

## Chapter 5

### Forecasting

Forecasting models for the yield series will be considered in this chapter. The yield series is divided into two parts, the sample forecast range (1960: Q1 to 1996: Q4) and the post sample forecast range (1997: Q1 to 1997: Q4). The Sample Forecast Range (SFR) is used to fit the model, whereas, the Post Sample Forecasting Range (PSFR) is used to assess the forecasting performance of the fitted model.

The analysis of patterns of the yield ( $Y_t$ ) series showed that the series has trend and seasonal components. The series also has an increasing variance of error terms. To achieve stationarity in variance the following transformations are applied. They were, logarithm ( $\lambda = 0$ ), negative square root reciprocal ( $\lambda = -0.5$ ) and negative reciprocal ( $\lambda = -1$ ). The series given in Table 5.1 will be used in developing forecasting models.

**Table 5.1: Series Used In Developing Forecasting Model**

	Transformation	Series
Original Series		$Y_t$
Logarithm	$\log Y_t$	$LY_t$
Negative Square Root Reciprocal	$1/\sqrt{Y_t}$	$Y1_t$
Negative Reciprocal	$1/Y_t$	$Y2_t$

Holt-Winters trend and seasonality method which is based on three smoothing constants, one for level, one for trend and one for seasonality will

sed. The Holt-Winters' Multiplicative Seasonality (HWMS) method will be fitted to original series ( $Y_t$ ). This method is suitable for increasing variance series. Holt-Winters' Additive Seasonality(HWAS) method will be fitted to the power-transformed series,  $LY_t$ ,  $Y1_t$  and  $Y2_t$  which is suitable for series having constant seasonality. ARIMA methodology requires the series to have constant variance. Therefore, ARIMA methodology will be only fitted to  $Y_t$ ,  $Y1_t$  and  $Y2_t$ .

### **5.1 Forecasting Models: By Holt-Winters Method**

In order to fit this method, the method requires initial estimates of the three smoothing constants. EViews software determines the initial values of the three smoothing constants and then obtains the appropriate estimates of smoothing constant for level,  $\alpha$ ; trend,  $\gamma$ ; and seasonal,  $\delta$ . The EViews output of the optimal values of the three levels of the exponential smoothing parameters for  $Y_t$ ,  $LY_t$ ,  $Y1_t$  and  $Y2_t$  are given in Table B1, B2, B3 and B4 (Appendix B) respectively. Table 5.2 shows the summary output of the optimal values of the  $\alpha$  (level),  $\gamma$ (trend) and  $\delta$ (seasonal) for Holt-Winters' Multiplicative Seasonality (HWMS) method on the original series and Holt-Winters' Additive Seasonality (HWAS) method on each of the power-transformation series. The optimal smoothing trend parameter for all series is zero. The implication is that the updating trend equation becomes  $T_t = T_{t-1}$  (see, Section 5.1)

The initial estimates and the optimal values of the exponential smoothing parameters which were determined earlier and together with the updating procedure is used for updating until quarter 4 of 1996. The outputs

end values of the three levels of quarter 4 of 1996 for  $Y_t$ ,  $LY_t$ ,  $Y1_t$  and  $Y2_t$  are given in Table B1, B2, B3 and B4 (Appendix B) respectively.

**Table 5.2: Optimal Values Of Smoothing Parameters**

METHOD	HWMS	HWAS	HWAS	HWAS
SERIES	$Y_t$	$LY_t$	$Y1_t$	$Y2_t$
LEVEL ( $\alpha$ )	0.4600	0.4500	0.4400	0.4700
TREND ( $\gamma$ )	0.0000	0.0000	0.0000	0.0000
SEASONAL ( $\delta$ )	0.3901	0.3701	0.2300	0.0000

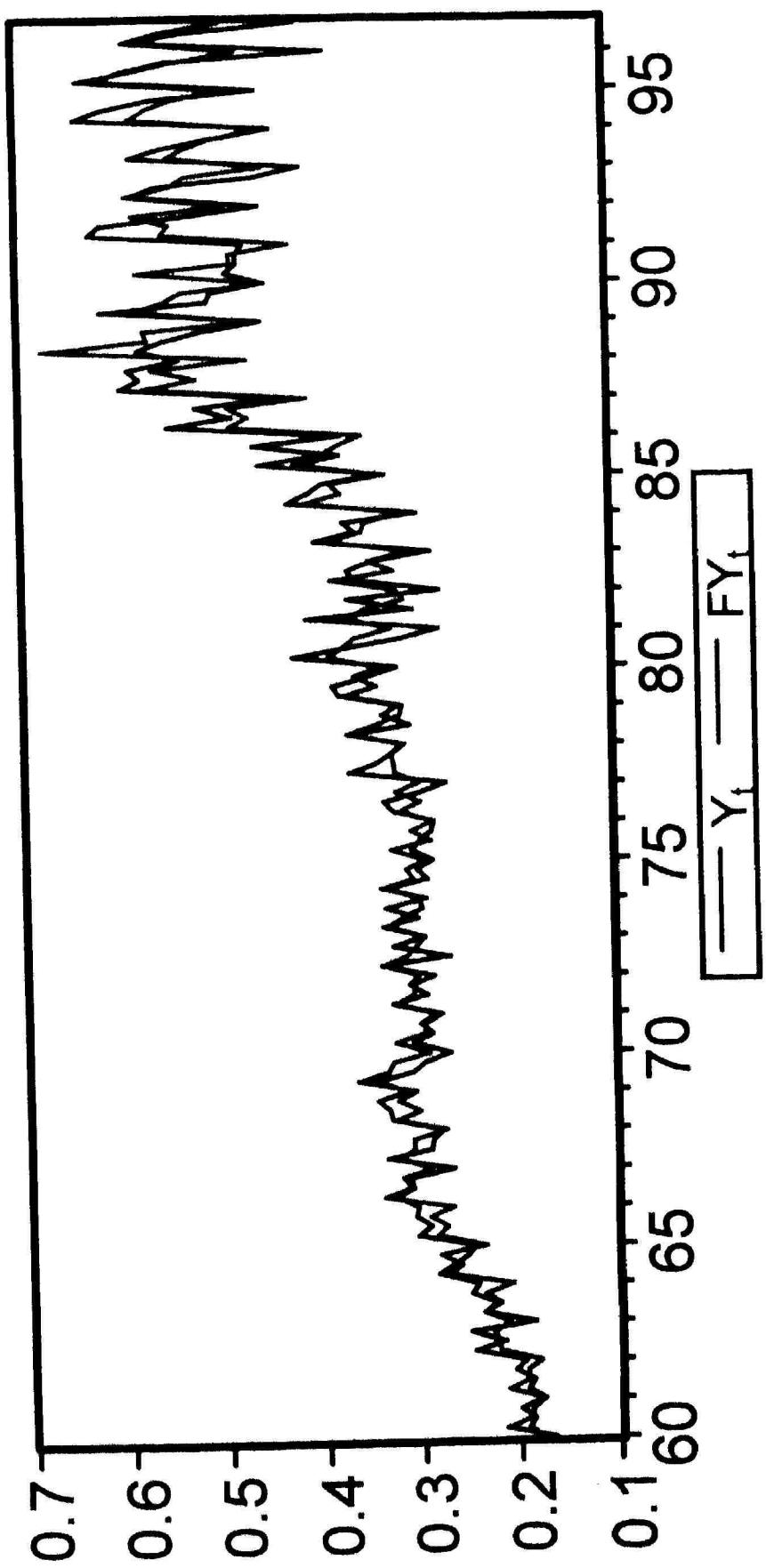
### **i.1.1 Holt-Winters Multiplicative Seasonality (HWMS)**

**Method on  $Y_t$**

#### *Sample Forecast Range (SFR): Model is Fitted*

Table B5 (Appendix B) shows the forecast (fitted) values in sample forecast range from the fitting of HWMS method. Figure 5.1 shows the plot of actual yield series ( $Y_t$ ) and forecast yield series ( $FY_t$ ). The correlogram of the residuals ( $Y_t - FY_t$ ) is shown in Exhibit B1(Appendix B). Ljung-Box Q-statistic is  $Q_{36} = 42.146$  with a p-value = 0.22, which is greater than 0.05. Therefore, it is reasonable to conclude that the fitted model is adequate and residuals of the fitted model follows white noise model. The root mean sum squared error and mean absolute percentage error from this fitted model is:

$$RMSE_{psfr} = 0.033, MAPE_{psfr} = 6.7$$



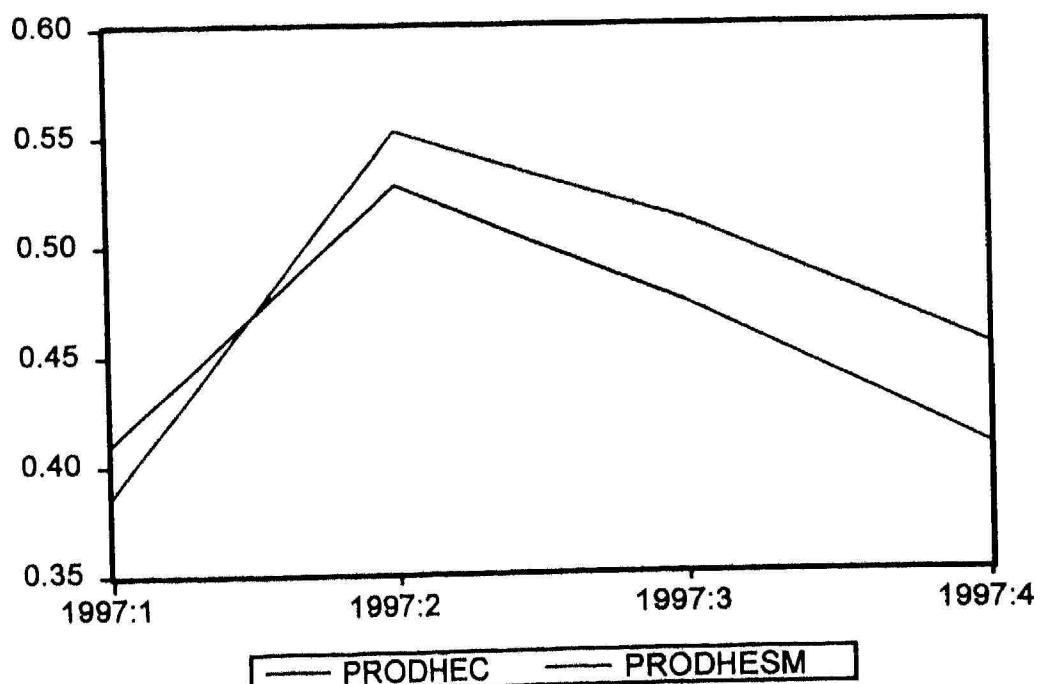
### *Post Sample Forecast Range (PSFR): Forecasting*

The quarterly forecast values for 1997 are obtained by using updating equations to update the end values of the three levels of quarter 4 of 1996 over into the quarters of 1997. Table 5.3 shows the actual ( $Y_t$ ) and forecast values ( $FY_t$ ) in post sample forecast range. Figure 5.2 shows the plot of  $Y_t$  and  $FY_t$  from 1997:Q1 to 1997:Q4.  $MAPE_{\text{sf}} = 7.4$  and  $RSME_{\text{sf}} = 43$

**Table 5.3: Actual ( $Y_t$ ) And Forecast ( $FY_t$ ) By HWMS:  
1997:Q1 To 1997:Q4**

	Q1	Q2	Q3	Q4
1997	0.410390	0.527636	0.473456	0.407115
t 1997	0.386691	0.552312	0.511290	0.453109

**Figure 5.2: Plot of  $Y_t$  And  $FY_t$  From 1997:Q1 To 1997:Q4  
By HWMS**



**Holt-Winters Additive Seasonality (HWAS) Method  
on LY<sub>t</sub>**

**Sample Forecast Range (SFR): Model is Fitted**

Table B6 (Appendix B) shows the forecast (fitted) values in sample forecast range from the fitting of HWMS method. Figure 5.3 shows the plot of actual yield series (LY<sub>t</sub>) and forecast yield series (FLY<sub>t</sub>). The correlogram of residuals (LY<sub>t</sub> - FLY<sub>t</sub>) is shown in Exhibit B2 (Appendix B). Ljung-Box statistic is Q<sub>36</sub> = 33.452 with a p-value = 0.59, which is greater than 0.05. Therefore, it is reasonable to conclude that the fitted model is adequate and residuals of the fitted model follows white noise model. The root mean sum squared error and mean absolute percentage error from this fitted model is: RMSE<sub>sfr</sub> = 0.083, MAPE<sub>sfr</sub> = 7.2.

**Post Sample Forecast Range (PSFR): Forecasting**

The quarterly forecast values for 1997 are obtained by using dating equations to update the end values of the three levels of quarter 4 of 1996 further into the quarters of 1997. Table 5.4 shows the actual (LY<sub>t</sub>) and forecast values (FLY<sub>t</sub>) in post sample forecast range. Figure 5.4 shows the plot of LY<sub>t</sub> and FLY<sub>t</sub> from 1997:Q1 to 1997:Q4. MAPE<sub>sfr</sub> = 9.7 and RMSE<sub>sfr</sub> = 0.082.

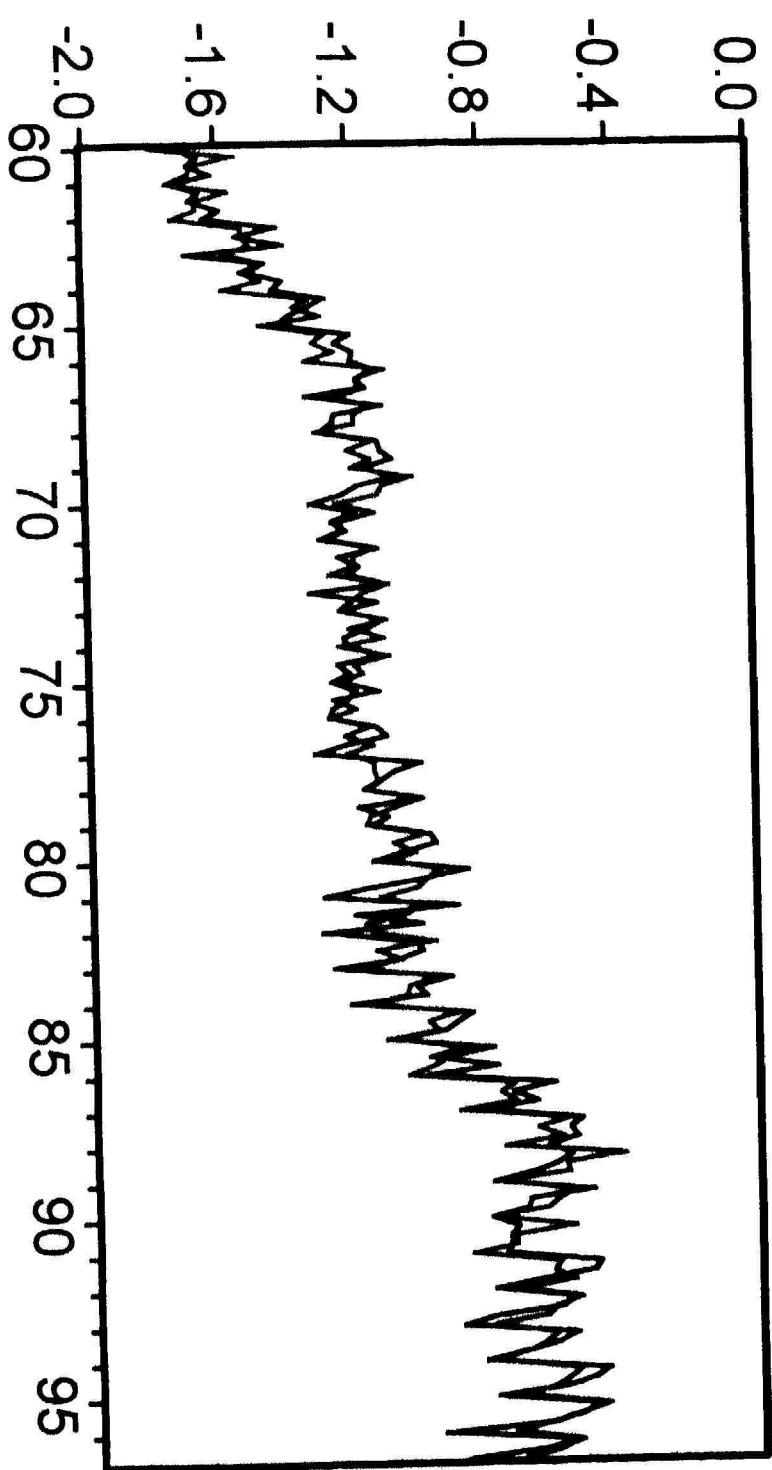
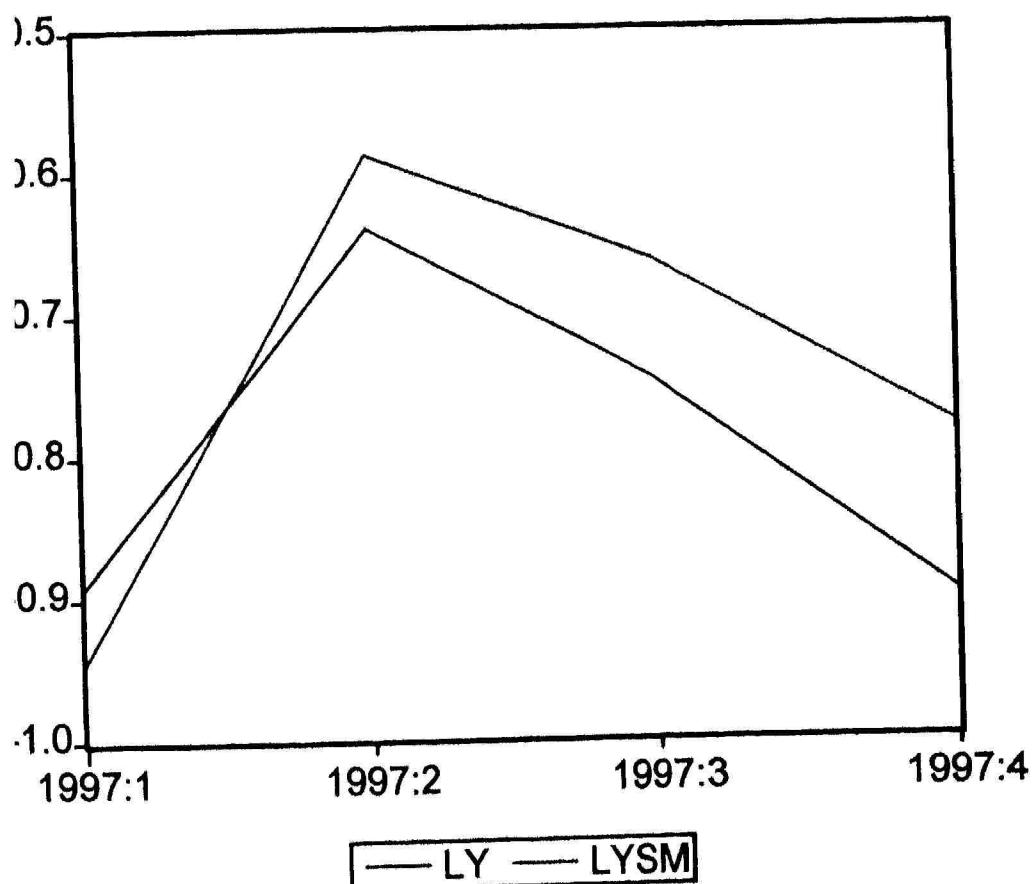


Figure 5.3: Plot of Yield Series ( $LY_t$ ) and Forecast Series ( $FLY_t$ ) by HWMS Method

**| 5.4: Actual ( $LY_t$ ) And Forecast ( $FLY_t$ ) From 1997:Q1 To 1997:Q4  
By HWAS**

	Q1	Q2	Q3	Q4
1997	-0.890647	-0.639349	-0.747696	-0.898660
1997	-0.943764	-0.587635	-0.663022	-0.779643

**| re 5.4: Plot Of  $LY_t$  And Forecast  $FLY_t$  From 1997:Q1 To 1997:Q4  
By HWAS:**



### **Holt-Winters Additive Seasonality (HWAS) Method**

*in Y<sub>t</sub>,*

#### *In Forecast Range (SFR): Model is Fitted*

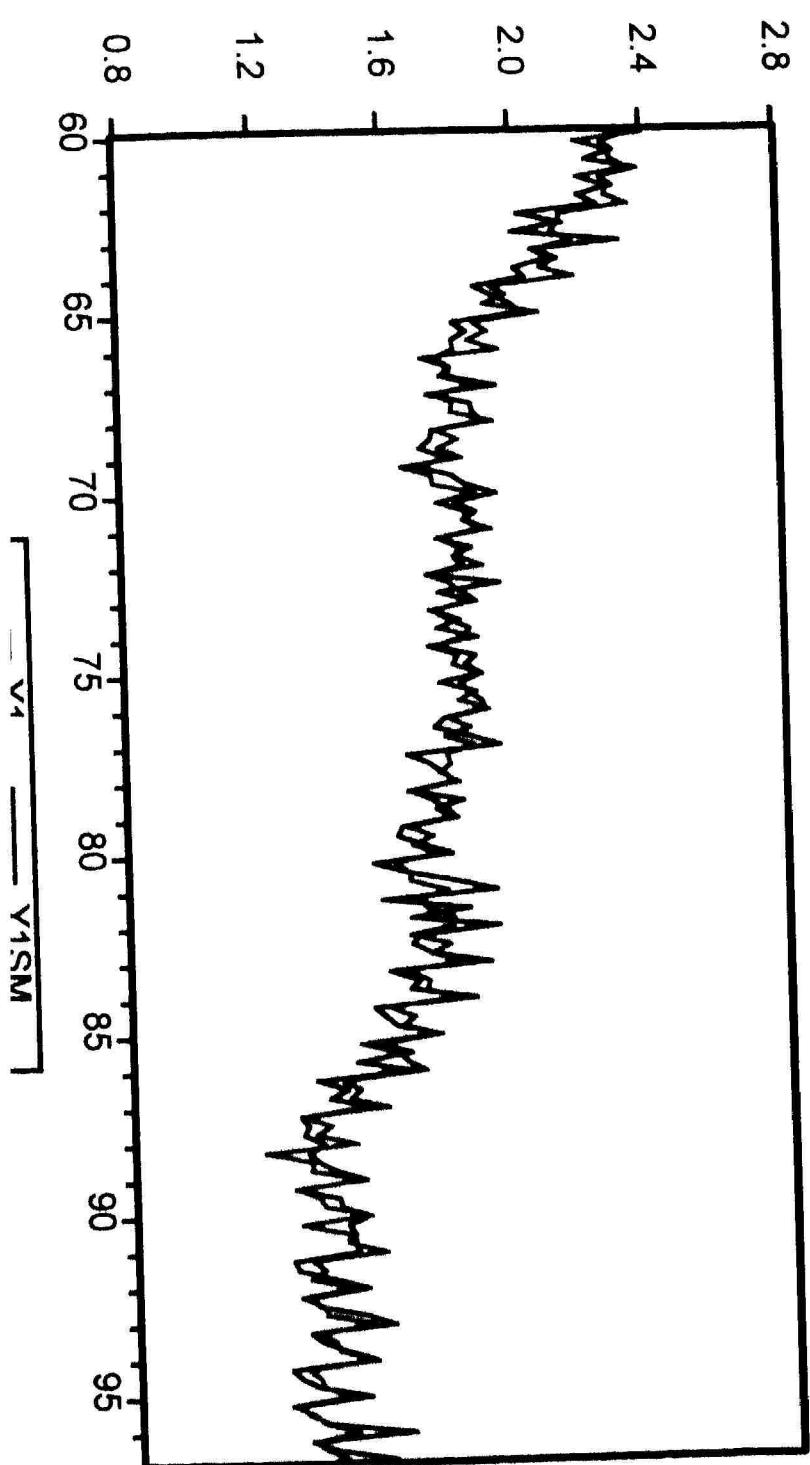
Table B7 (Appendix B) shows the forecast (fitted) values in sample post range from the fitting of HWMS method. Figure 5.5 shows the plot of actual yield series ( $Y_t$ ) and forecast yield series ( $FY_{t,1}$ ). The correlogram of residuals ( $Y_t - FY_{t,1}$ ) is shown in Exhibit B1(Appendix B). Ljung-Box statistic is  $Q_{36} = 37.65$  with a p-value = 0.394, which is greater than 0.05. Therefore, it is reasonable to conclude that the fitted model is adequate and residuals of the fitted model follows white noise model. The root mean squared error and mean absolute percentage error from this fitted model is:

$$E_{psfr} = 0.071, MAPE_{psfr} = 3.3$$

#### *at Sample Forecast Range (PSFR): Forecasting*

The quarterly forecast values for 1997 are obtained by using updating equations to update the end values of the three levels of quarter 4 of 1996 further into the quarters of 1997. Table 5.5 shows the actual ( $Y_{t,1}$ ) forecast values ( $FY_{t,1}$ ) in post sample forecast range. Figure 5.6 shows the plot of  $Y_{t,1}$  and  $FY_{t,1}$  from 1997:Q1 to 1997:Q4.  $MAPE_{sfir} = 3.4$  and  $SE_{sfir} = 0.061$ .

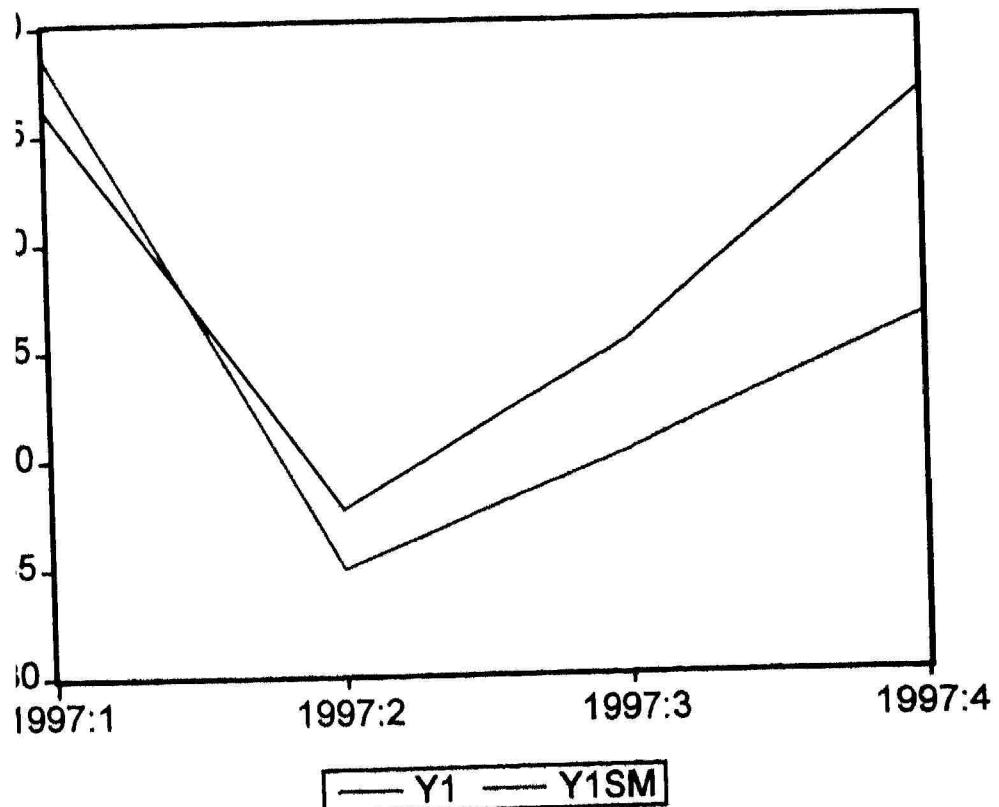
Figure 5.5: Plot of Yield Series ( $Y_1$ ) and Forecast Series ( $\hat{Y}_1$ ) by HWAS Method



**.5: Actual ( $Y_{1t}$ ) And Forecast ( $FY_{1t}$ ) From 1997:Q1 To 1997:Q4  
By HWAS**

	Q1	Q2	Q3	Q4
1997	1.560995	1.376679	1.453316	1.567261
1997	1.584868	1.348984	1.402161	1.463507

**Figure 5.6: Plot Of  $Y_{1t}$  And  $FY_{1t}$  From 1997:Q1 To 1997:Q4 By HWAS**



### **Winters Additive Seasonality (HWAS) Method**

$Y_{2,t}$

#### *Forecast Range (SFR): Model is Fitted*

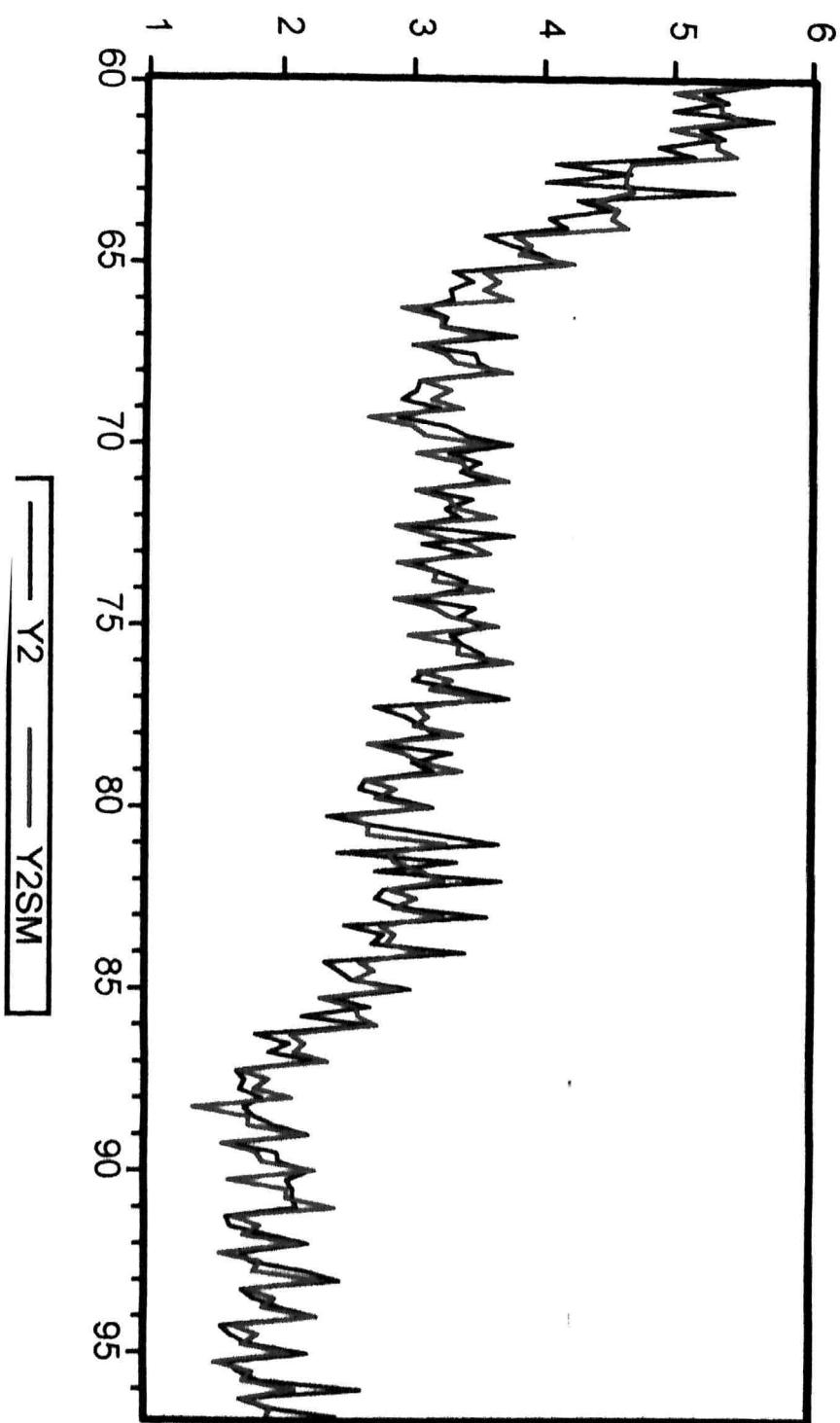
Table B8 (Appendix B) shows the forecast (fitted) values in sample t range from the fitting of HWMS method. Figure 5.7 shows the plot of yield series ( $Y_{2,t}$ ) and forecast yield series ( $FY_{2,t}$ ). The correlogram of residuals ( $Y_{2,t} - FY_{2,t}$ ) is shown in Exhibit B4 (Appendix B). Ljung-Box statistic is  $Q_{36} = 39.459$  with a p-value = 0.318, which is greater than 0.05. Therefore, it is reasonable to conclude that the fitted model is adequate and residuals of the fitted model follows white noise model. The root mean sum of squared error and mean absolute percentage error from this fitted model is:

$$RMSE_{psfr} = 0.254, MAPE_{psfr} = 6.7$$

#### *Sample Forecast Range (PSFR): Forecasting*

The quarterly forecast values for 1997 are obtained by using fitting equations to update the end values of the three levels of quarter 4 of 1996 further into the quarters of 1997. Table 5.6 shows the actual ( $Y_{2,t}$ ) forecast values ( $FY_{2,t}$ ) in post sample forecast range. Figure 5.8 shows plot of  $Y_{2,t}$  and  $FY_{2,t}$  from 1997:Q1 to 1997:Q4.  $MAPE_{sfir} = 5.3$  and  $RMSE_{sfir} = 0.197$

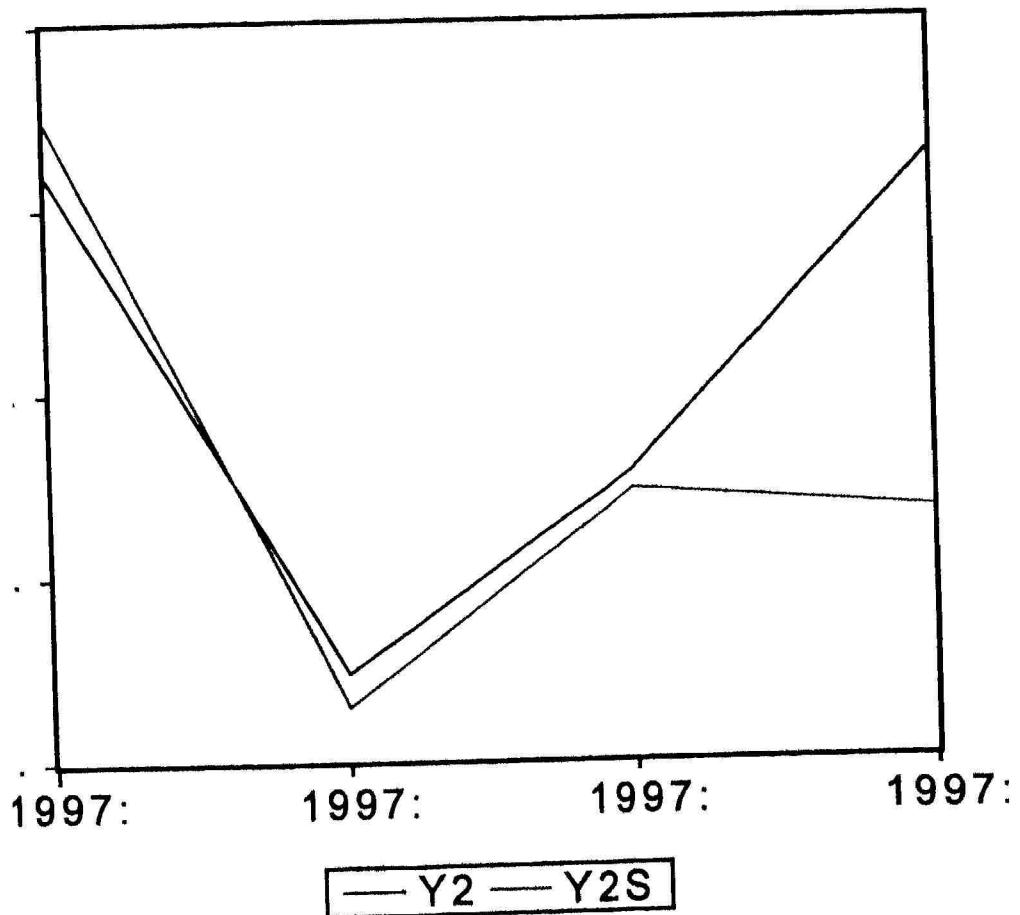
Figure 5.7: Plot of Yield Series ( $Y_{2,t}$ ) and Forecast Series( $\hat{Y}_{2,t}$ ) by HWAS Method



.6: Actual ( $Y_{2t}$ ) And Forecast ( $FY_{2t}$ ) From 1997:Q1 To 1997:Q4  
By HWAS

	Q1	Q2	Q3	Q4
1997	2.436707	1.895246	2.112129	2.456308
1997	2.495474	1.858865	2.091943	2.068142

5.8: Plot Of  $Y_{2t}$  And  $FY_{2t}$  From 1997:Q1 To 1997:Q4 By HWAS



### **forecast Yield From Holt-Winters Method**

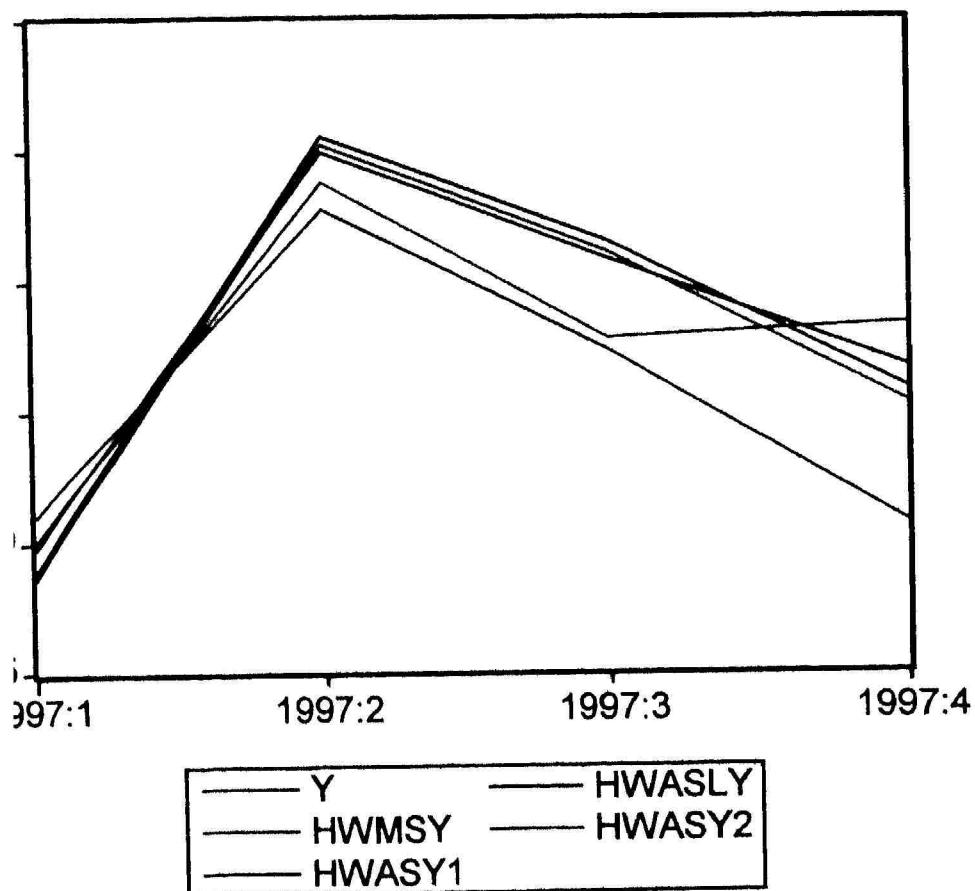
Table 5.7 shows the comparison of actual yield and the forecast yield by Exponential Smoothing Method in the post sample forecast. The forecasts given in Table 5.7 obtained for transformed series ( $LY_t$ ,  $Y_2_t$ ) are converted back to the original scale. Figure 5.9 shows the actual yield and forecasted yield by Holt-Winters' Multiplicative method (HWMS) on original series ( $Y_t$ ) and Holt-Winters' Seasonality(HWAS) method on power-transformed series,  $LY_t$ ,  $Y1_t$ , which has constant seasonality.

**5.7: Actual Yield And Forecast Yield Generated By Holt-Winters Method In Post Sample Forecast Range**

-1997	Q1	Q2	Q3	Q4
$\bar{Y}_t$	0.410390	0.527630	0.473456	0.407115
$\bar{s}(Y_t)$	0.386691	0.552312	0.511290	0.453109
$\bar{s}(LY_t)$	0.389160	0.555639	0.515292	0.458569
$\bar{s}(Y1_t)$	0.398120	0.549524	0.508633	0.466886
$\bar{s}(Y2_t)$	0.400725	0.537963	0.478024	0.483526

For first, third and fourth quarter, the fitted models are underestimating, whereas, for second quarter the fitted models are overestimating. The fitted models above, except for HWAS on  $Y2_t$  follow the seasonal pattern of the actual yield series. As for HWAS on  $Y2_t$ , it does not follow the seasonal pattern because the forecasted yield for quarter 4 shows an increase instead of a decline.

F 5.9: Plot Of Actual Yield And Forecast Yield  
By Holt-Winters Method Models



**Forecast Performance in the Post Sample Forecast Range:  
Models by Holt-Winters Method**

In order to compare the forecasting performance, the transformed first yield are be brought back to original scale. Table 5.8 shows the mean sum square error and mean absolute percentage error of the models. These performance measures indicated that forecasted values generated from these fitted models do not differ. HWAS on  $Y_1_t$  give the best performance. The performance of HWAS on  $Y_2_t$  is also good but the plot in figure 5.9 shows the generated forecasts by this fitted model

ot follow the predicted seasonal pattern. For Holt-Winters method, on  $Y_1$ , able to forecast the yield very close to the actual values.

#### **5.8: Forecast Performance In Post Sample Forecast Range: Holt-Winters Method**

	RMSE	MAPE
S on $Y_t$	0.0343	7.435
S on $LY_t$	0.0819	9.655
S on $Y_1t$	0.0607	3.420
S on $Y_2t$	0.1974	5.272

### **Forecasting: Box-Jenkins Methodology**

#### ***Model Building***

##### **.1 Identification**

Autocorrelation plots of  $LY_t$ ,  $Y_1t$  and  $Y_2t$  series is shown in Exhibit 5.1 to Exhibit 5.3 respectively. Exhibit 5.1 to 5.3 suggest that these series are non-stationary and need to be differenced. The autocorrelation plots for the differenced series for  $LY_t$ ,  $Y_1t$  and  $Y_2t$  are shown in Exhibit 5.4 to 5.6 respectively. These plots suggest that seasonal differencing is required. The series differenced once for trend and once for seasonality are shown in Exhibit 5.7 to 5.9. These plots are not very different and the series appear to be stationary and many of the dominant seasonal spikes have disappeared. Thus, the study has identified the model to be an ARIMA  $(p, 1, q)(P, 1, Q)_4$ . The values for  $p$ ,  $q$ ,  $P$  and  $Q$  are yet to be determined.

**Exhibit 5.1: Output of the SAC and SPAC for LY<sub>t</sub>**

Correlogram of LY

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	0.849	0.849	108.77 0.000
		2	0.840	0.427	216.01 0.000
		3	0.797	0.114	313.21 0.000
		4	0.869	0.494	429.60 0.000
		5	0.742	-0.422	515.16 0.000
		6	0.728	-0.069	597.92 0.000
		7	0.679	0.005	670.60 0.000
		8	0.751	0.245	760.15 0.000
		9	0.632	-0.206	824.01 0.000
		10	0.619	-0.036	885.56 0.000
		11	0.574	0.027	938.86 0.000
		12	0.653	0.196	1008.5 0.000
		13	0.540	-0.160	1056.4 0.000
		14	0.523	-0.080	1101.8 0.000
		15	0.480	0.033	1140.3 0.000
		16	0.546	-0.006	1190.4 0.000
		17	0.444	-0.022	1223.8 0.000
		18	0.446	0.129	1257.7 0.000
		19	0.406	-0.001	1286.0 0.000
		20	0.462	-0.110	1323.1 0.000
		21	0.368	-0.033	1346.8 0.000
		22	0.368	0.040	1370.6 0.000
		23	0.328	0.003	1389.8 0.000
		24	0.395	0.068	1417.7 0.000
		25	0.321	0.088	1436.2 0.000
		26	0.327	-0.002	1455.7 0.000
		27	0.305	0.056	1472.7 0.000
		28	0.365	-0.025	1497.4 0.000
		29	0.278	-0.171	1511.8 0.000
		30	0.284	-0.041	1527.0 0.000
		31	0.249	-0.051	1538.8 0.000
		32	0.300	0.034	1556.0 0.000
		33	0.216	-0.007	1565.0 0.000
		34	0.218	-0.002	1574.2 0.000
		35	0.183	0.005	1580.8 0.000
		36	0.241	0.072	1592.3 0.000

**Exhibit 5.2: Output of the SAC and SPAC for  $Y_{1,t}$**

Correlogram of  $Y_{1,t}$

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	0.850	0.850	109.23 0.000
		2	0.847	0.446	218.22 0.000
		3	0.793	0.065	314.38 0.000
		4	0.857	0.453	427.72 0.000
		5	0.733	-0.410	511.06 0.000
		6	0.723	-0.053	592.72 0.000
		7	0.666	0.013	662.49 0.000
		8	0.729	0.221	746.82 0.000
		9	0.612	-0.207	806.62 0.000
		10	0.605	-0.016	865.53 0.000
		11	0.553	0.038	915.16 0.000
		12	0.626	0.182	979.05 0.000
		13	0.512	-0.183	1022.2 0.000
		14	0.499	-0.101	1063.3 0.000
		15	0.451	0.060	1097.3 0.000
		16	0.509	0.009	1140.9 0.000
		17	0.412	-0.002	1169.6 0.000
		18	0.419	0.120	1199.6 0.000
		19	0.376	-0.003	1223.9 0.000
		20	0.427	-0.109	1255.5 0.000
		21	0.338	-0.017	1275.5 0.000
		22	0.341	0.021	1296.0 0.000
		23	0.302	0.010	1312.1 0.000
		24	0.365	0.086	1336.0 0.000
		25	0.296	0.083	1351.8 0.000
		26	0.305	-0.010	1368.7 0.000
		27	0.282	0.044	1383.3 0.000
		28	0.339	-0.024	1404.6 0.000
		29	0.256	-0.189	1416.8 0.000
		30	0.267	-0.011	1430.2 0.000
		31	0.231	-0.031	1440.3 0.000
		32	0.278	0.041	1455.2 0.000
		33	0.201	0.012	1463.0 0.000
		34	0.209	0.014	1471.5 0.000
		35	0.175	-0.003	1477.5 0.000
		36	0.233	0.067	1488.3 0.000

**Exhibit 5.3: Output of the SAC and SPAC for Y2<sub>t</sub>**

Correlogram of Y2

				AC	PAC	Q-Stat	Prob
tocorrelation		Partial Correlation					
				1	0.848	0.848	108.70 0.000
				2	0.849	0.461	218.29 0.000
				3	0.783	0.016	312.10 0.000
				4	0.842	0.414	421.46 0.000
				5	0.716	-0.395	501.04 0.000
				6	0.710	-0.036	579.86 0.000
				7	0.644	0.024	645.20 0.000
				8	0.700	0.189	722.96 0.000
				9	0.582	-0.205	777.05 0.000
				10	0.582	0.006	831.49 0.000
				11	0.524	0.050	876.04 0.000
				12	0.590	0.160	932.83 0.000
				13	0.475	-0.192	969.91 0.000
				14	0.464	-0.119	1005.6 0.000
				15	0.413	0.083	1034.0 0.000
				16	0.463	0.012	1070.2 0.000
				17	0.370	0.020	1093.4 0.000
				18	0.382	0.109	1118.3 0.000
				19	0.338	-0.008	1138.0 0.000
				20	0.383	-0.100	1163.4 0.000
				21	0.298	-0.006	1179.0 0.000
				22	0.305	0.003	1195.4 0.000
				23	0.267	0.010	1208.0 0.000
				24	0.326	0.109	1227.1 0.000
				25	0.263	0.074	1239.6 0.000
				26	0.276	-0.013	1253.4 0.000
				27	0.252	0.027	1265.1 0.000
				28	0.305	-0.015	1282.3 0.000
				29	0.226	-0.204	1291.9 0.000
				30	0.242	0.011	1302.9 0.000
				31	0.207	-0.010	1311.0 0.000
				32	0.250	0.042	1322.9 0.000
				33	0.180	0.033	1329.2 0.000
				34	0.193	0.026	1336.4 0.000
				35	0.161	-0.011	1341.5 0.000
				36	0.217	0.056	1350.8 0.000

**Exhibit 5.4: Output of the SAC and SPAC for  $LY_t$  by using the Transformation  $ZLY_t = LY_t - LY_{t-1}$**

Correlogram of  $ZLY$

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	-0.527	-0.527	41.662 0.000
		2	0.115	-0.225	43.655 0.000
		3	-0.398	-0.645	67.725 0.000
		4	0.698	0.292	142.36 0.000
		5	-0.403	0.104	167.46 0.000
		6	0.115	0.041	169.51 0.000
		7	-0.418	-0.323	196.89 0.000
		8	0.689	0.137	271.78 0.000
		9	-0.357	0.145	291.97 0.000
		10	0.091	0.053	293.29 0.000
		11	-0.423	-0.214	322.15 0.000
		12	0.659	0.002	392.56 0.000
		13	-0.323	0.060	409.58 0.000
		14	0.082	0.020	410.69 0.000
		15	-0.368	0.037	433.12 0.000
		16	0.608	0.107	494.99 0.000
		17	-0.364	-0.157	517.37 0.000
		18	0.141	-0.031	520.74 0.000
		19	-0.360	0.078	542.89 0.000
		20	0.549	0.066	594.79 0.000
		21	-0.319	-0.047	612.49 0.000
		22	0.135	-0.040	615.66 0.000
		23	-0.372	-0.082	640.16 0.000
		24	0.519	-0.075	689.09 0.000
		25	-0.288	0.018	703.01 0.000
		26	0.097	-0.084	704.69 0.000
		27	-0.305	-0.044	721.64 0.000
		28	0.516	0.080	770.58 0.000
		29	-0.308	0.079	788.15 0.000
		30	0.119	0.066	790.80 0.000
		31	-0.308	-0.037	808.74 0.000
		32	0.486	0.031	853.77 0.000
		33	-0.293	0.026	870.28 0.000
		34	0.137	0.079	873.91 0.000
		35	-0.338	-0.060	896.20 0.000
		36	0.527	0.130	951.06 0.000

**Exhibit 5.5: Output of the SAC and SPAC for  $Y_1$ , by using the Transformation  $ZY_{1,t} = Y_{1,t} - Y_{1,t-1}$**

Correlogram of  $ZY_1$

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	-0.560	-0.560	47.055 0.000
		2	0.181	-0.193	52.014 0.000
		3	-0.428	-0.628	79.850 0.000
		4	0.686	0.270	152.00 0.000
		5	-0.427	0.091	180.08 0.000
		6	0.171	0.025	184.64 0.000
		7	-0.435	-0.308	214.23 0.000
		8	0.681	0.162	287.36 0.000
		9	-0.377	0.181	309.97 0.000
		10	0.139	0.066	313.08 0.000
		11	-0.433	-0.192	343.29 0.000
		12	0.640	0.003	409.67 0.000
		13	-0.332	0.097	427.73 0.000
		14	0.115	0.017	429.92 0.000
		15	-0.373	0.015	452.97 0.000
		16	0.593	0.109	511.84 0.000
		17	-0.380	-0.155	536.12 0.000
		18	0.179	-0.019	541.54 0.000
		19	-0.369	0.080	564.83 0.000
		20	0.531	0.045	613.47 0.000
		21	-0.319	-0.038	631.18 0.000
		22	0.155	-0.041	635.41 0.000
		23	-0.378	-0.088	660.61 0.000
		24	0.511	-0.059	707.11 0.000
		25	-0.292	0.032	722.41 0.000
		26	0.114	-0.089	724.78 0.000
		27	-0.307	-0.051	742.03 0.000
		28	0.504	0.091	788.76 0.000
		29	-0.317	0.048	807.42 0.000
		30	0.140	0.054	811.11 0.000
		31	-0.313	-0.032	829.60 0.000
		32	0.469	0.025	871.55 0.000
		33	-0.296	0.009	888.40 0.000
		34	0.161	0.081	893.45 0.000
		35	-0.349	-0.055	917.22 0.000
		36	0.521	0.131	970.78 0.000

**Exhibit 5.6: Output of the SAC and SPAC for  $Y_{2,t}$  by using the Transformation  $ZY_{2,t} = Y_{2,t} - Y_{2,t-1}$**

Correlogram of  $ZY_{2,t}$ 

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	-0.591	-0.591	52.483 0.000
		2	0.249	-0.155	61.864 0.000
		3	-0.460	-0.601	94.067 0.000
		4	0.674	0.252	163.60 0.000
		5	-0.448	0.074	194.59 0.000
		6	0.224	-0.005	202.41 0.000
		7	-0.444	-0.280	233.19 0.000
		8	0.670	0.201	303.83 0.000
		9	-0.396	0.222	328.66 0.000
		10	0.182	0.073	333.94 0.000
		11	-0.432	-0.158	363.98 0.000
		12	0.606	-0.003	423.53 0.000
		13	-0.331	0.131	441.48 0.000
		14	0.142	0.020	444.78 0.000
		15	-0.364	-0.003	466.74 0.000
		16	0.563	0.109	519.75 0.000
		17	-0.383	-0.155	544.45 0.000
		18	0.206	-0.008	551.63 0.000
		19	-0.365	0.076	574.41 0.000
		20	0.501	0.022	617.67 0.000
		21	-0.311	-0.030	634.44 0.000
		22	0.168	-0.039	639.38 0.000
		23	-0.369	-0.104	663.46 0.000
		24	0.488	-0.046	705.86 0.000
		25	-0.288	0.046	720.80 0.000
		26	0.129	-0.088	723.82 0.000
		27	-0.301	-0.060	740.32 0.000
		28	0.478	0.104	782.42 0.000
		29	-0.317	0.017	801.08 0.000
		30	0.156	0.045	805.62 0.000
		31	-0.307	-0.016	823.38 0.000
		32	0.438	0.015	859.98 0.000
		33	-0.291	-0.013	876.29 0.000
		34	0.181	0.077	882.62 0.000
		35	-0.346	-0.048	906.06 0.000
		36	0.496	0.119	954.57 0.000

**Exhibit 5.7: Output of the SAC and SPAC for  $LY_t$ , by using the Transformation  $ZZLY_t = LY_t - LY_{t-1} - LY_{t-4} + LY_{t-5}$**

Correlogram of  $ZZLY$

Ito correlation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	-0.383	-0.383	21.368 0.000
		2	0.006	-0.164	21.374 0.000
		3	0.234	0.209	29.455 0.000
		4	-0.511	-0.421	68.393 0.000
		5	0.136	-0.255	71.177 0.000
		6	0.029	-0.092	71.303 0.000
		7	0.005	0.212	71.307 0.000
		8	0.021	-0.191	71.377 0.000
		9	0.020	-0.153	71.437 0.000
		10	-0.017	-0.051	71.481 0.000
		11	-0.108	0.009	73.318 0.000
		12	0.065	-0.130	73.991 0.000
		13	0.108	0.136	75.835 0.000
		14	-0.102	-0.009	77.518 0.000
		15	0.048	-0.118	77.885 0.000
		16	0.030	-0.065	78.035 0.000
		17	-0.164	0.051	82.448 0.000
		18	0.115	0.036	84.659 0.000
		19	0.048	0.022	85.041 0.000
		20	-0.042	0.041	85.340 0.000
		21	0.028	-0.048	85.475 0.000
		22	0.040	0.069	85.749 0.000
		23	-0.176	-0.109	91.112 0.000
		24	-0.010	-0.107	91.130 0.000
		25	0.122	0.011	93.764 0.000
		26	-0.195	-0.135	100.52 0.000
		27	0.218	0.000	108.98 0.000
		28	-0.008	-0.030	108.99 0.000
		29	-0.092	0.018	110.52 0.000
		30	0.094	-0.095	112.14 0.000
		31	-0.036	0.072	112.39 0.000
		32	-0.085	-0.146	113.74 0.000
		33	0.119	0.117	116.39 0.000
		34	0.019	0.043	116.46 0.000
		35	-0.072	0.056	117.46 0.000
		36	0.123	0.034	120.38 0.000

**xhibit 5.8: Output of the SAC and SPAC for Y1, by using the  
Transformation  $ZZY1_t = Y1_t - Y1_{t-1} - Y1_{t-4} + Y1_{t-5}$**

Correlogram of  $ZZY1$ 

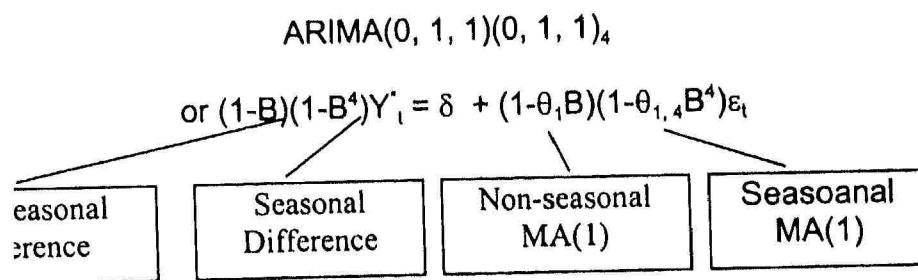
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
-0.395	-0.395	22.732	0.000		
0.016	-0.165	22.769	0.000		
0.216	0.192	29.659	0.000		
-0.509	-0.430	68.355	0.000		
0.146	-0.272	71.575	0.000		
0.017	-0.108	71.619	0.000		
0.014	0.191	71.650	0.000		
0.029	-0.187	71.778	0.000		
0.017	-0.156	71.825	0.000		
-0.007	-0.040	71.832	0.000		
-0.094	0.028	73.232	0.000		
0.043	-0.141	73.527	0.000		
0.122	0.145	75.896	0.000		
-0.128	-0.019	78.539	0.000		
0.049	-0.111	78.926	0.000		
0.049	-0.047	79.313	0.000		
-0.193	0.032	85.416	0.000		
0.153	0.028	89.317	0.000		
0.034	0.039	89.516	0.000		
-0.067	0.026	90.264	0.000		
0.051	-0.067	90.707	0.000		
0.018	0.067	90.764	0.000		
-0.161	-0.082	95.261	0.000		
-0.001	-0.138	95.261	0.000		
0.126	0.011	98.047	0.000		
-0.184	-0.115	104.05	0.000		
0.197	-0.011	111.01	0.000		
0.014	-0.025	111.05	0.000		
-0.106	0.018	113.10	0.000		
0.078	-0.094	114.21	0.000		
-0.009	0.063	114.23	0.000		
-0.122	-0.159	117.01	0.000		
0.123	0.105	119.88	0.000		
0.019	0.023	119.94	0.000		
-0.074	0.047	121.00	0.000		
0.130	0.029	124.28	0.000		

**hibit 5.9: Output of the SAC and SPAC for  $Y_{2,t}$  by using the Transformation  $ZZY_{2,t} = Y_{2,t} - Y_{2,t-1} - Y_{2,t-4} + Y_{2,t-5}$**

Correlogram of  $ZZY_{2,t}$

correlation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1	-0.396	-0.396	22.914 0.000
		2	0.032	-0.149	23.060 0.000
		3	0.180	0.165	27.883 0.000
		4	-0.507	-0.451	66.280 0.000
		5	0.156	-0.287	69.942 0.000
		6	0.002	-0.113	69.943 0.000
		7	0.034	0.168	70.123 0.000
		8	0.043	-0.181	70.410 0.000
		9	0.007	-0.159	70.419 0.000
		10	-0.002	-0.035	70.419 0.000
		11	-0.082	0.045	71.466 0.000
		12	0.015	-0.144	71.501 0.000
		13	0.137	0.149	74.507 0.000
		14	-0.142	-0.026	77.745 0.000
		15	0.049	-0.102	78.140 0.000
		16	0.062	-0.024	78.772 0.000
		17	-0.212	0.016	86.147 0.000
		18	0.179	0.019	91.439 0.000
		19	0.022	0.058	91.517 0.000
		20	-0.080	0.010	92.605 0.000
		21	0.068	-0.084	93.390 0.000
		22	-0.006	0.060	93.395 0.000
		23	-0.143	-0.058	96.915 0.000
		24	0.004	-0.166	96.918 0.000
		25	0.124	0.011	99.606 0.000
		26	-0.164	-0.103	104.36 0.000
		27	0.175	-0.028	109.85 0.000
		28	0.032	-0.015	110.04 0.000
		29	-0.114	0.022	112.40 0.000
		30	0.061	-0.088	113.08 0.000
		31	0.009	0.060	113.09 0.000
		32	-0.145	-0.152	117.00 0.000
		33	0.123	0.088	119.86 0.000
		34	0.012	-0.001	119.88 0.000
		35	-0.068	0.025	120.76 0.000
		36	0.129	0.025	123.98 0.000

erved Exhibit 5.7 to 5.9 show that a single peak in the first few lags ACF with a decay in the PACF suggest an MA(1) for the non-seasonal part. Similarly, the single peak in the seasonal part of the ACF with a bonding decay in the PACF suggest an MA(1) for the seasonal part. The study comes up with the tentative identification for all the three parts shown below:



$$Y_t^* = LY_t \text{ or } Y1_t \text{ or } Y2_t \quad \text{and} \quad \delta = \text{constant term}$$

### *? Estimation of Parameters and Diagnostic Checking*

Table 5.9, 5.10 and 5.11 shows the summary of criterion measures for choosing the best ARIMA model for  $LY_t$ ,  $Y1_t$ , and  $Y2_t$  respectively. Each table is supported by Diagnostic Checking measures based on Ljung-Box Q-statistic value. The table also includes columns for p-value and decision on whether the residuals follow a white noise model at 5% level of significance.

**9: ARIMA Models For LY<sub>t</sub> Series And Criterion  
Measures For Choosing The Best Model**

MODEL	m	Log(L)	AIC	Ljung-Box Q-Statistic (36 obs.)	p-value	White Noise Model (Y/N)
I(0,1,1) <sub>4</sub>	2	152.1536	-4.9239	34.4	0.461	Y
I(0,1,1) <sub>4</sub>	3	152.1561	-4.9099	34.1	0.410	Y
I(0,1,2) <sub>4</sub>	3	152.1781	-4.9103	34.1	0.380	Y
I(0,1,1) <sub>4</sub>	3	151.0589	-4.9091	34.8	0.380	Y
I(1,1,1) <sub>4</sub>	3	147.2669	-4.8993	38.3	0.242	Y
I(1,1,1) <sub>4</sub>	4	146.2363	-4.8848	38.7	0.194	Y
I(1,1,0) <sub>4</sub>	2	133.7862	-4.7333	78.6	0.000	N

Number of parameters in the estimation) = p+q+P+Q

Y: Yes  
N: No

Results in Table 5.9 suggest that the parsimonious ARIMA model for LY<sub>t</sub>

is IMA(0,1,1)(0,1,1)<sub>4</sub>

**10: ARIMA Models For Y1, Series And Criterion  
Measures For Choosing The Best Model**

MODEL	m	Log(L)	AIC	Ljung-Box Q-Statistic (36 obs.)	p-value	White Noise Model (Y/N)
$(0,1,1)_4$	2	175.4775	-5.2502	36.2	0.367	Y
$(0,1,1)_4$	3	175.4883	-5.2363	36.3	0.322	Y
$(0,1,2)_4$	3	175.6212	-5.2382	35.1	0.369	Y
$(0,1,1)_4$	3	174.8040	-5.2435	35.9	0.330	Y
$(1,1,1)_4$	3	158.8513	-5.0821	67.1	0.000	N
$(1,1,1)_4$	4	169.3165	-5.2193	39.6	0.168	Y
$(1,1,0)_4$	2	157.1039	-5.0713	83.1	0.000	N

Number of parameters in the estimation) = p+q+P+Q

Model at 5% Level of Significance

↓: No

Results in Table 5.10 suggest that the parsimonious ARIMA model for the series is ARIMA(0,1,1)(0,1,1)<sub>4</sub>

**11: ARIMA Models For Y<sub>2,t</sub>, Series And Criterion  
Measures For Choosing The Best Model**

MODEL	m	Log(L)	AIC	Ljung-Box Q-Statistic (36 obs.)	p-value	White Noise Model* (Y/N)
(0,1,1) <sub>4</sub>	2	-6.0473	-2.7113	36.3	0.362	Y
(0,1,1) <sub>4</sub>	3	-6.0227	-2.6977	36.2	0.323	Y
(0,1,2) <sub>4</sub>	3	-5.5005	-2.7030	37.4	0.275	Y
(0,1,1) <sub>4</sub>	3	-4.91846	-2.7123	38.7	0.226	Y
(1,1,1) <sub>4</sub>	3	-13.4706	-2.4998	62.6	0.001	N
(1,1,1) <sub>4</sub>	4	-4.7626	-2.69963	37.0	0.248	Y
(1,1,0) <sub>4</sub>	2	-15.4524	-2.5043	83.1	0.000	N

er of parameters in the estimation) = p+q+P+Q

d at 5% Level of Significance

↓: No

The criterion measures of choosing the best model in Table 5.11 that, ARIMA(0,1,1)(0,1,1)<sub>4</sub> Y<sub>2,t</sub> model has the highest likelihood value tallest AIC. However an increase in two additional parameters in this does justify the change in AIC as compared AIC of the ARIMA(0, 1, 1, 1)<sub>4</sub> model. The change is very small and therefore, ARIMA(0, 1, 1)<sub>4</sub> is also taken to be the best model for Y<sub>2,t</sub>.

### *ing and Forecasting*

The parsimonious estimated model for all the three series is  $\text{LY}_t \text{ : ARIMA}(0, 1, 1)(0, 1, 1)_4$ . Eviews output of these estimated models, together correlogram of the residuals is given in Table B9 to B11 and Exhibit B7 (Appendix B). The results of these estimated models is used and shown in Table 5.12. The constant term in the estimated f all three series is insignificant at 5 % level of significance.

#### **.12: Estimated Parameters For The Parsimonious Models Of $\text{LY}_t$ , $\text{Y1}_t$ And $\text{Y2}_t$ Series**

##### **$\text{LY}_t$ Series : ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub> Model**

Parameter	Coefficient	Std. Error	T-Statistic	Prob.
	-0.000793	0.000955	-0.830621	0.4076
	-0.461671	0.071385	-6.467342	0.0000
	-0.774250	0.052652	-14.70517	0.0000

##### **$\text{Y1}_t$ Series : ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub> Model**

Parameter	Coefficient	Std. Error	T-Statistic	Prob.
	0.000729	0.000771	0.944626	0.3465
	-0.473520	0.069396	-6.823477	0.0000
	-0.784360	0.050182	-15.63035	0.0000

##### **$\text{Y2}_t$ Series : ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub> Model**

Parameter	Coefficient	Std. Error	T-Statistic	Prob.
	0.002904	0.002794	1.039399	0.3004
	-0.487696	0.068007	-7.171214	0.0000
	-0.772290	0.049680	-15.54537	0.0000

odel for the three transformed series that will be fitted and used to  
e forecast values are:

1. Series: Logarithmic Transformation  $LY_t = \log(Y_t)$

$$(1 - B)(1 - B^4) LY_t = (1 + 0.461671B)(1 + 0.774250B^4)\varepsilon_t$$

2. Forecast Range (SFR): Model is Fitted

Table B12 (Appendix) shows the forecast values in sample forecast  
Figure 5.10 shows the plot of actual yield series ( $LY_t$ ) and forecast  
eries ( $FLY_t$ ). The root mean sum square error and mean absolute  
tage error from this fitted model is:  $RMSE_{psfr} = 0.083$ ,  $MAPE_{psfr} = 7.2$ .

3. Sample Forecast Range (PSFR): Forecasting

Table 5.13 shows the actual ( $LY_t$ ) and forecast values( $FLY_t$ ) in post  
e forecast range. Figure 5.11 shows the plot of  $LY_t$  and  $FLY_t$  from  
Q1 to 1997:Q4. The root mean sum square error and mean absolute  
tage error from this fitted model is:  $RMSE_{sfr} = 0.067$ ,  $MAPE_{sfr} = 4.2$ .

4. Series: Power Transformation  $Y1_t = Y_t^{(-0.5)}$

$$(1 - B)(1 - B^4) Y1_t = (1 + 0.473520B)(1 + 0.784360 B^4)\varepsilon_t$$

5. Forecast Range (SFR) : Model is Fitted

Table B13 (Appendix) shows the forecast values in sample forecast  
3. Figure 5.12 shows the plot of actual yield series ( $Y1_t$ ) and forecast  
eries ( $FY1_t$ ). The root mean sum squared error and mean absolute  
tage error from this fitted model is:  $RMSE_{psfr} = 0.071$ ,  $MAPE_{psfr} = 3.3$

### Sample Forecast Range (PSFR): Forecasting

Table 5.14 shows the actual ( $Y_{1,t}$ ) and forecast values ( $FY_{1,t}$ ) in post forecast range. Figure 5.13 shows the plot of  $Y_{1,t}$  and  $FY_{1,t}$  from 1991 to 1997:Q4. The root mean sum squared error and mean absolute percentage error from this fitted model is:  $RMSE_{sfr} = 0.048$ ,  $MAPE_{sfr} = 2.7$ .

### Series: Power Transformation $Y_{2,t} = Y_{1,t}^{(-1)}$

$$(1 - B^4) Y_{2,t} = (1 + 0.487696 B)(1 + 0.772290 B^4) \epsilon_t$$

The constant is insignificant to be included in the model.

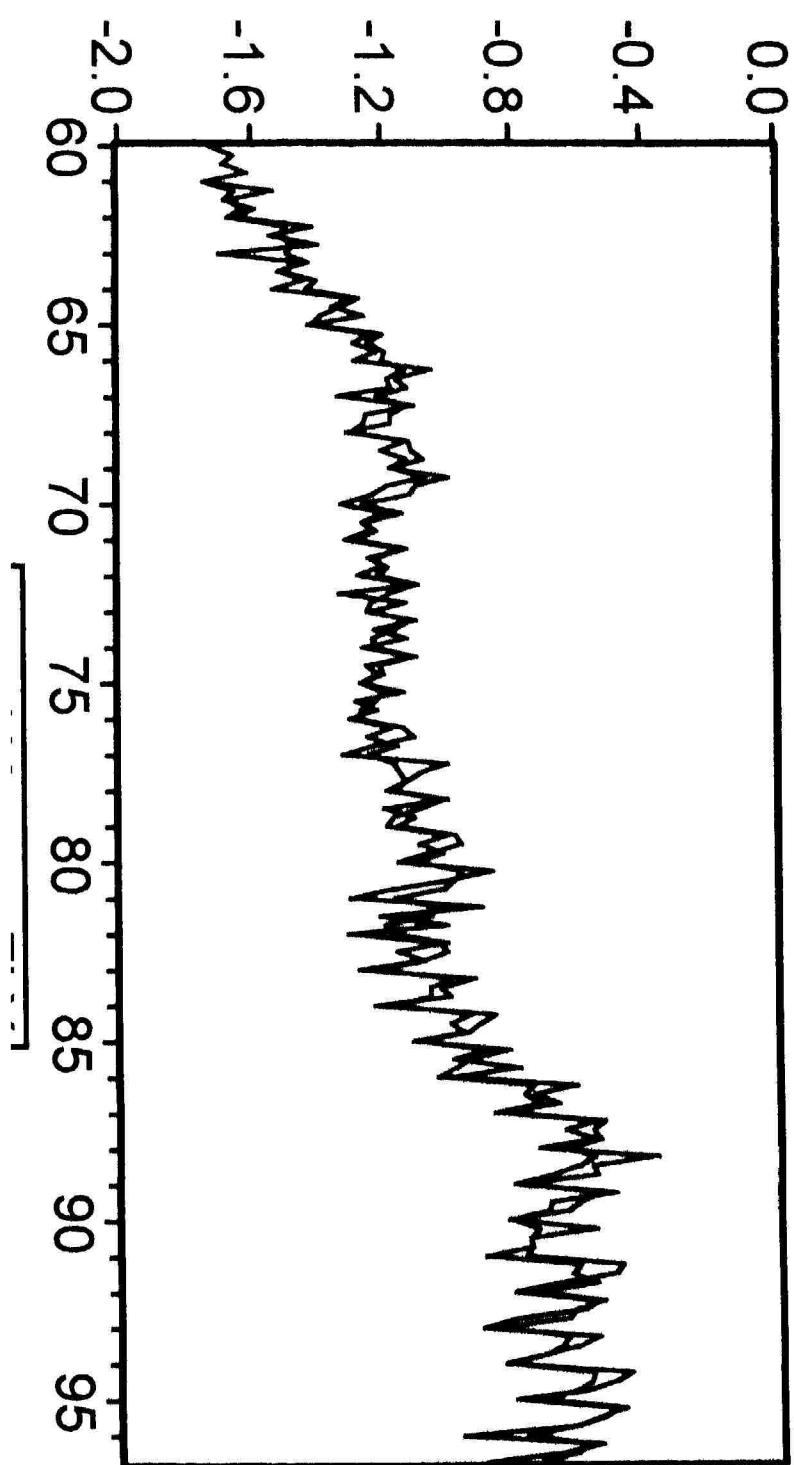
### Forecast Range (SFR): Model is Fitted

Table B14 shows the forecast values in sample forecast range. Figure 5.14 shows the plot of actual yield series ( $Y_{2,t}$ ) and forecast yield series ( $FY_{2,t}$ ). The root mean sum squared error and mean absolute percentage error from this fitted model is:  $RMSE_{perfr} = 0.254$ ,  $MAPE_{perfr} = 6.9$ .

### Sample Forecast Range (PSFR): Forecasting

Table 5.15 shows the actual ( $Y_{2,t}$ ) and forecast values ( $FY_{2,t}$ ) in post forecast range. Figure 5.15 shows the plot of  $Y_{2,t}$  and  $FY_{2,t}$  from 1991 to 1997:Q4. The root mean sum squared error and mean absolute percentage error from this fitted model is:  $RMSE_{sfr} = 0.144$ ,  $MAPE_{sfr} = 4.9$ .

Figure 5.10: Plot of Yield Series ( $LY_t$ ) and Forecast Series ( $FLY_t$ ) by  $(1 - B)(1 - B^4) LY_t = (1 + 0.461671B)(1 + 0.774250B^4)\epsilon_t$



3: Actual ( $LY_t$ ) And Forecast ( $FLY_t$ ) From 1997:Q1 To 1997:Q4  
By ARIMA (0,1,1)(0,1,1)<sub>4</sub>

	Q1	Q2	Q3	Q4
97	-0.890647	-0.639349	-0.747696	-0.898660
97	-1.002820	-0.615724	-0.702357	-0.847943

.11: Plot Of  $LY_t$  And  $FLY_t$  From 1997:Q1 To 1997:Q4 By  
ARIMA (0,1,1)(0,1,1)<sub>4</sub>  $LY_t$

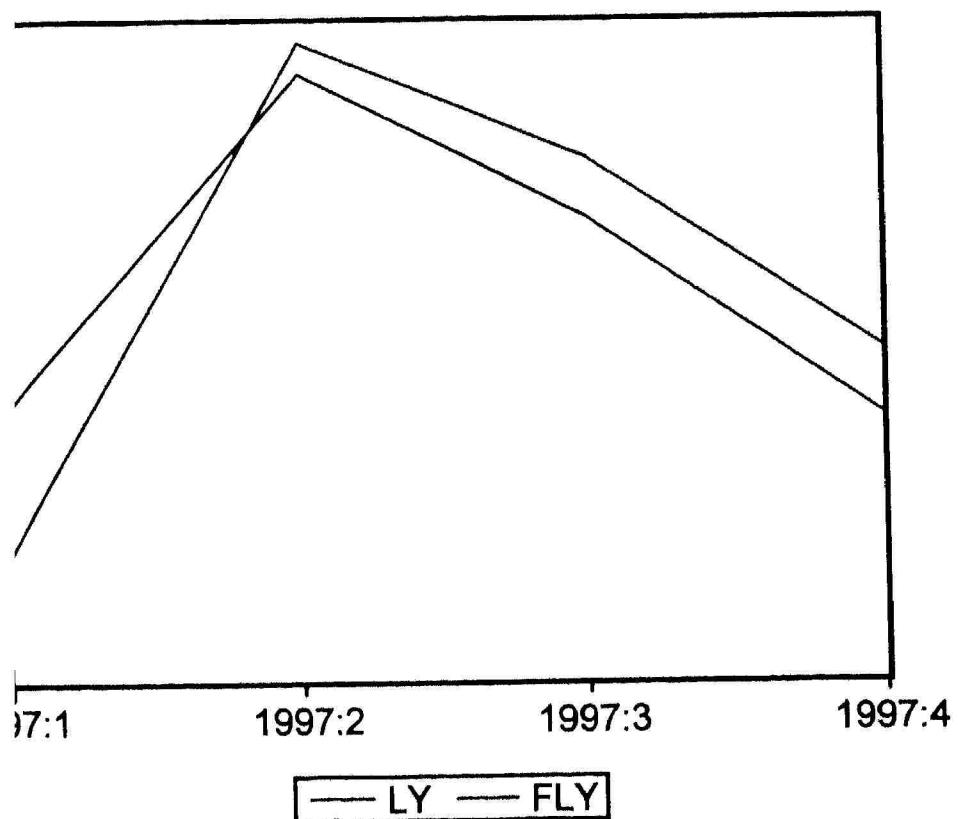
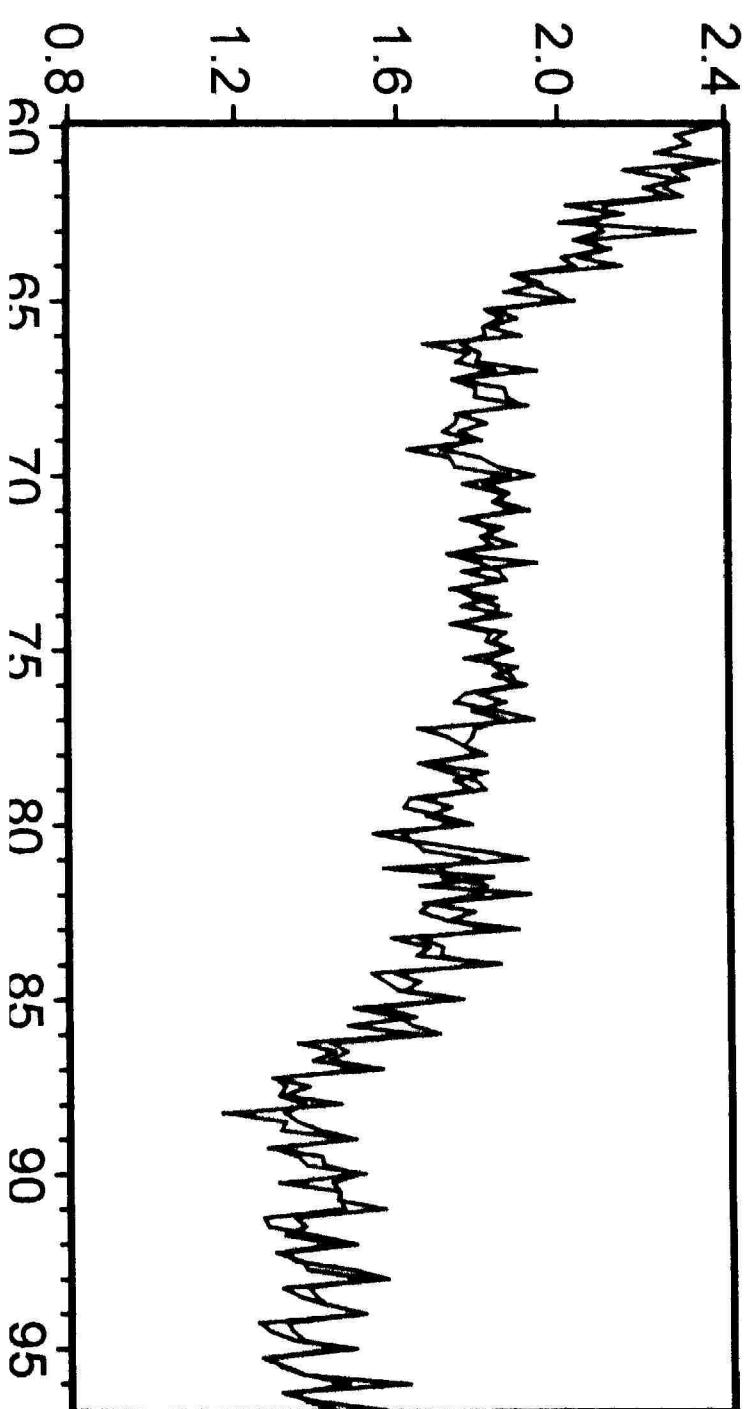


Figure 5.12: Plot of Yield Series ( $Y_{1,t}$ ) and Forecast Series ( $FY_{1,t}$ ) by  $(1 - B)(1 - B^4) Y_{1,t} = (1 + 0.473520B)(1 + 0.784360 B^4)\epsilon_t$



: Actual ( $Y_{1,t}$ ) And Forecast ( $FY_{1,t}$ ) From 1997:Q1 To 1997:Q4  
By ARIMA (0,1,1)(0,1,1)<sub>4</sub>

	<b>Q1</b>	<b>Q2</b>	<b>Q3</b>	<b>Q4</b>
7	1.560995	1.376679	1.453316	1.567261
7	1.640770	1.367804	1.423720	1.522993

Figure 5.13: Plot Of  $Y_{1,t}$  And  $FY_{1,t}$  From 1997:Q1 To 1997:Q4  
By ARIMA (0,1,1)(0,1,1)<sub>4</sub>  $Y_{1,t}$

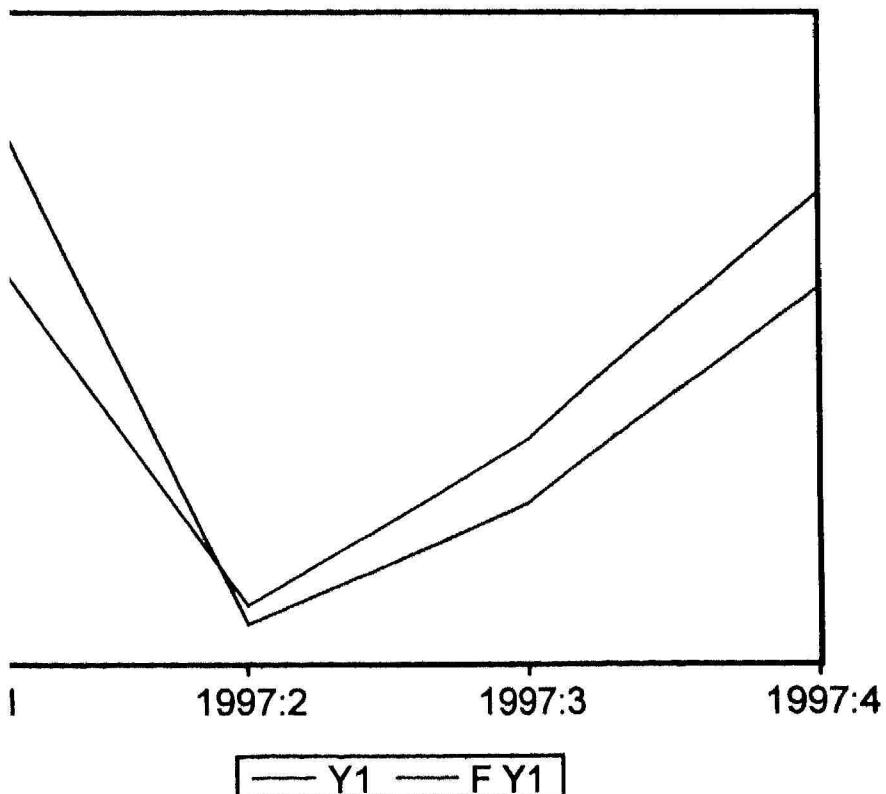
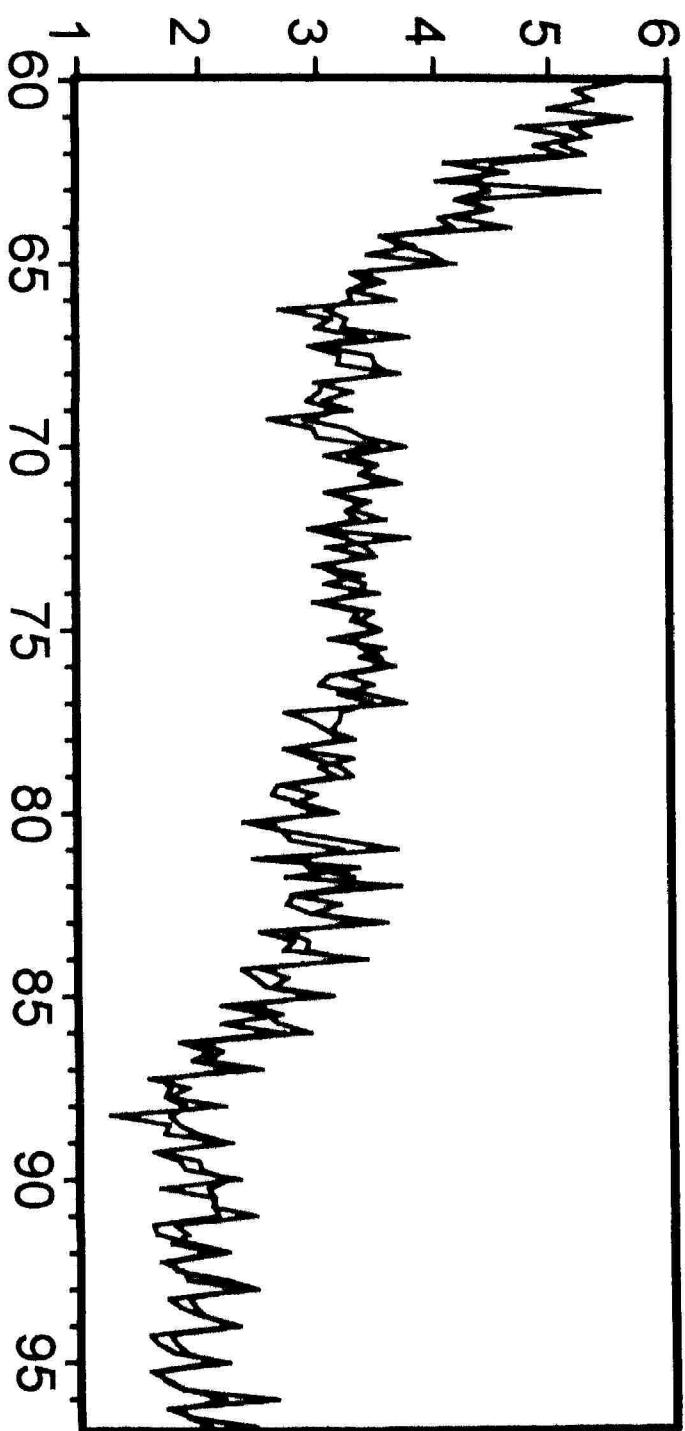


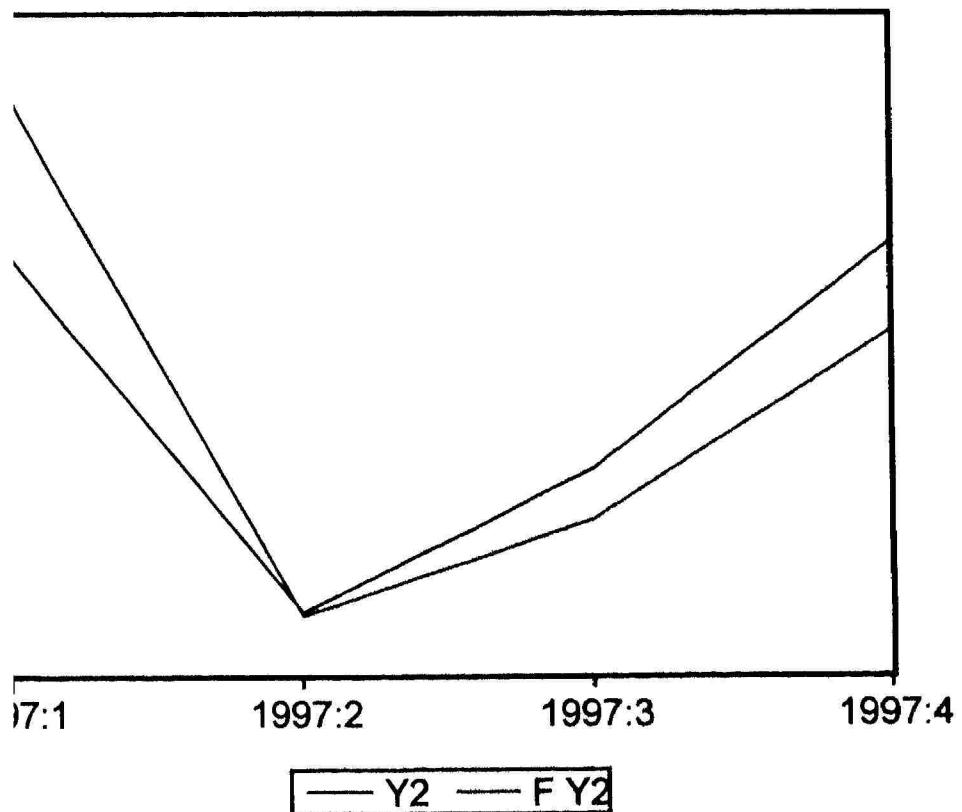
Figure 5.14: Plot of Yield Series ( $Y_{2t}$ ) and Forecast Series ( $FY_{2t}$ ) by  $(1 - B)(1 - B^4) Y_{2t} = (1 + 0.487696 B)(1 + 0.772290 B^4) \epsilon_t$



: Actual ( $Y_{2t}$ ) And Forecast ( $FY_{2t}$ ) From 1997:Q1 To 1997:Q4  
By ARIMA (0,1,1)(0,1,1)<sub>4</sub>

	Q1	Q2	Q3	Q4
1997:1	2.436707	1.895246	2.112129	2.456308
1997:2	2.679005	1.889669	2.034875	2.321050

figure 5.15: Plot Of  $Y_{2t}$  And  $FY_{2t}$  From 1997:Q1 To 1997:Q4  
By ARIMA (0,1,1)(0,1,1)<sub>4</sub>  $Y_{2t}$



### 5.2.3 Forecast Yield by ARIMA Models

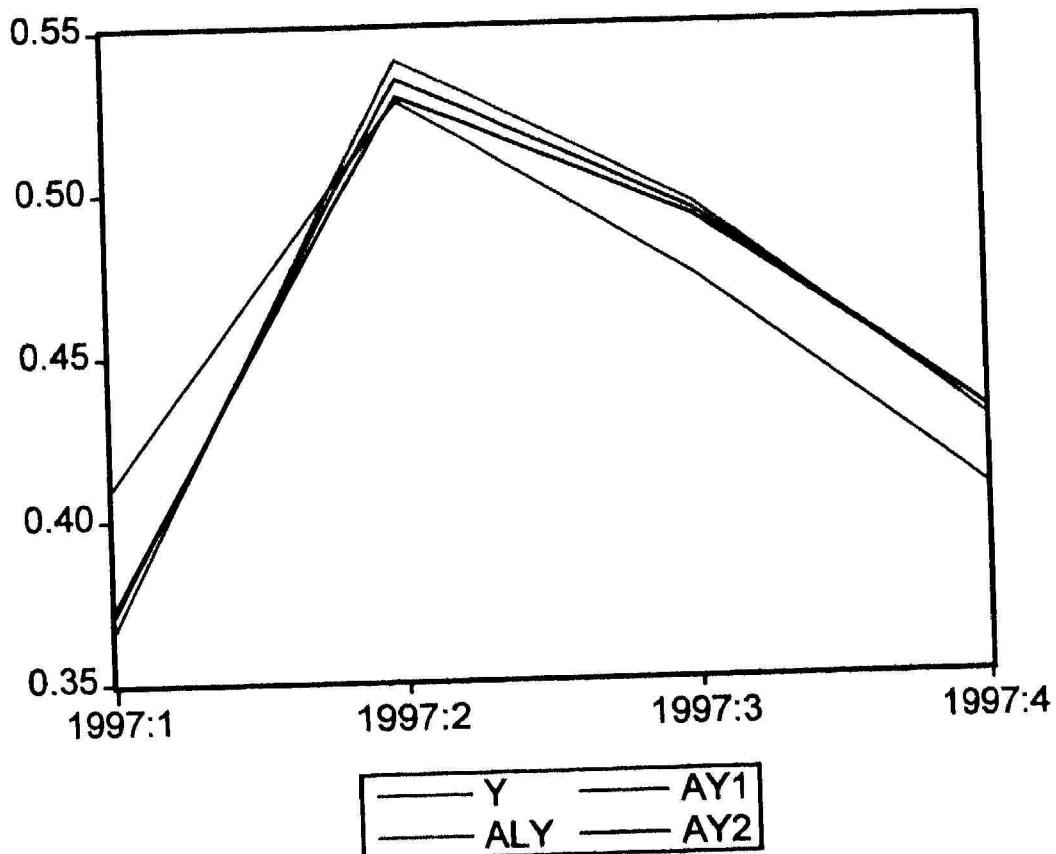
Table 5.16 shows the comparison of actual yield and the forecast yield obtained by Box-Jenkins methodology in the post sample forecast range. The forecasts obtained for transformed series ( $LY_t$ ,  $Y1_t$  and  $Y2_t$ ) are converted back to the original. Figure 5.16 shows the plot of actual yield and forecast yield by ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub> on the power-transformed series,  $LY_t$ ,  $Y1_t$  and  $Y2_t$ .

**Table 5.16: Actual Yield And Forecasts Generated By ARIMA Models In The Post Sample Forecast Range**

ARIMA Model	Q1	Q2	Q3	Q4
Actual	0.410390	0.527636	0.473456	0.407115
(0, 1, 1)(0, 1, 1) <sub>4</sub> $LY_t$	0.366843	0.540249	0.495416	0.428295
(0, 1, 1)(0, 1, 1) <sub>4</sub> $Y1_t$	0.371454	0.534506	0.493345	0.431126
(0, 1, 1)(0, 1, 1) <sub>4</sub> $Y2_t$	0.373273	0.529193	0.491431	0.430839

The forecasted yield obtained from all models for first, third and fourth quarter is under forecasting, whereas, for second quarter, the models are over forecasting. The forecast yield exhibit the same seasonal pattern as the actual yield.

**Figure 5.16: Plot Of Actual Yield And Forecast Yield Generated By ARIMA Models**



ALY: ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub>LY; Forecast yield  
 AY1: ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub>Y1; Forecast yield  
 AY2: ARIMA(0, 1, 1)(0, 1, 1)<sub>4</sub>Y2; Forecast yield

#### **5.2.4 Forecast Performance in the Post Sample Forecast Range : ARIMA Models**

In order to compare the forecasting performance, the transformed forecast yield are brought back to original scale. Table 5.17 shows the root mean sum square error and mean absolute percentage error of the fitted models. These performance measures indicate that forecast values

generated from these fitted models do not differ greatly. ARIMA (0, 1, 1)(0, 1, 1)<sub>4</sub> Y<sub>2,t</sub> give the best performance.

**Table 5.17: Forecast Performance In Post Sample Forecast Range: ARIMA Models**

ARIMA Model	RMSE	MAPE
(0, 1, 1)(0, 1, 1) <sub>4</sub> LY <sub>t</sub>	<b>0.0273</b>	<b>5.7</b>
(0, 1, 1)(0, 1, 1) <sub>4</sub> Y <sub>1,t</sub>	<b>0.0252</b>	<b>5.2</b>
(0, 1, 1)(0, 1, 1) <sub>4</sub> Y <sub>2,t</sub>	<b>0.0238</b>	<b>4.7</b>

### 5.3 Summary

The models developed to generate forecasts for the yield series are:

#### Exponential Smoothing Method

Holt-Winters' Multiplicative Seasonality(HWMS) on Y<sub>t</sub>  
using  $\alpha = 0.46$ ;  $\gamma = 0$ ;  $\delta = 0.39$

Holt-Winters' Additive Seasonality(HWAS) on LY<sub>t</sub>  
using  $\alpha = 0.45$ ;  $\gamma = 0$ ;  $\delta = 0.37$

Holt-Winters' Additive Seasonality(HWAS) on Y<sub>1,t</sub>  
using  $\alpha = 0.44$ ;  $\gamma = 0$ ;  $\delta = 0.23$

Holt-Winters' Additive Seasonality(HWAS) on Y<sub>2,t</sub>  
using  $\alpha = 0.47$ ;  $\gamma = 0$ ;  $\delta = 0$

#### Box-Jenkins Methodology

**LY<sub>t</sub> Series : Logarithmic Transformation LY<sub>t</sub> = Log(Y<sub>t</sub>)**

$$(1 - B)(1 - B^4) LY_t = (1 + 0.461671B)(1 + 0.774250B^4)\epsilon_t$$

**Y<sub>1,t</sub> Series : Power Transformation Y<sub>1,t</sub> = Y<sub>t</sub><sup>(-0.5)</sup>**

$$(1 - B)(1 - B^4) Y_{1,t} = (1 + 0.473520B)(1 + 0.784360 B^4)\epsilon_t$$

**Y<sub>2t</sub> Series : Power Transformation  $Y_{2t} = Y_t^{(-1)}$**

$$(1 - B)(1 - B^4) Y_{2t} = (1 + 0.487696 B)(1 + 0.772290 B^4)\varepsilon_t$$

where the constant is insignificant to be included in the model.

Based on the plot fit and forecast performance, Holt-Winters Additive Seasonality method fitted to  $Y_{1t}$ , ARIMA(0,1,1)(0,1,1)<sub>4</sub>Y<sub>1t</sub> and ARIMA(0,1,1)(0,1,1)<sub>4</sub>Y<sub>2t</sub> are the appropriate models to forecast quarterly yield of tea in Peninsular Malaysia. Holt-Winters Multiplicative Seasonality method fitted to  $Y_t$  (original series) would be also appropriate if data transformed is not considered.