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**DIFFERENTIAL GEOMETRY OF TANGENT BUNDLES
OF RIEMANNIAN MANIFOLDS**

by

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ABSTRACT

A survey of the geometry on the tangent bundles of Riemannian manifolds is conducted in the first two chapters. Specifically, the vertical lifts, the complete lifts and the horizontal lifts of tensor fields are considered. The nullity distribution on the tangent bundle of a Riemannian manifold is studied. The vertical and complete lifts of a vector field belonging to the nullity distribution on the base manifold are shown to be in the nullity distribution on the tangent bundle. Under certain regularity condition, the dimension of the nullity distribution on the tangent bundle of a Riemannian manifold is found to be twice that of the nullity distribution on base manifold. Geodesics on the tangent bundle with the Sasaki metric as its Riemannian metric are studied. The relation between the tangent vector field of a geodesic and the tangent vector field of the liftings of the curve is obtained.

ABSTRAK

Tinjauan bagi berkas tangen manifold Riemannian diberi dalam bab pertama dan bab kedua. Khususnya, lif cancangan, lif lengkap dan lif ufukan bagi medan tensor dipertimbangkan. Taburan kenolan pada berkas tangen manifold Riemannian dikaji. Lif cancangan dan lengkap suatu medan vektor dalam taburan kenolan pada manifold asas didapati berada dalam taburan kenolan pada berkas tangennya. Dengan syarat sekata tertentu, dimensi bagi taburan kenolan pada berkas tangen suatu manifold Riemannian didapati adalah dua kali ganda dimensi bagi taburan kenolan manifold asas. Geodesik pada berkas tangen dengan metrik Sasaki sebagai metrik Riemannannya dikaji. Hubungan diantara medan vektor tangen suatu geodesik dan medan vector tangen lif lengkungan itu diperolehi.

INTRODUCTION

The main purpose of this thesis is to consider some aspects of the geometry of the tangent bundle of a Riemannian manifold.

The first chapter consists of some definitions and basic concepts of manifold theory.

We review some basic tools used in the study of the tangent bundle in the second chapter. In particular, we will study the vertical lifts, complete lifts and horizontal lifts of tensor fields. Our main reference is the book *Tangent and Cotangent Bundles* by Kentaro Yano and Shigeru Ishihara. We give the proofs of some of the theorems which are not provided in the reference.

In the third chapter, we shall discuss the nullity distribution on an affine manifold. The concept of nullity distribution was first introduced by S.S. Chern and N. Kuiper in their paper, *Some Theorems on the Isometric Imbedding of Compact Riemannian Manifolds in Euclidean Spaces*. Nullity distribution on an affine manifold was studied by Sin-Leng Tan in his paper, *On Nullity Distributions*. We consider here the nullity distribution on the tangent bundle of a Riemannian manifold and look at the relationship between the nullity distributions on the base manifold and its tangent bundle. The vertical and complete lifts of a vector field belonging to the nullity distribution on an affine manifold can be shown to belong to the nullity distribution on its tangent bundle. With a regularity assumption, the dimension of the nullity distribution on the tangent bundle is twice that of the nullity distribution on the base manifold.

We will next look at the Sasaki metric \tilde{g}_* , which can be considered as a natural Riemannian metric on the tangent bundle of a Riemannian manifold. We consider the adapted frame of the tangent bundle and study the geodesics on (TM, \tilde{g}_*) with respect to the adapted frame in the final chapter. In particular, we can relate the liftings of the tangent vector field of a geodesic with the tangent vector field of the liftings of the curve.

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