Chapter 3

Wavefront Reconstruction and Digital Holography

3.1 Wavefront Reconstruction

In this section the mathematical description of wavefront reconstruction is presented. The wavefront reconstruction of holographically recorded optical wave involved diffraction. In both holography and holographic interferometry, laser light is used due to its highly monochromatic and coherent properties. Because of these nearly ideal properties, the diffraction of laser light can be described adequately in terms of elementary diffraction theory.

The concept of intensity of a wave field is introduced, and the Fresnel-Kirchhoff diffraction integral, from which the numerical wavefront reconstruction is derived, is presented.

3.1.1 Intensity of a wave field

Holography is a technique for recording and reconstructing light wave. The wave that is to be recorded is the object wave. In order to reconstruct, it is sufficient to reproduce its complex amplitude $\Psi_o$ at one plane in space. Once this has been
reproduced, the light propagating away from this plane will be identical to the original object wave. The distribution of both real amplitude and phase in the plane must be recorded. However, practical photodetectors such as photographic film, photodiodes, CCD camera, or any other photosensitive sensors respond only to the intensity $I$ of the light falling on its surface. Thus, it is important to relate such intensity $I$ to the complex amplitude $\Psi_o$ of the object wave.

The intensity $I$ of a scalar monochromatic wave at point $(x,y)$ is given by the squared magnitude of the complex amplitude $\Psi_o(x,y)$ of the disturbance [14],

$$I(x,y) = |\Psi_o(x,y)|^2 \quad (3.1)$$

When calculating a diffraction pattern, the intensity of the pattern is the quantity we are seeking.

### 3.1.2 The Fresnel-Kirchhoff diffraction integral

Consider an amplitude transmittance $t(x,y)$, which is placed in the plane $z = 0$ as shown in Figure 3-1. The transmittance is illuminated by a plane wave $\Psi_i(x,y) = a \exp(ik \cdot r)$ propagating parallel to z-axis.

The wave field at the point $P_o$ that is at a distance $d$ away from the transmittance plane, is given by integration over all spherical waves emitted from the $(x,y,0)$ plane, and can be described by the Fresnel-Kirchhoff diffraction integral as follow [16]:

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\[ \Psi(\xi, \eta, d) = \frac{A}{i\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t(x, y) \frac{\exp(ikr)}{r} \cos \Theta \, dx \, dy \] (3.2)

with

\[ r = \sqrt{d^2 + (\xi - x)^2 + (\eta - y)^2} \] (3.3)

where \((x, y)\) and \((\xi, \eta)\) are the coordinates in the hologram plane and in the plane of the real image respectively as illustrated in Figure 3-1; \(t(x, y)\) is the amplitude transmittance of the hologram, \(A\) is the amplitude of the incident wave, \(k\) is the propagation constant (= \(2\pi/\lambda\)) and \(\cos \Theta\) is the obliquity factor.

In order to simplify the evaluation of this integral, a few assumptions have to be made. This approximation will be discussed in the next section.

Figure 3-1 Plane wave illumination of a transmittance.
3.2 Classification of Holograms

Holograms can be classified according to the way of forming the object and the reference waves and also according to the way of recording the interference pattern [17]. Depending on the arrangement of the object and the hologram, and also on the presence of optical elements between them, the relationship between the amplitude distribution in the plane of the hologram and the corresponding distribution directly after the object can be described by the Fresnel-Kirchhoff diffraction integral [Equation (3.2)].

In order to simplify the evaluation of this integral, a few assumptions have to be made. First assumption is that the distance $d$ between the object and hologram plane is very much greater than the maximum linear dimension of the object plane and hologram plane. With this paraxial assumption, the obliquity factor in the Equation (3.2) is readily approximated by $\cos \Theta \equiv 1$. Under similar condition, the denominator quantity in the Equation (3.2) will not differ significantly from $d$ and can be replaced by $d$. A convenient means of approximation is offered by a binomial expansion of the square root

$$\sqrt{1+b} = 1 + \frac{1}{2}b - \frac{1}{8}b^2 + \ldots \quad |b| < 1$$

(3.4)

To apply the binomial expansion to the problem at hand, factor $d$ outside the expression for $r$ in the Equation (3.3), yielding

$$r = d\sqrt{1 + \left(\frac{\xi - x}{d}\right)^2 + \left(\frac{\eta - y}{d}\right)^2}$$

(3.5)
3.2.1 Fresnel hologram

The most general kind of hologram is the Fresnel hologram as show in Figure 3-2. It is formed when a hologram is in the near-field diffraction region. Thus, the square root in Equation (3.5) is adequately approximated by the first two terms of its expansion,

\[ r \equiv d \left[ 1 + \frac{1}{d} \left( \frac{\xi - x}{d} \right)^2 + \frac{1}{2} \left( \frac{\eta - y}{d} \right)^2 \right] \]  \hspace{1cm} (3.6)

This assumption is referred to as the Fresnel approximation [14,16,17].

![Diagram of Fresnel hologram](image)

Figure 3-2 Recording a Fresnel hologram.
For the $r^2$ term appearing in the denominator of Equation (3.2), the error introduced by dropping all terms but $d$ is generally acceptably small. However, for the quantity in the argument of the exponent, the error is much more critical. Firstly, they are multiplied by a very large wave number $k$. A typical value for which might be greater than $10^7$ in the visible region of the spectrum (e.g. $\lambda = 5 \times 10^{-7} m$). Secondly, phase changes of as little as a fraction of a radian can change the value of the exponent significantly. For this reason, both terms of the binomial approximation are retained in the exponent. The resulting expression for the Fresnel-Kirchhoff diffraction integral now becomes

$$
\Psi(\xi, \eta) = \frac{e^{ikd}}{i\lambda d} \exp \left[ \frac{i\pi}{\lambda d} \left( \xi^2 + \eta^2 \right) \right] \\
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t(x, y) \exp \left[ \frac{i\pi}{\lambda d} \left( x^2 + y^2 \right) \right] \exp \left[ -i2\pi \left( \frac{\xi}{\lambda d} x + \frac{\eta}{\lambda d} y \right) \right] dx dy
$$

Equation (3.7) is recognized, aside from multiplication of amplitude and phase factors that are independent of $(x, y)$, to be the Fourier transform of the product of the complex field just to the right of the aperture and a quadratic phase exponential, $t(x, y) \exp \left[ \frac{i\pi}{\lambda d} \left( x^2 + y^2 \right) \right]$. It is referred to as the Fresnel transformation (FRT).

When the approximation is valid, the observer is said to be in the region of Fresnel diffraction, or equivalently in the near field of the aperture.
3.2.2 Fraunhofer hologram

When a hologram is at an infinite distance from the object, i.e. in the far field
diffraction region, it is called a Fraunhofer hologram. In this case, each point of the
object sends a parallel light beam to the hologram and the relationship between the
amplitude-phase distribution of the object wave in the plane of the hologram and in
the plane of the object is given by a Fourier transformation. To obtain such a
hologram, the object either has to be small compared to the dimensions of the
holographic arrangement, or in the focus of the lens, as shown in Figure 3-3, or the
object must be sufficiently far from the photographic plate, that is [14]

\[ d >> \frac{x^2 + y^2}{\lambda} \]  

(3.8)

![Diagram of Fraunhofer hologram]

Figure 3-3 Recording a Fraunhofer hologram.
Thus, the Equation (3.7) can be written as

$$\Psi(\xi, \eta) = \frac{e^{ikd}}{i\lambda d} \exp\left[i\pi \left(\frac{\xi^2}{\lambda d} + \frac{\eta^2}{\lambda d}\right)\right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{t}(x, y) \exp\left[-i2\pi \left(\frac{x}{\lambda d} + \frac{\eta}{\lambda d}\right)\right] dx dy$$

$$\quad = \frac{e^{ikd}}{i\lambda d} \exp\left[i\pi \left(\frac{\xi^2}{\lambda d} + \frac{\eta^2}{\lambda d}\right)\right] \mathfrak{F}\{\mathbf{t}(x, y)\}$$

(3.9)

where $\mathfrak{F}$ is the Fourier transformation. If we multiply Equation (3.9) by its complex conjugate, we find that the intensity in the far field of $\mathbf{t}(x, y)$ is exactly the square of the absolute value of the Fourier transform of $\mathbf{t}(x, y)$.

3.2.3 Image hologram

If the object is focused into the plane of the hologram, the amplitude-phase distribution on the hologram will be the same as in the plane of the object (Figure 3-4). Here the real image of the object is recorded instead of the wave field reflected or scattered by the object. The corresponding hologram is called an image hologram.

Figure 3-4  Recording an image hologram.
3.2.4 Fourier hologram

If the object and the reference source are arranged in the focal plane of the lens as show in Figure 3-5, a Fourier hologram will be produced. If the complex amplitude leaving the object plane is \( \psi_o(x, y) \), then the complex amplitude of the wave field at the holographic plate located in the back focal plane of the lens is the Fourier transform of \( \psi_o(x, y) \)

\[
\psi_o(\xi, \eta) = \mathcal{F}\{\psi_o(x, y)\} \tag{3.10}
\]

where \( \mathcal{F} \) is Fourier transformation.

![Figure 3-5 Typical arrangement used to record a Fourier hologram.](image)
3.2.5 Lensless Fourier hologram

Lensless Fourier transform hologram is possible if the object point and the reference source are at a finite but the same distance from the holographic plate, as shown in Figure 3-6.

Figure 3-6 Recording of a lensless Fourier transform hologram.
3.3 Digital Holography

As mentioned in Section 3.2, Fresnel holography is the most general kind of holography methods. It includes Fraunhofer holography as a special case with infinite distance \( d \). Therefore, in this work a Fresnel hologram is generated on CCD-sensor, which acts as the recording medium allowing digitization and storage in a digital image processing system memory. In reconstruction of the real image, the digitized holograms are processed numerically. This process involved numerical realization of the Fresnel-Kirchhoff diffraction integral.

In this section, after a short description on the requirements of the directly recording of Fresnel hologram on CCD-sensor, a detailed description on the numerical reconstruction based on the Fresnel-Kirchhoff diffraction integral is discussed. For the numerical evaluation the discrete representation of the Fresnel-Kirchhoff diffraction integral is used to calculate the complex amplitude of the diffracted wave in the real image plane. After the description of the digitization of the Fresnel-Kirchhoff diffraction integral, the numerical algorithm of reconstruction based on this discrete representation equation is discussed.

3.3.1 Digital recording on the CCD-sensor

The photosensitive material for recording holograms must resolve the complicated intensity distribution resulting from the interference between the waves scattered from all object points and the reference wave. The maximum spatial frequency \( f_{\text{max}} \), which has to be resolved, is determined by the maximum angle, \( \theta_{\text{max}} \), between these waves [18]:

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\[ f_{\text{max}} = \frac{2}{\lambda} \sin \left( \frac{\theta_{\text{max}}}{2} \right) \]  

(3.11)

where \( \lambda \) denotes the wavelength.

In this work CCD camera is used as a holographic recording medium. The maximum spatial frequency, which can be recorded using a CCD camera, is limited by the pixel size, \( \Delta x \). To record a hologram of the entire object, the resolution of the camera must be sufficient to record the fringes formed by the reference wave and the wave from the object point farther from the reference point. Since for the recording of a hologram it is necessary to have at least two sampled points for each fringe, the maximum spatial frequency that can be recorded is given by [18]:

\[ f_{\text{max}} = \frac{1}{2\Delta x} \]  

(3.12)

This means that the interference fringes obtained between the light coming from the object and the reference should have a period greater than \( 2\Delta x \). If the distance between neighboring pixels for the CCD is \( 10\mu m \), then according to Equation (3.12), the maximum spatial frequency that can be resolved by the CCD-sensor is

\[ f_{\text{max}} = \frac{1}{2 \times 10\mu m} = 50\text{mm}^{-1} \]
According to Equation (3.11) and for $\lambda = 632.8\,nm$, the angle $\theta_{\text{max}}$ between the reference and the object waves should be no greater than

$$\theta_{\text{max}} = 2 \sin^{-1} \left( \frac{\lambda f_{\text{max}}}{2} \right) \approx 1.8^\circ$$

This means that only small objects at a large distance from the CCD-target are resolvable.

### 3.3.2 Numerical reconstruction of Fresnel holograms

Illuminating a hologram with the reference wave alone performs the optical reconstruction of the real image. Since in digital holography the holograms are recorded with a CCD camera and stored electronically, the reconstruction of the hologram cannot be done optically but has to take place using a computer. The real image can be constructed from the digitally sampled hologram if the diffraction of the reconstructing wave at the microstructure of the hologram is carried out by numerical methods.

Presently, the standard CCD cameras have resolution of about 100 lines/mm; therefore only small objects at large distance from the CCD-target are resolvable. For these reasons, the recorded Fresnel hologram on CCD-sensor will practically fulfill the paraxial and the Fresnel approximations. Thus the distance between the hologram and the real image, $d$ is much greater than the maximum dimensions of the CCD-chip, and precisely as follow [14,19]
\[ d^3 \gg \frac{\pi}{4\lambda} \left[ (x - \xi)^2 + (y - \eta)^2 \right]_{\text{max}} \]  \hspace{1cm} (3.13)

Thus, mathematically the amplitude and phase distribution, \( \Gamma(\xi, \eta) \), in the plane of the real image can be described by the Fresnel-Kirchhoff diffraction integral as follow [19,20]

\[
\Gamma(\xi, \eta) = \frac{e^{i k d}}{i \lambda d} \exp \left[ \frac{i \pi}{\lambda d} \left( \xi^2 + \eta^2 \right) \right] \\
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t(x, y) \exp \left[ \frac{i \pi}{\lambda d} \left( x^2 + y^2 \right) \right] \exp \left[ -i 2 \pi \left( \frac{\xi}{\lambda d}, \frac{\eta}{\lambda d} \right) \right] dx dy \]  \hspace{1cm} (3.14)

where \((x, y)\) and \((\xi, \eta)\) are the coordinates in the hologram plane and in the real image plane as illustrated in Figure 3-7, and \( t(x, y) \) is the amplitude transmittance of the hologram. Basically, the reconstruction of the real image from the diffraction image is based on the evaluation of this integral. The diffracted field is the Fourier transform of the hologram transmittance multiplied by the chirp function \( \exp \left[ \frac{i \pi}{\lambda d} \left( x^2 + y^2 \right) \right] \). The result for this Fourier transform is multiplied by a phase factor and the spatially constant intensity factor \( \frac{1}{i \lambda d} \).
3.3.2.1 Digitization of Fresnel holograms

We now consider a discrete representation of a Fresnel holograms [20,21]. For a numerical evaluation the discrete finite form of Equation (3.14), omitting the spatially constant factors

\[
\Gamma(\xi, \eta) = \exp \left[ \frac{i\pi}{\lambda d} (\xi^2 + \eta^2) \right] \\
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t(x, y) \exp \left[ \frac{i\pi}{\lambda d} \left( x^2 + y^2 \right) \right] \exp \left[ -i2\pi \left( \frac{\xi}{\lambda d} x + \frac{\eta}{\lambda d} y \right) \right] dx dy
\]

(3.15)
The computational method used for this discrete synthesis depends on the form of the digitization of the amplitude transmission of the hologram, $t(x, y)$. The simplest way to digitize $t(x, y)$ is to specify it as a matrix of numbers $t(k, l)$, taken on a rectangular raster with certain steps $\Delta x$ and $\Delta y$ along the coordinates $(k, l)$. This description is based on the sampling theorem [18]. To transform from this discrete description to a continuous description of $t(x, y)$ a linear interpolation of the readings is used. Mathematically, the interpolation can be described as the transformation of the series [20]:

$$
t(x, y) = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} t(k, l) \delta(x - k\Delta x, y - l\Delta y) \tag{3.16}
$$

where $\delta$ is the delta function. The ranges of $k, l, N_x$, and $N_y$ are governed by the dimensions of the hologram and by the digitization step. Practically, $t(x, y)$ is non-vanishing in the rectangular $[-X_{\text{max}}, X_{\text{max}}; -Y_{\text{max}}, Y_{\text{max}}]$, then

$$
N_x = \frac{2X_{\text{max}}}{\Delta x}, \quad N_y = \frac{2Y_{\text{max}}}{\Delta y} \tag{3.17}
$$

An accurate interpolation of $t(x, y)$ from $t(k, l)$ is possible if

$$
\Delta x = \frac{\lambda d}{2\xi_{\text{max}}}; \quad \Delta y = \frac{\lambda d}{2\eta_{\text{max}}} \tag{3.18}
$$
Substitution of Equations (3.16), (3.17), and (3.18) into Equation (3.15) shows that the Fresnel hologram of the discrete object \( t(x, y) \) can be calculated as the finite sum

\[
\Gamma(\xi, \eta) = \exp\left[ \frac{i\pi}{\lambda d} (\xi^2 + \eta^2) \right] \\
\times \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} t(k, l) \exp\left[ \frac{i\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2) \right] \\
\times \exp\left[ -\frac{i2\pi}{\lambda d} (k\Delta x \xi + l\Delta y \eta) \right]
\] (3.19)

The digitization in Equation (3.19) is now extended to \( \xi \) and \( \eta \). This procedure can be justified on the basis that the dimensions of the hologram \( t(x, y) \) are bounded. It follows from the boundedness of the region in which \( t(x, y) \) is specified that the function \( \Gamma(\xi, \eta) \) can be reconstructed by a linear interpolation of the discrete function [20]

\[
\Gamma(\xi, \eta) = \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} \Gamma(m\Delta \xi, n\Delta \eta) \delta(\xi - m\Delta \xi, \eta - n\Delta \eta)
\] (3.20)

where \( \Gamma(m\Delta \xi, n\Delta \eta) \) are the reading of \( \Gamma(\xi, \eta) \) taken on a rectangular raster with step \( \Delta \xi \) and \( \Delta \eta \) along the coordinates \( (\xi, \eta) \). Thus, the digitization of Equation (3.20) becomes
\( \Gamma(m, n) = \exp \left[ \frac{i\pi}{\lambda d} \left( m^2 \Delta \xi^2 + n^2 \Delta \eta^2 \right) \right] \) 
\[ \times \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} \mathbf{t}(k, l) \exp \left[ \frac{i\pi}{\lambda d} \left( k^2 \Delta x^2 + l^2 \Delta y^2 \right) \right] \] 
\[ \times \exp \left[ -\frac{i2\pi}{\lambda d} \left( k \Delta x m \Delta \xi + l \Delta y n \Delta \eta \right) \right] \quad (3.21) \]

The maximum values of \( N_x \) and \( N_y \) for which the sum in Equation (3.21) must be calculated are determined by

\[ N_x = \frac{\lambda d}{\Delta \xi \Delta x} \quad ; \quad N_y = \frac{\lambda d}{\Delta \eta \Delta y} \quad (3.22) \]

Thus, we finally yield

\[ \Gamma(m, n) = \exp \left[ i\pi \lambda d \left( \frac{m^2}{N_x^2 \Delta x^2} + \frac{n^2}{N_y^2 \Delta y^2} \right) \right] \] 
\[ \times \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} \mathbf{t}(k, l) \exp \left[ \frac{i\pi}{\lambda d} \left( k^2 \Delta x^2 + l^2 \Delta y^2 \right) \right] \] 
\[ \times \exp \left[ -\frac{i2\pi}{\lambda d} \left( \frac{mk}{N_x} + \frac{nl}{N_y} \right) \right] \quad (3.23) \]

For the case \( N_x = N_y = N \), the Equation (3.23) can be rewritten as follow
\[ \Gamma(m, n) = \exp \left[ i\pi \lambda d \left( \frac{m^2}{N^2 \Delta x^2} + \frac{n^2}{N^2 \Delta y^2} \right) \right] \times \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} t(k, l) \exp \left[ \frac{i\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2) \right] \times \exp \left[ -i2\pi \left( \frac{mk}{N} + \frac{nl}{N} \right) \right] \] (3.24)

3.3.2.2 Reconstruction algorithm

In most applications, only the intensity and the phase different of the reconstructed holograms are of interest, thus the phase factor \[ \exp \left[ i\pi \lambda d \left( \frac{m^2}{N^2 \Delta x^2} + \frac{n^2}{N^2 \Delta y^2} \right) \right] \] in Equation (3.24) can be neglected. This phase factor is independent of the evaluated hologram, thus at each point it gives the same phase shift in all reconstruction of different object states and cancels out in the phase-subtraction process of holographic interferometry. For this reason Equation (3.24) can be rewritten as follow

\[ \Gamma(m, n) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} t(k, l) \exp \left[ \frac{i\pi}{\lambda d} (k^2 \Delta x^2 + l^2 \Delta y^2) \right] \times \exp \left[ -i2\pi \left( \frac{mk}{N} + \frac{nl}{N} \right) \right] \] (3.25)

From the mathematical standpoint, Equation (3.25) is a representation of the Fresnel approximation in terms of an inverse discrete Fourier transformation (DFT). The function to be Fourier transformed is the product consisting of the hologram
amplitude transmittance $t(k,l)$ multiplied with a first exponential factor in the sum. In the calculation of the transformation, the standard Fast Fourier transform (FFT) algorithm can be applied [18], which will provide an effective calculation. The factor before the sum is only a multiplication constant.

The reconstructed real image $\Gamma(m,n)$ is a complex function, so both the intensity and the phase can be calculated. This is in contrast to the optical reconstruction making only the intensity visible. The intensity is determined by [19]

$$I(m,n) = |\Gamma(m,n)|^2 = \Re^2[\Gamma(m,n)] + \Im^2[\Gamma(m,n)]$$

(3.26)

and the phase distribution is calculated by [19]

$$\Phi(m,n) = \arctan \frac{\Im[\Gamma(m,n)]}{\Re[\Gamma(m,n)]}$$

(3.27)

where $\Re$ denotes the real part and $\Im$ is the imaginary part.

If we take into account the signs of $\Im[\Gamma(m,n)]$ and $\Re[\Gamma(m,n)]$ separately, $\Phi(m,n)$ take the values in $[-\pi, +\pi]$, the principal values of the arctan function. As a consequence of the surface roughness of the object, the phase varies randomly.
3.4 Digital Holographic Interferometry

In this section a method of digital holographic interferometry [19,22-25] is described. Two Fresnel holograms, which represent the undistorted and the distorted state of the object, are recorded on a CCD-target and stored digitally in the computer memory. Then they are added points-wisely, and the numerical reconstruction of this sum wave fields leads to a holographic interferogram. In the numerical reconstruction process not only the intensity, but also the phase of a holographically stored wavefront can be calculated. This mean that the interference phase, which represents the deformation field quantitatively, can be calculated directly by subtracting the reconstructed phases of the undistorted and the distorted object waves.

Clearly, there are two approaches to obtain the interference phase in digital holographic interferometry. The first approach is carried out by first superimposing the two holograms. Then, the interference phase is determined from the resultant interferogram with techniques such as in phase shifting hologram interferometry, heterodyne holography, and etc. [26].

The second approach is to directly calculate the interference phase by comparing the phases of digitally stored holograms.

3.4.1 Interference phase determination from interferogram

If the two Fresnel holograms with the amplitude transmittances \( t_1(k,l) \) and \( t_2(k,l) \), which represent the undistorted and the distorted state of the object, are superimposed together, yielding
\[ t(k,l) = t_1(k,l) + t_2(k,l) \] (3.28)

By inserting this result into the Equations (3.25) and (3.26), and after carried out the numerical reconstruction, it will lead to an interferogram. This is the numerical counterpart of the double exposure holography.

The resulting intensity exhibits the cosine-shaped interference pattern according to Equation (2.12). For quantitative measurements, several techniques have been developed to determine the interference phase from the intensity [26]. In phase shifting hologram interferometry, three or more phase-shifted interferograms are reconstructed with the use of mutual phase shifts in the reference wave. The interference phase can be calculated from the phase-shifted intensity distributions. In heterodyne holography, the interference phase is measured electronically by modulation of the interference signal in time [27]. All these evaluation techniques have the principal drawback that additional experimental effort is necessary and that the intensity fields are disturbed by noise.

3.4.2 Digital interference phase

If the two holograms \( t_1(k,l) \) and \( t_2(k,l) \) are not added but are reconstructed separately, their phase distribution \( \Phi_1(m,n) \) and \( \Phi_2(m,n) \) can be calculated separately with Equation (3.27). The interference phase, which is the phase difference between the wave field of the object before and after a change of the loading, is then calculated by [19]
\[ \Delta \Phi(m,n) = \begin{cases} 
\Phi_1 - \Phi_2 & \text{if } \Phi_1 \geq \Phi_2 \\
\Phi_1 - \Phi_2 + 2\pi & \text{if } \Phi_1 < \Phi_2 
\end{cases} \] (3.29)

where \( \Phi_1 \) and \( \Phi_2 \) are the individual phases, respectively.

Although the individual phases \( \Phi_1(m,n) \) and \( \Phi_2(m,n) \) are randomly fluctuating from point to point, the difference modulo \( 2\pi \) is deterministic. This difference is the interference phase distribution governed by the deformation of the object surface. Equation (3.29) permits the calculation of the interference phase directly from the digitally sampled holograms. The evaluation of a fringe pattern, as in conventional hologram interferometry, is not necessary.

The two possible approaches to achieve the interference phase distribution in the digital holographic interferometry are summarized in Figure 3-8.
Figure 3-8 Two approaches to determine the interference phase $\Delta \Phi$. 