Appendix

Error Analysis

Assume that \( F \) is multiples function of numerical value \( x, y \) and \( z \) while \( K, a, b \) and \( c \) is constant. The standard error of function \( F \) can be derived by applying the differential method as below:

\[
F = Kx^a y^b z^c
\]

applying natural logarithms,

\[
LnF = LnK + aLnx + bLny + cLnz
\]

differential,

\[
\frac{\delta F}{F} = a \left( \frac{\delta x}{x} \right) + b \left( \frac{\delta y}{y} \right) + c \left( \frac{\delta z}{z} \right)
\]

the magnitude of error,

\[
\left[ \frac{\delta F}{F} \right]^2 = \left[ a \frac{\delta x}{x} \right]^2 + \left[ b \frac{\delta y}{y} \right]^2 + \left[ c \frac{\delta z}{z} \right]^2
\]

1. Refractive Index, \( n \)

The refractive index of thin film is defined as,

\[
n(\lambda) = \left[ N + \left( N^2 - n_o^2 n_i^2 \right)^{1/2} \right]^{1/2}
\]

with

\[
N = \frac{n_o^2 + n_i^2}{2} + 2n_o n_i \left[ \frac{T_{Max} - T_{min}}{T_{Max} T_{Min}} \right]
\]
The maximum error in $T_{\text{max}}$ and $T_{\text{min}}$ is,

$$\left[\frac{\Delta T_{\text{Max}}}{T_{\text{Max}}}\right] = \left[\frac{\Delta T_{\text{Min}}}{T_{\text{Min}}}\right] = 0.01$$

$$\left[\frac{\Delta N(\lambda)}{N(\lambda)}\right]^2 \approx \left[\frac{\Delta T_{\text{Max}} + \Delta T_{\text{Min}}}{T_{\text{Max}}}\right]^2 + \left[\frac{\Delta T_{\text{Max}}}{T_{\text{Max}}}\right]^2 + \left[\frac{\Delta T_{\text{Min}}}{T_{\text{Min}}}\right]^2 = 0.0006$$

$$\left[\frac{\Delta n(\lambda)}{n(\lambda)}\right]^2 = \left[\frac{\Delta N}{N}\right]^2 = 0.0006$$

Hence

$$\left[\frac{\Delta n(\lambda)}{n(\lambda)}\right] = 0.024$$

Since mean $n(\lambda)$ is 2.12, thus $\Delta n(\lambda) = 2.12 \times 0.024 \approx 0.05$

2. Films Thickness, $d$ and Deposition Rate

$d$ is defined by equation as below,

$$d = \left(\frac{m \lambda}{2n}\right)$$

Hence

$$\left(\frac{\Delta d}{d}\right) = \left(\frac{\Delta n}{n}\right) = 0.024$$

Since mean $d$ is 1023nm, thus $\Delta d = 1023 \times 0.024 \approx 25$nm

The deposition rate is defined as $R = d/s$, where $s$ is the period in seconds.

Hence

$$\left[\frac{\Delta R}{R}\right] = \left[\frac{\Delta d}{d}\right] = 0.024$$

Since mean $R$ is 2.67Å/s, $\Delta R = 2.67 \times 0.024 \approx 0.06$Å/s
3. Optical Energy Gap, $E_g$

The optical energy gap is determined from the intercept of the linear trends at energy axis of the graph $(\alpha \hbar \nu)^{1/2}$ versus $\hbar \nu$. Thus $E_g = C/m$ where $C$ is the intercept on the energy axis and $m$ is the gradient of the linear portion.

\[
\left( \frac{\Delta E_g}{E_g} \right)^2 = \left( \frac{\Delta C}{C} \right)^2 + \left( \frac{\Delta m}{m} \right)^2
\]

from the least square method,

\[
\left( \frac{\Delta C}{C} \right) = 0.002
\]

\[
\left( \frac{\Delta m}{m} \right) = 0.01
\]

Thus

\[
\left( \frac{\Delta E_g}{E_g} \right) = 0.01
\]

4. Urbach Tail Bandwidth, $E_e$

The Urbach Tail bandwidth is determined from the gradient of the linear part of plot $\ln \alpha$ versus $E$ near the absorption edge. Therefore, $E_e = 1/m$ where $m$ is the gradient.

\[
\left( \frac{\Delta E_e}{E_e} \right) = \left( \frac{\Delta m}{m} \right)
\]

from the least square method,

\[
\left( \frac{\Delta m}{m} \right) = 0.01
\]

Thus

\[
\left( \frac{\Delta E_e}{E_e} \right) = 0.01
\]
Appendix

5. Surface Roughness

Since the scanning condition of Atomic Force Microscope in this experiment uses the *Van Der Waals* force and *Tapping mode*, the roughness can be determined to an accuracy of 0.001nm. However the accuracy also depends on the scanned roughness and the error is about 1% of surface roughness. The maximum surface roughness in this experiment is 32.076nm, so the maximum error is $0.01 \times 32.076\text{nm} \approx 0.3\text{nm}$.

6. Mean Grain Diameters

Since the mean grain diameter is determined from the average of 20 measured grain diameters, the standard error, $R$ is derived from the statistic method.

$$R = \sqrt{\frac{\sum_{i=1}^{n} (X_n - \overline{X})^2}{n}}$$

where $n = 20$ and $\overline{X}$ is mean grain diameter.