EXTENSIONS OF RESIDUALLY FINITE GROUPS

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ABSTRACT

This thesis is about some extensions of residual finiteness on generalised free products, tree products and HNN extensions. These extensions include residual p-finiteness, weak potency and cyclic subgroup separability.

We begin the thesis by proving sufficient conditions for the generalised free products and tree products of residually p-finite groups and weakly potent groups to be residually p-finite and weakly potent respectively. Our results are then applied to generalised free products and tree products of polycyclic-by-finite groups and free-by-finite groups. Then we give characterisations for certain one-relator groups to be residually p-finite and weakly potent.

Next we study the weak potency and cyclic subgroup separability of HNN extensions. We show that certain HNN extensions of weakly potent groups and cyclic subgroup separable are again weakly potent and cyclic subgroup separable respectively. These results are applied to HNN extensions of polycyclic-by-finite groups and free-by-finite groups. Then we give a characterisation for the one-relator groups $\langle h, t; t^{-1}h^{\gamma}t = h^{\delta}\rangle$ to be weakly potent and cyclic subgroup separable. Finally we show that all one-relator groups with non-trivial centre are weakly potent and cyclic subgroup separable.

ABSTRAK

Tesis ini adalah berkenaan beberapa perluasan sifat terhingga tersisa terhadap berbagai perluasan kumpulan seperti hasil darab bebas teritlak, hasil darab pokok dan perluasan HNN. Perluasan-perluasan sifat tersisa terhingga tersebut termasuk sifat tersisa terhingga secara p, sifat poten lemah dan sifat terpisahkan secara subkumpulan kitaran.

Kita mulakan tesis ini dengan membuktikan syarat-syarat cukup bagi hasil darab bebas teritlak kumpulan bersifat tersisa terhingga secara p dan sifat poten lemah supaya masing-masing mempunyai sifat tersisa terhingga secara p dan sifat potent lemah. Seterusnya kita menggunakan keputusan ini ke atas hasil darab bebas teritlak dan hasil darab pokok bagi kumpulan-kumpulan polycyclic-by-finite dan free-by-finite. Kemudian kita menentukan sifat-sifat tertentu bagi pelbagai kumpulan satu penghubung supaya bersifat tersisa terhingga secara p dan sifat poten lemah.

Selanjutnya, kita mengaji tentang sifat potent lemah dan sifat terpisahkan secara subkumpulan kitaran bagi perluasan HNN. Kita buktikan bahawa perluasan HNN bagi kumpulan tertentu yang bersifat potent lemah dan terpisahkan secara subkumpulan kitaran juga masing-masing adalah bersifat potent lemah dan terpisahkan secara subkumpulan kitaran. Keputusan ini digunakan ke atas perluasan HNN bagi kumpulan-kumpulan polycyclic-by-finite dan free-by-finite. Kemudian, kita menentukan sifat-sifat tertentu bagi kumpulan $\langle h, t; t^{-1}h^{\gamma}t = h^{\delta}\rangle$ supaya ia berpoten lemah dan terpisahkan secara subkumpulan. Akhir sekali, kita tunjukkan bahawa semua kumpulan satu penghubung dengan pusat tak remeh adalah berpoten lemah dan terpisahkan secara subkumpulan.