell vertex $\Gamma_{\mu,R}(on-shell)$ using the on-shell condition expressed in Eq.(4.26). Besides, we also make use of the following identities

$$\left.\begin{aligned} \gamma_\mu \not{k} &= k_\mu + i\sigma_{\mu\nu}k^\nu \\ \not{k}\gamma_\mu &= k_\mu - i\sigma_{\mu\nu}k^\nu \end{aligned}\right\} \tag{5.1}$$

$$\text{where} \quad \sigma_{\mu\nu} = \frac{1}{2i}\left[\gamma_\mu, \gamma_\nu\right] \tag{5.2}$$

After putting the external s and d quarks on mass shell, Eq.(4.31) becomes

$$\Gamma_{\mu,R}(on-shell)=\frac{gG_F}{4\sqrt{2}\pi^2\cos\theta_w}\sum_j\lambda_j\Big\{(k_\mu k\!\!\!/-k^2\gamma_\mu)(A_j^L L+A_j^R R)$$
$$+i\sigma_{\mu\nu}k^\nu(m_d B_j^L L+m_s B_j^R R)+M_w^2(C_j^L\gamma_\mu L+C_j^R\gamma_\mu R)\Big\} \tag{5.3}$$

where $G_F=\frac{1}{4\sqrt{2}}\left(\frac{g}{M_w}\right)^2$ and

$$A_j^L=\frac{1}{\hat m_s^2-\hat m_d^2}\Big[\hat m_d^2\left(\widetilde E_4^L+\widetilde E_6^L\right)+\hat m_s^2\left(\widetilde E_2^L+\widetilde E_5^L\right)+\hat m_s\hat m_d\left(\widetilde E_2^R+\widetilde E_4^R+\widetilde E_5^R+\widetilde E_6^R\right)$$
$$+\hat m_d\left(\widetilde E_7^L+\widetilde E_9^L+\widetilde E_{10}^L\right)+\hat m_s\left(\widetilde E_7^R+\widetilde E_9^R+\widetilde E_{10}^R\right)+2\left(\widetilde E_3^L+\widetilde E_4^L\right) \tag{5.4}$$

$$A_j^R=\frac{1}{\hat m_s^2-\hat m_d^2}\Big[\hat m_d^2\left(\widetilde E_4^R+\widetilde E_6^R\right)+\hat m_s^2\left(\widetilde E_2^R+\widetilde E_5^R\right)+\hat m_s\hat m_d\left(\widetilde E_2^L+\widetilde E_4^L+\widetilde E_5^L+\widetilde E_6^L\right)$$
$$+\hat m_d\left(\widetilde E_7^R+\widetilde E_9^R+\widetilde E_{10}^R\right)+\hat m_s\left(\widetilde E_7^L+\widetilde E_9^L+\widetilde E_{10}^L\right)+2\left(\widetilde E_3^R+\widetilde E_4^R\right) \tag{5.5}$$

$$B_j^L=\widetilde E_4^L-\widetilde E_6^L-\frac{m_s}{m_d}\left(\widetilde E_2^R+\widetilde E_5^R\right)+\frac{1}{\hat m_d}\left(\widetilde E_{10}^L-\widetilde E_7^L-\widetilde E_9^L\right) \tag{5.6}$$

$$B_j^R=\frac{m_d}{m_s}\left(\widetilde E_4^R-\widetilde E_6^R\right)-\widetilde E_2^L-\widetilde E_5^L+\frac{1}{\hat m_s}\left(\widetilde E_{10}^R-\widetilde E_7^R-\widetilde E_9^R\right) \tag{5.7}$$

$$C_j^L=\widetilde E_1^L+\frac{k^2}{M_w^2}\left(\widetilde E_3^L+\widetilde E_4^L-\widetilde E_6^L\right)+\hat m_s\hat m_d\widetilde E_2^R+\hat m_d\widetilde E_7^L+\hat m_s\widetilde E_8^R$$
$$+\frac{\hat m_d k^2}{m_s^2-m_d^2}\Big[\hat m_d\left(\widetilde E_4^L+\widetilde E_6^L\right)+\hat m_s\left(\widetilde E_2^R+\widetilde E_5^R\right)+\widetilde E_7^L+\widetilde E_9^L+\widetilde E_{10}^L\Big]$$
$$+\frac{\hat m_s k^2}{m_s^2-m_d^2}\Big[\hat m_d\left(\widetilde E_4^R+\widetilde E_6^R\right)+\hat m_s\left(\widetilde E_2^L+\widetilde E_5^L\right)+\widetilde E_7^R+\widetilde E_9^R+\widetilde E_{10}^R\Big]$$
$$+\beta_L+\frac{1}{3}s^2\hat m_j^2 \tag{5.8}$$

$$C_j^R = \widetilde{E}_1^R + \frac{k^2}{M_w^2}\left(\widetilde{E}_3^R + \widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s\hat{m}_d\widetilde{E}_2^L + \hat{m}_d\widetilde{E}_7^R + \hat{m}_s\widetilde{E}_8^L$$
$$+\frac{\hat{m}_d k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^R + \widetilde{E}_6^R\right) + \hat{m}_s\left(\widetilde{E}_2^L + \widetilde{E}_5^L\right) + \widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right] \tag{5.9}$$
$$+\frac{\hat{m}_s k^2}{m_s^2 - m_d^2}\left[\hat{m}_d\left(\widetilde{E}_4^L + \widetilde{E}_6^L\right) + \hat{m}_s\left(\widetilde{E}_2^R + \widetilde{E}_5^R\right) + \widetilde{E}_7^L + \widetilde{E}_9^L + \widetilde{E}_{10}^L\right] + \beta_R .$$

with $\widetilde{E}_i^L = E_i^L(on-shell)$ and $\widetilde{E}_i^R = E_i^R(on-shell)$.

(5.10)

For practical applications, we approximate by neglecting k^2, m_s^2 and m_d^2 compared to M_w^2. We then find that

$$D_w = \left(m_j^2 - M_w^2\right)x + M_w^2, \tag{5.11}$$

$$D_Q = \left(M_w^2 - m_j^2\right)x + m_j^2. \tag{5.12}$$

On carrying out the y integration for the $\widetilde{E}_i^L$'s and the $\widetilde{E}_i^R$'s, we find that

$$\widetilde{E}_2^L + \widetilde{E}_4^L + \widetilde{E}_5^L \approx 0 , \tag{5.13}$$

$$\widetilde{E}_2^R + \widetilde{E}_4^R + \widetilde{E}_5^R \approx 0 , \tag{5.14}$$

$$\widetilde{E}_6^L \approx \widetilde{E}_6^R \approx 0 , \tag{5.15}$$

$$m_s\widetilde{E}_7^L \approx m_d\widetilde{E}_8^R , \tag{5.16}$$

$$m_s \widetilde{E}_8^L \approx m_d \widetilde{E}_7^R \ , \tag{5.17}$$

$$m_d\left(\widetilde{E}_7^R + \widetilde{E}_9^R\right) + m_s \widetilde{E}_{10}^L \approx 0 \ , \tag{5.18}$$

$$m_s\left(\widetilde{E}_7^L + \widetilde{E}_9^L\right) + m_d \widetilde{E}_{10}^R \approx 0 \ . \tag{5.19}$$

Using Eqs. (5.13) through (5.19), we find that

$$A_j^L \approx \frac{1}{m_s}\left(\widetilde{E}_7^R + \widetilde{E}_9^R + \widetilde{E}_{10}^R\right) + 2\widetilde{E}_3^L + \widetilde{E}_4^L \tag{5.20}$$

$$A_j^R \ll A_j^L \ (= A_j) \tag{5.21}$$

$$\begin{aligned} B_j^L &\approx B_j^R \ (= B_j) \\ &\approx \widetilde{E}_4^L + \frac{1}{m_d}\left(\widetilde{E}_{10}^L - \widetilde{E}_7^L - \widetilde{E}_9^L\right) \end{aligned} \tag{5.22}$$

$$C_j^L \approx \widetilde{E}_1^L + \beta_L + \frac{1}{3} s^2 \hat{m}_j^2 \tag{5.23}$$

$$C_j^R \ll C_j^L \ (= C_j) \tag{5.24}$$

In arriving at the expressions shown above, we have made use of the following fact:

$$\left.\begin{aligned} &E_7^L,\ E_8^L,\ E_9^L,\ E_{10}^L \propto \hat{m}_d \\ &E_1^R,\ E_2^R,\ E_3^R,\ E_4^R,\ E_5^R,\ E_6^R \propto \hat{m}_s \hat{m}_d \\ &E_7^R,\ E_8^R,\ E_9^R,\ E_{10}^R \propto \hat{m}_s \ . \end{aligned}\right\} \tag{5.25}$$

Using Eqs.(5.21) through (5.24), the vertex function in Eq.(5.3) can be rewritten as:

$$\Gamma_{\mu,n}(on-shell) = \frac{gG_F}{4\sqrt{2}\pi^2 M_w^2 \cos\theta_w}\sum_j \lambda_j \Big\{(k_\mu \not{k} - k^2\gamma_\mu)A_j L + i\sigma_{\mu\nu}k^\nu B_j\big(m_d L + m_s R\big) + M_w^2 C_j \gamma_\mu L\Big\}. \tag{5.26}$$

The explicit expressions for the form factors A_j, B_j and C_j are

$$\begin{aligned} A_j \approx \int_0^1 dx \int_0^{1-x} dy \Bigg\{ & \Bigg[\frac{1}{2}\left(1-2s^2\right)\hat{m}_j^2\left(x+\frac{1}{2}(2y-1)(1+x+2y)\right)+ys^2 \\ & +\left(4y^2-3y+x+2xy-1\right)\left(1-s^2\right)\Bigg]\hat{D}_w^{-1}+\Bigg[\left(\frac{2}{3}s^2\left(2y^2-xy+x-1\right)-\frac{y}{2}\right)\hat{m}_j^2 \\ & +(1-y)(x-2y)\left(\frac{4}{3}s^2-1\right)\Bigg]\Bigg\} \\ \approx & \frac{\hat{m}_j^2}{4\left(1-\hat{m}_j^2\right)^3}\left\{\frac{1}{2}+\frac{1}{3}\left(\frac{13}{2}-\frac{25}{3}s^2\right)\hat{m}_j^2-\frac{1}{3}\left(5-\frac{19}{3}s^2\right)\hat{m}_j^4\right\} \\ & +\frac{\ln\hat{m}_j^2}{\left(1-\hat{m}_j^2\right)^4}\left\{\frac{1}{3}-\frac{4}{9}s^2-\left(\frac{5}{4}-\frac{16}{9}s^2\right)\hat{m}_j^2+3\left(\frac{3}{4}-s^2\right)\hat{m}_j^4\right. \\ & \left.-\frac{1}{3}\left(\frac{7}{2}-5s^2\right)\hat{m}_j^6+\frac{1}{6}\left(\frac{1}{2}-s^2\right)\hat{m}_j^8\right\} \end{aligned} \tag{5.27}$$

$$\begin{aligned} B_j \approx \int_0^1 dx \int_0^{1-x} dy \Bigg\{ & \Bigg[\left(1-2s^2\right)\left(\frac{1}{4}(2y-1)(x-1)-\frac{x}{2}\right) \\ & +(x+2xy-y-1)\left(1-s^2\right)+(1-x-y)s^2\Bigg]\hat{D}_w^{-1} \\ & +\left[\hat{m}_j^2\left(\frac{2}{3}s^2 y(1-x)+\frac{1}{2}(1-\frac{4}{3}s^2)(x+y-1)\right)+x(1-y)\left(\frac{4}{3}s^2-1\right)\hat{D}_Q^{-1}\right\} \end{aligned}$$

$$\approx -\frac{\hat{m}_j^2}{\left(1-\hat{m}_j^2\right)^4}\left\{\left[\frac{7}{12}s^2+\frac{1}{4}\left(\frac{1}{2}-\frac{5}{3}s^2\right)\hat{m}_j^2+\left(\frac{5}{8}-\frac{2}{3}s^2\right)\hat{m}_j^4\right]\right.$$
$$\left.+\left[\frac{1}{4}-\left(\frac{3}{4}-s^2\right)\hat{m}_j^2+\frac{1}{4}\left(5-6s^2\right)\hat{m}_j^4\right]\ln\hat{m}_j^2\right\} \tag{5.28}$$

$$C_j \approx \int_0^1 dx\int_0^{1-x} dy\left\{\left[\left(2(1-s^2)+\frac{1}{4}(1-2s^2)\hat{m}_j^2\right)\left(2x(x-1)p^2+y(2y-1)k^2\right)\right.\right.$$
$$\left.+[xy(1-2s^2)\hat{m}_j^2-2(x+2y-4xy-1)(1-s^2)]k\cdot p+2\hat{m}_j^2\right]\hat{D}_w^{-1}$$
$$+\left[6(1-s^2)+\frac{1}{2}(1-2s^2)\hat{m}_j^2\right]\ln\hat{D}_w-\hat{m}_j^2\left[\frac{4}{3}s^2+\frac{1}{2}\left(\frac{4}{3}s^2-1\right)\hat{m}_j^2\right]D_Q^{-1}$$
$$\left.+\left(\frac{4}{3}s^2-1+\frac{2}{3}s^2\hat{m}_j^2\right)\ln\hat{D}_Q^{-1}\right\}+\left(\frac{2}{3}s^2-1\right)\left(1+\frac{1}{2}\hat{m}_j^2\right)\int_0^1 x\left[x+(1-x)\hat{m}_j^2\right]dx+\frac{1}{3}s^2\hat{m}_j^2 \tag{5.29}$$

$$\approx \frac{\hat{m}_j^2}{2\left(1-\hat{m}_j^2\right)^2}\left[\left(6-\hat{m}_j^2\right)\left(1-\hat{m}_j^2\right)+\left(2+3\hat{m}_j^2\right)\ln\hat{m}_j^2\right]$$

The vertex function as given in Eq. (5.26) involves six form factors, namely A_j, B_j and C_j $(j=u,c,t)$. This is more general than those obtained by earlier calculations [27,32], which involved only the C_j form factors.

5.3 PROPERTIES OF THE FORM FACTORS A_j, B_j and C_j.

We consider the asymptotic properties of A_j, B_j and C_j in the limit when m_j takes on very small and very large values.

For $m_j \to 0$,

$$A_j \to \frac{1}{3}\left(1-\frac{4}{3}s^2\right)\ln \hat{m}_j^2 , \tag{5.30}$$

$$B_j \to -\frac{1}{4}\hat{m}_j^2 \ln \hat{m}_j^2 , \tag{5.31}$$

$$C_j \to 0 . \tag{5.32}$$

We note that while A_j diverges as $m_j \to 0$, this divergence is spurious because as $m_j \to 0$, the integral in Eq. (5.27) is regulated by the masses m_s and m_d which we have set to be very small in our approximation. We also note that C_j actually approach constant values which we have neglected since these values do not contribute to $\Gamma_{\mu,R}(on-shell)$ because of the unitarity property of the KM matrix. Anyway, for region where $\hat{m}_j$ is small, the contribution of A_j dominates over the contributions from B_j and C_j.

For $m_j \to \infty$,

$$A_j \to \frac{19}{36}s^2 - \frac{5}{12} + \frac{1}{6}\left(\frac{1}{2} - s^2\right)\ln \hat{m}_j^2 , \tag{5.33}$$

$$B_j \to \frac{3}{2}s^2 - \frac{5}{8} , \tag{5.34}$$

$$C_j \to \frac{1}{2}\hat{m}_j^2 . \tag{5.35}$$

We see that for large $\hat{m}_j^2$, the contribution of C_j dominates over the contributions from B_j and A_j. B_j approaches to a constant value whereas A_j only increase as $\ln \hat{m}_j^2$. The behaviours of the form factors as functions of m_j are tabulated in Table 1 and displayed in Fig. 5.1.

Table 1: The variation of the form factors A_j, B_j and C_j with m_j ($M_w = 80.22$ GeV).

m_j (GeV)	A_j	B_j	C_j
1	-2.0268	0.0003	-0.0009
5	-1.2842	0.0049	-0.0102
10	-0.9662	0.0145	-0.0211
15	-0.7818	0.0262	-0.0244
20	-0.6520	0.0390	-0.0179
25	-0.5524	0.0522	-0.0008
30	-0.4719	0.0656	0.0266
35	-0.4046	0.0788	0.0637
40	-0.3471	0.0918	0.1100
45	-0.2970	0.1045	0.1649
50	-0.2528	0.1167	0.2278
55	-0.2135	0.1285	0.2981
60	-0.1781	0.1399	0.3753
65	-0.1461	0.1508	0.4591
70	-0.1170	0.1612	0.5489
75	-0.0903	0.1713	0.6444
80	-0.0657	0.1808	0.7454
85	-0.0429	0.1900	0.8516
90	-0.0220	0.1988	0.9628
95	-0.0025	0.2072	1.0788
100	0.0157	0.2152	1.1993
105	0.0328	0.2229	1.3243
110	0.0487	0.2302	1.4536
115	0.0638	0.2373	1.5870
120	0.0779	0.2440	1.7246
125	0.0912	0.2505	1.8662
130	0.1038	0.2566	2.0118
135	0.1157	0.2626	2.1612
140	0.1270	0.2682	2.3144
145	0.1378	0.2737	2.4714
150	0.1480	0.2789	2.6321
155	0.1577	0.2839	2.7965
160	0.1670	0.2888	2.9646
165	0.1758	0.2934	3.1363
170	0.1843	0.2979	3.3116
175	0.1924	0.3022	3.4905
180	0.2001	0.3063	3.6730
185	0.2076	0.3103	3.8590
190	0.2147	0.3141	4.0486
195	0.2216	0.3178	4.2417
200	0.2282	0.3213	4.4383

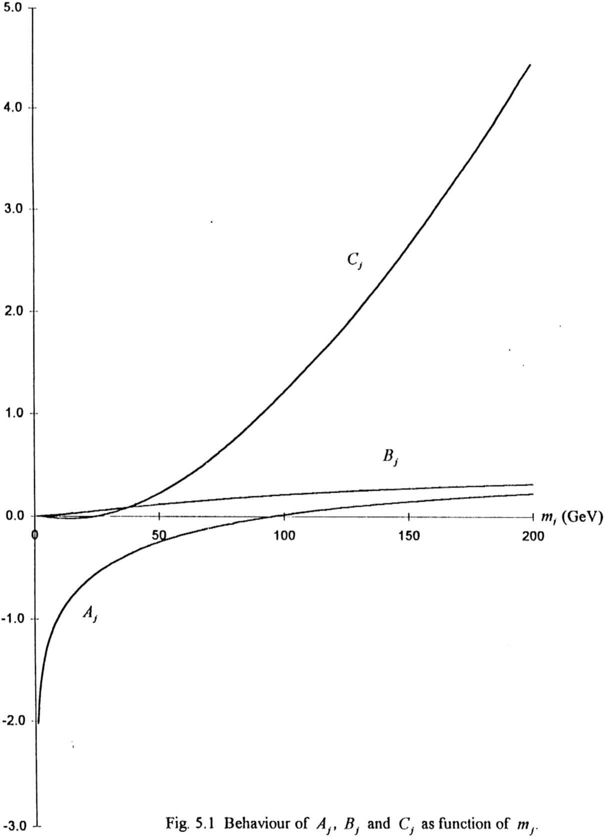

Fig. 5.1 Behaviour of A_j, B_j and C_j as function of m_j.