Appendix A

Normalized natural frequencies due to different locations of damaged element and the value of flexural stiffness, *EI* for the models of free-free and fixed roller supported beams

Flexural mode 1 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Ele2	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Ele3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Ele4	1.000	1.000	1.000	1.000	1.000	0.999	0.999
Ele5	1.000	1.000	1.000	1.000	0.999	0.999	0.998
Ele6	1.000	1.000	1.000	0.999	0.999	0.997	0.996
Ele7	1.000	1.000	0.999	0.999	0.998	0.996	0.993
Ele8	1.000	1.000	0.999	0.998	0.996	0.993	0.989
Ele9	1.000	1.000	0.999	0.997	0.994	0.989	0.984
Ele10	1.000	0.999	0.998	0.996	0.992	0.984	0.977
Ele11	1.000	0.999	0.998	0.994	0.989	0.978	0.968
Ele12	1.000	0.999	0.997	0.993	0.985	0.971	0.958
Ele13	1.000	0.998	0.996	0.991	0.982	0.964	0.948
Ele14	0.999	0.998	0.995	0.988	0.977	0.955	0.936
Ele15	0.999	0.998	0.994	0.986	0.972	0.947	0.924
Ele16	0.999	0.997	0.993	0.983	0.968	0.938	0.911
Ele17	0.999	0.997	0.992	0.981	0.963	0.929	0.899
Ele18	0.999	0.996	0.991	0.978	0.958	0.920	0.888
Ele19	0.999	0.996	0.989	0.976	0.953	0.911	0.877
Ele20	0.999	0.996	0.988	0.973	0.948	0.904	0.867
Ele21	0.998	0.995	0.987	0.971	0.944	0.896	0.858
Ele22	0.998	0.995	0.986	0.969	0.940	0.890	0.850
Ele23	0.998	0.994	0.986	0.967	0.937	0.885	0.843
Ele24	0.998	0.994	0.985	0.966	0.934	0.880	0.838
Ele25	0.998	0.994	0.984	0.964	0.932	0.877	0.834
Ele26	0.998	0.994	0.984	0.964	0.931	0.875	0.831
Ele27	0.998	0.994	0.984	0.963	0.930	0.874	0.830

Flexural mode 2 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	1.000	1.000	1.000	1.000	0.999	0.999
Ele2	1.000	1.000	1.000	1.000	1.000	0.999	0.999
Ele3	1.000	1.000	1.000	1.000	0.999	0.999	0.998
Ele4	1.000	1.000	1.000	0.999	0.998	0.997	0.995
Ele5	1.000	1.000	0.999	0.998	0.996	0.992	0.988
Ele6	1.000	0.999	0.998	0.996	0.992	0.985	0.978
Ele7	1.000	0.999	0.997	0.994	0.988	0.975	0.963
Ele8	1.000	0.999	0.996	0.991	0.981	0.962	0.945
Ele9	0.999	0.998	0.994	0.987	0.974	0.948	0.925
Ele10	0.999	0.997	0.993	0.983	0.966	0.934	0.905
Ele11	0.999	0.997	0.991	0.979	0.958	0.920	0.888
Ele12	0.999	0.996	0.989	0.975	0.951	0.909	0.874
Ele13	0.998	0.995	0.988	0.972	0.945	0.900	0.865
Ele14	0.998	0.995	0.986	0.969	0.941	0.894	0.859
Ele15	0.998	0.995	0.986	0.968	0.939	0.892	0.858
Ele16	0.998	0.994	0.986	0.967	0.939	0.893	0.860
Ele17	0.998	0.995	0.986	0.968	0.940	0.897	0.866
Ele18	0.998	0.995	0.986	0.970	0.944	0.904	0.875
Ele19	0.998	0.995	0.988	0.972	0.949	0.913	0.887
Ele20	0.999	0.996	0.989	0.976	0.955	0.924	0.901
Ele21	0.999	0.997	0.991	0.980	0.963	0.936	0.917
Ele22	0.999	0.997	0.993	0.984	0.971	0.950	0.934
Ele23	0.999	0.998	0.995	0.989	0.979	0.963	0.952
Ele24	1.000	0.999	0.997	0.993	0.986	0.976	0.968
Ele25	1.000	0.999	0.998	0.996	0.992	0.987	0.982
Ele26	1.000	1.000	0.999	0.998	0.997	0.994	0.993
Ele27	1.000	1.000	1.000	1.000	0.999	0.999	0.998

Flexural mode 3 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	1.000	0.999	0.999	0.999	0.998	0.997
Ele2	1.000	1.000	1.000	0.999	0.999	0.998	0.998
Ele3	1.000	1.000	0.999	0.999	0.998	0.996	0.994
Ele4	1.000	0.999	0.999	0.997	0.994	0.988	0.983
Ele5	1.000	0.999	0.997	0.994	0.988	0.975	0.963
Ele6	0.999	0.998	0.996	0.989	0.979	0.956	0.935
Ele7	0.999	0.997	0.993	0.984	0.968	0.936	0.908
Ele8	0.999	0.997	0.991	0.978	0.957	0.917	0.886
Ele9	0.999	0.996	0.988	0.973	0.948	0.904	0.872
Ele10	0.998	0.995	0.987	0.969	0.942	0.898	0.867
Ele11	0.998	0.995	0.986	0.968	0.940	0.898	0.870
Ele12	0.998	0.995	0.986	0.968	0.942	0.903	0.878
Ele13	0.998	0.995	0.987	0.970	0.947	0.913	0.891
Ele14	0.998	0.996	0.988	0.975	0.955	0.926	0.908
Ele15	0.999	0.996	0.991	0.980	0.964	0.941	0.927
Ele16	0.999	0.998	0.994	0.986	0.974	0.958	0.947
Ele17	0.999	0.999	0.996	0.991	0.984	0.974	0.967
Ele18	1.000	0.999	0.998	0.996	0.993	0.987	0.984
Ele19	1.000	1.000	1.000	0.999	0.998	0.996	0.995
Ele20	1.000	1.000	1.000	1.000	0.999	0.999	0.998
Ele21	1.000	1.000	0.999	0.998	0.996	0.993	0.991
Ele22	1.000	0.999	0.997	0.994	0.989	0.981	0.975
Ele23	0.999	0.998	0.995	0.989	0.980	0.964	0.953
Ele24	0.999	0.997	0.993	0.983	0.969	0.947	0.931
Ele25	0.999	0.996	0.990	0.978	0.960	0.931	0.912
Ele26	0.999	0.996	0.989	0.974	0.953	0.920	0.897
Ele27	0.998	0.995	0.988	0.972	0.949	0.914	0.890

Flexural mode 4 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	1.000	0.999	0.998	0.997	0.994	0.993
Ele2	1.000	1.000	0.999	0.998	0.997	0.996	0.995
Ele3	1.000	0.999	0.999	0.997	0.994	0.989	0.985
Ele4	1.000	0.999	0.997	0.993	0.987	0.973	0.959
Ele5	0.999	0.998	0.995	0.988	0.975	0.950	0.923
Ele6	0.999	0.997	0.992	0.980	0.961	0.923	0.894
Ele7	0.999	0.996	0.989	0.974	0.949	0.908	0.879
Ele8	0.998	0.995	0.987	0.970	0.943	0.904	0.879
Ele9	0.998	0.995	0.986	0.969	0.944	0.909	0.888
Ele10	0.998	0.995	0.987	0.971	0.949	0.920	0.903
Ele11	0.999	0.996	0.989	0.976	0.958	0.935	0.922
Ele12	0.999	0.997	0.992	0.983	0.970	0.953	0.943
Ele13	0.999	0.998	0.995	0.990	0.982	0.972	0.965
Ele14	1.000	0.999	0.998	0.996	0.992	0.987	0.984
Ele15	1.000	1.000	1.000	0.999	0.998	0.997	0.996
Ele16	1.000	1.000	1.000	0.999	0.998	0.996	0.995
Ele17	1.000	0.999	0.998	0.995	0.991	0.983	0.978
Ele18	0.999	0.998	0.995	0.989	0.978	0.962	0.951
Ele19	0.999	0.997	0.992	0.981	0.965	0.941	0.924
Ele20	0.999	0.996	0.989	0.975	0.955	0.925	0.905
Ele21	0.998	0.995	0.988	0.972	0.950	0.918	0.897
Ele22	0.998	0.995	0.988	0.973	0.951	0.921	0.901
Ele23	0.999	0.996	0.989	0.976	0.958	0.932	0.915
Ele24	0.999	0.997	0.992	0.983	0.969	0.949	0.936
Ele25	0.999	0.998	0.995	0.989	0.981	0.968	0.960
Ele26	1.000	0.999	0.998	0.995	0.992	0.986	0.982
Ele27	1.000	1.000	1.000	0.999	0.998	0.997	0.995

Flexural mode 5 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	0.999	0.998	0.996	0.994	0.989	0.984
Ele2	1.000	0.999	0.998	0.997	0.994	0.992	0.990
Ele3	1.000	0.999	0.998	0.995	0.990	0.980	0.969
Ele4	0.999	0.998	0.995	0.989	0.977	0.952	0.929
Ele5	0.999	0.997	0.992	0.981	0.961	0.925	0.899
Ele6	0.999	0.996	0.989	0.974	0.950	0.913	0.891
Ele7	0.998	0.995	0.987	0.970	0.947	0.915	0.898
Ele8	0.998	0.995	0.987	0.972	0.952	0.927	0.913
Ele9	0.999	0.996	0.989	0.977	0.962	0.943	0.933
Ele10	0.999	0.997	0.993	0.985	0.975	0.962	0.955
Ele11	1.000	0.999	0.997	0.993	0.987	0.980	0.976
Ele12	1.000	1.000	0.999	0.998	0.996	0.994	0.992
Ele13	1.000	1.000	1.000	0.999	0.998	0.996	0.994
Ele14	1.000	0.999	0.998	0.995	0.990	0.983	0.976
Ele15	0.999	0.998	0.994	0.986	0.975	0.956	0.944
Ele16	0.999	0.996	0.990	0.978	0.960	0.934	0.917
Ele17	0.999	0.995	0.988	0.974	0.953	0.924	0.907
Ele18	0.998	0.995	0.988	0.974	0.954	0.928	0.913
Ele19	0.999	0.996	0.990	0.978	0.963	0.942	0.930
Ele20	0.999	0.997	0.994	0.986	0.975	0.961	0.953
Ele21	1.000	0.999	0.997	0.993	0.988	0.981	0.976
Ele22	1.000	1.000	0.999	0.998	0.997	0.995	0.993
Ele23	1.000	1.000	0.999	0.999	0.997	0.995	0.993
Ele24	1.000	0.999	0.997	0.994	0.989	0.981	0.976
Ele25	0.999	0.998	0.994	0.986	0.975	0.959	0.949
Ele26	0.999	0.996	0.990	0.978	0.961	0.938	0.924
Ele27	0.999	0.995	0.988	0.974	0.953	0.925	0.909

Flexural mode 6 (Free-free beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.000	0.999	0.997	0.994	0.990	0.980	0.968
Ele2	1.000	0.999	0.998	0.995	0.992	0.986	0.982
Ele3	1.000	0.999	0.996	0.992	0.984	0.967	0.950
Ele4	0.999	0.998	0.993	0.984	0.968	0.937	0.914
Ele5	0.999	0.996	0.990	0.976	0.955	0.924	0.906
Ele6	0.998	0.995	0.988	0.973	0.953	0.928	0.915
Ele7	0.999	0.995	0.988	0.975	0.959	0.941	0.931
Ele8	0.999	0.997	0.991	0.982	0.971	0.958	0.951
Ele9	0.999	0.998	0.995	0.990	0.984	0.976	0.972
Ele10	1.000	0.999	0.999	0.997	0.994	0.991	0.988
Ele11	1.000	1.000	0.999	0.998	0.997	0.994	0.992
Ele12	1.000	0.999	0.997	0.993	0.987	0.978	0.971
Ele13	0.999	0.997	0.993	0.984	0.970	0.950	0.937
Ele14	0.999	0.996	0.989	0.976	0.958	0.933	0.919
Ele15	0.998	0.995	0.988	0.975	0.957	0.935	0.922
Ele16	0.999	0.996	0.990	0.979	0.965	0.948	0.939
Ele17	0.999	0.998	0.994	0.987	0.978	0.967	0.961
Ele18	1.000	0.999	0.998	0.995	0.991	0.985	0.982
Ele19	1.000	1.000	0.999	0.999	0.997	0.995	0.993
Ele20	1.000	0.999	0.998	0.996	0.993	0.987	0.983
Ele21	0.999	0.998	0.995	0.988	0.979	0.965	0.957
Ele22	0.999	0.996	0.991	0.979	0.964	0.943	0.930
Ele23	0.999	0.995	0.988	0.975	0.956	0.932	0.918
Ele24	0.999	0.996	0.989	0.976	0.959	0.938	0.925
Ele25	0.999	0.997	0.992	0.983	0.971	0.955	0.946
Ele26	1.000	0.999	0.996	0.992	0.985	0.977	0.972
Ele27	1.000	1.000	0.999	0.998	0.996	0.993	0.990

Flexural mode 1 (Fixed roller supported beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
Ele2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9991	0.9986
Ele3	1.0000	0.9999	0.9997	0.9994	0.9988	0.9976	0.9965
Ele4	0.9999	0.9998	0.9995	0.9988	0.9977	0.9955	0.9934
Ele5	0.9999	0.9997	0.9992	0.9981	0.9963	0.9927	0.9893
Ele6	0.9999	0.9996	0.9988	0.9973	0.9946	0.9893	0.9845
Ele7	0.9998	0.9994	0.9984	0.9963	0.9926	0.9854	0.9789
Ele8	0.9998	0.9992	0.9979	0.9951	0.9904	0.9811	0.9727
Ele9	0.9997	0.9990	0.9974	0.9939	0.9880	0.9764	0.9661
Ele10	0.9996	0.9988	0.9968	0.9926	0.9854	0.9715	0.9590
Ele11	0.9996	0.9986	0.9962	0.9912	0.9827	0.9664	0.9518
Ele12	0.9995	0.9983	0.9956	0.9898	0.9799	0.9611	0.9445
Ele13	0.9994	0.9981	0.9950	0.9883	0.9771	0.9558	0.9372
Ele14	0.9993	0.9979	0.9943	0.9869	0.9743	0.9506	0.9300
Ele15	0.9992	0.9976	0.9937	0.9854	0.9715	0.9454	0.9230
Ele16	0.9991	0.9974	0.9931	0.9840	0.9688	0.9405	0.9163
Ele17	0.9991	0.9972	0.9925	0.9826	0.9662	0.9358	0.9100
Ele18	0.9990	0.9970	0.9919	0.9813	0.9637	0.9314	0.9040
Ele19	0.9989	0.9967	0.9914	0.9801	0.9614	0.9273	0.8986
Ele20	0.9989	0.9965	0.9909	0.9790	0.9593	0.9236	0.8937
Ele21	0.9988	0.9964	0.9904	0.9780	0.9575	0.9203	0.8894
Ele22	0.9988	0.9962	0.9900	0.9771	0.9558	0.9174	0.8856
Ele23	0.9987	0.9961	0.9897	0.9763	0.9544	0.9149	0.8824
Ele24	0.9987	0.9960	0.9894	0.9757	0.9533	0.9130	0.8799
Ele25	0.9987	0.9959	0.9892	0.9752	0.9524	0.9115	0.8780
Ele26	0.9986	0.9959	0.9891	0.9749	0.9518	0.9105	0.8767
Ele27	0.9986	0.9958	0.9890	0.9748	0.9515	0.9100	0.8761

Flexural mode 2 (Fixed roller supported beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9991
Ele2	1.0000	0.9999	0.9997	0.9993	0.9985	0.9971	0.9957
Ele3	0.9999	0.9997	0.9993	0.9982	0.9964	0.9928	0.9895
Ele4	0.9998	0.9995	0.9986	0.9968	0.9936	0.9872	0.9815
Ele5	0.9997	0.9993	0.9980	0.9951	0.9903	0.9809	0.9726
Ele6	0.9996	0.9990	0.9972	0.9934	0.9869	0.9744	0.9634
Ele7	0.9995	0.9987	0.9964	0.9917	0.9835	0.9681	0.9547
Ele8	0.9995	0.9984	0.9958	0.9901	0.9805	0.9625	0.9471
Ele9	0.9994	0.9982	0.9951	0.9887	0.9778	0.9577	0.9407
Ele10	0.9994	0.9980	0.9947	0.9875	0.9756	0.9540	0.9359
Ele11	0.9993	0.9978	0.9942	0.9866	0.9741	0.9515	0.9328
Ele12	0.9993	0.9977	0.9940	0.9861	0.9732	0.9501	0.9313
Ele13	0.9993	0.9977	0.9939	0.9860	0.9729	0.9499	0.9314
Ele14	0.9993	0.9977	0.9940	0.9862	0.9734	0.9509	0.9330
Ele15	0.9993	0.9978	0.9942	0.9866	0.9744	0.9530	0.9360
Ele16	0.9994	0.9980	0.9946	0.9875	0.9760	0.9560	0.9401
Ele17	0.9994	0.9982	0.9950	0.9885	0.9780	0.9598	0.9454
Ele18	0.9994	0.9983	0.9956	0.9898	0.9805	0.9642	0.9514
Ele19	0.9995	0.9985	0.9961	0.9912	0.9831	0.9691	0.9580
Ele20	0.9996	0.9988	0.9968	0.9926	0.9860	0.9742	0.9649
Ele21	0.9997	0.9991	0.9975	0.9942	0.9889	0.9794	0.9719
Ele22	0.9998	0.9993	0.9981	0.9956	0.9915	0.9844	0.9787
Ele23	0.9998	0.9995	0.9987	0.9970	0.9941	0.9890	0.9850
Ele24	0.9999	0.9997	0.9992	0.9981	0.9963	0.9931	0.9905
Ele25	1.0000	0.9998	0.9995	0.9990	0.9980	0.9963	0.9949
Ele26	1.0000	0.9999	0.9998	0.9996	0.9993	0.9985	0.9980
Ele27	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995

Flexural mode 3 (Fixed roller supported beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	0.9999	0.9997	0.9994	0.9988	0.9981	0.9971	0.9962
Ele2	0.9999	0.9998	0.9995	0.9991	0.9989	0.9986	0.9985
Ele3	0.9999	0.9998	0.9995	0.9992	0.9988	0.9984	0.9980
Ele4	0.9999	0.9997	0.9993	0.9986	0.9975	0.9955	0.9937
Ele5	0.9998	0.9995	0.9986	0.9969	0.9939	0.9881	0.9826
Ele6	0.9997	0.9990	0.9973	0.9938	0.9875	0.9751	0.9614
Ele7	0.9994	0.9983	0.9955	0.9893	0.9787	0.9577	0.9391
Ele8	0.9992	0.9975	0.9933	0.9842	0.9686	0.9396	0.9155
Ele9	0.9989	0.9966	0.9910	0.9790	0.9592	0.9243	0.8974
Ele10	0.9987	0.9959	0.9891	0.9748	0.9520	0.9142	0.8870
Ele11	0.9985	0.9953	0.9877	0.9721	0.9479	0.9101	0.8842
Ele12	0.9984	0.9951	0.9872	0.9713	0.9475	0.9117	0.8882
Ele13	0.9984	0.9953	0.9877	0.9726	0.9504	0.9183	0.8977
Ele14	0.9986	0.9958	0.9890	0.9756	0.9563	0.9287	0.9113
Ele15	0.9989	0.9965	0.9909	0.9800	0.9643	0.9419	0.9278
Ele16	0.9992	0.9974	0.9933	0.9853	0.9736	0.9568	0.9460
Ele17	0.9995	0.9984	0.9958	0.9906	0.9830	0.9718	0.9645
Ele18	0.9997	0.9992	0.9979	0.9953	0.9913	0.9853	0.9811
Ele19	0.9999	0.9998	0.9993	0.9985	0.9971	0.9950	0.9934
Ele20	1.0000	1.0000	0.9999	0.9997	0.9994	0.9988	0.9983
Ele21	0.9999	0.9998	0.9994	0.9987	0.9975	0.9954	0.9938
Ele22	0.9998	0.9993	0.9981	0.9955	0.9916	0.9853	0.9806
Ele23	0.9995	0.9985	0.9960	0.9910	0.9832	0.9707	0.9619
Ele24	0.9992	0.9976	0.9937	0.9858	0.9737	0.9548	0.9417
Ele25	0.9990	0.9968	0.9915	0.9809	0.9649	0.9403	0.9237
Ele26	0.9987	0.9961	0.9898	0.9772	0.9582	0.9295	0.9102
Ele27	0.9986	0.9958	0.9889	0.9751	0.9546	0.9237	0.9030

Flexural mode 4 (Fixed roller supported beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	0.9998	0.9994	0.9987	0.9975	0.9961	0.9937	0.9915
Ele2	0.9998	0.9995	0.9989	0.9982	0.9974	0.9964	0.9958
Ele3	0.9998	0.9995	0.9988	0.9975	0.9957	0.9924	0.9894
Ele4	0.9997	0.9991	0.9977	0.9948	0.9899	0.9795	0.9694
Ele5	0.9995	0.9984	0.9957	0.9897	0.9791	0.9572	0.9368
Ele6	0.9991	0.9973	0.9928	0.9829	0.9656	0.9328	0.9057
Ele7	0.9988	0.9962	0.9899	0.9763	0.9537	0.9153	0.8877
Ele8	0.9985	0.9953	0.9876	0.9715	0.9466	0.9084	0.8836
Ele9	0.9983	0.9949	0.9865	0.9699	0.9455	0.9110	0.8900
Ele10	0.9984	0.9950	0.9870	0.9715	0.9498	0.9206	0.9035
Ele11	0.9986	0.9957	0.9889	0.9760	0.9582	0.9350	0.9215
Ele12	0.9990	0.9968	0.9918	0.9823	0.9692	0.9521	0.9420
Ele13	0.9994	0.9981	0.9950	0.9891	0.9810	0.9699	0.9631
Ele14	0.9997	0.9992	0.9978	0.9952	0.9913	0.9858	0.9821
Ele15	0.9999	0.9998	0.9995	0.9989	0.9979	0.9962	0.9948
Ele16	1.0000	0.9999	0.9997	0.9992	0.9984	0.9969	0.9956
Ele17	0.9998	0.9993	0.9982	0.9958	0.9921	0.9859	0.9813
Ele18	0.9994	0.9983	0.9955	0.9897	0.9807	0.9664	0.9564
Ele19	0.9991	0.9971	0.9924	0.9827	0.9680	0.9454	0.9301
Ele20	0.9987	0.9961	0.9897	0.9768	0.9576	0.9292	0.9105
Ele21	0.9985	0.9955	0.9881	0.9736	0.9523	0.9215	0.9018
Ele22	0.9985	0.9955	0.9882	0.9738	0.9530	0.9235	0.9050
Ele23	0.9987	0.9960	0.9897	0.9772	0.9592	0.9339	0.9181
Ele24	0.9990	0.9970	0.9923	0.9829	0.9694	0.9502	0.9380
Ele25	0.9994	0.9982	0.9954	0.9896	0.9813	0.9689	0.9609
Ele26	0.9998	0.9992	0.9980	0.9956	0.9918	0.9861	0.9820
Ele27	0.9999	0.9998	0.9996	0.9991	0.9982	0.9966	0.9953

Flexural mode 5 (Fixed roller support beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	0.9996	0.9990	0.9978	0.9958	0.9930	0.9878	0.9824
Ele2	0.9996	0.9992	0.9982	0.9967	0.9949	0.9925	0.9904
Ele3	0.9996	0.9990	0.9977	0.9950	0.9907	0.9819	0.9733
Ele4	0.9995	0.9984	0.9956	0.9897	0.9791	0.9567	0.9356
Ele5	0.9990	0.9971	0.9924	0.9819	0.9634	0.9293	0.9034
Ele6	0.9987	0.9959	0.9891	0.9745	0.9514	0.9155	0.8928
Ele7	0.9984	0.9951	0.9870	0.9710	0.9476	0.9162	0.8984
Ele8	0.9983	0.9950	0.9871	0.9720	0.9518	0.9266	0.9129
Ele9	0.9986	0.9957	0.9892	0.9771	0.9615	0.9425	0.9322
Ele10	0.9990	0.9971	0.9927	0.9845	0.9741	0.9610	0.9538
Ele11	0.9995	0.9986	0.9963	0.9922	0.9866	0.9793	0.9748
Ele12	0.9998	0.9996	0.9989	0.9978	0.9960	0.9931	0.9911
Ele13	0.9999	0.9998	0.9996	0.9987	0.9980	0.9962	0.9946
Ele14	0.9997	0.9991	0.9978	0.9949	0.9904	0.9831	0.9777
Ele15	0.9993	0.9979	0.9943	0.9871	0.9760	0.9586	0.9468
Ele16	0.9988	0.9964	0.9906	0.9790	0.9617	0.9363	0.9199
Ele17	0.9985	0.9954	0.9883	0.9741	0.9539	0.9258	0.9087
Ele18	0.9985	0.9954	0.9880	0.9740	0.9547	0.9289	0.9139
Ele19	0.9987	0.9961	0.9900	0.9785	0.9627	0.9423	0.9306
Ele20	0.9991	0.9974	0.9934	0.9857	0.9752	0.9613	0.9531
Ele21	0.9996	0.9987	0.9969	0.9932	0.9880	0.9807	0.9760
Ele22	0.9999	0.9996	0.9992	0.9983	0.9969	0.9945	0.9925
Ele23	0.9999	0.9997	0.9994	0.9987	0.9975	0.9922	0.9937
Ele24	0.9996	0.9989	0.9973	0.9940	0.9891	0.9817	0.9766
Ele25	0.9992	0.9976	0.9938	0.9863	0.9754	0.9600	0.9501
Ele26	0.9987	0.9962	0.9903	0.9786	0.9621	0.9393	0.9253
Ele27	0.9985	0.9954	0.9881	0.9740	0.9541	0.9269	0.9103

Flexural mode 6 (Fixed roller support beam model)

Damaged element	Normalized natural frequencies						
	0.92EI	**0.78EI**	**0.58EI**	**0.36EI**	**0.22EI**	**0.12EI**	**0.09EI**
Ele1	0.9995	0.9987	0.9970	0.9940	0.9895	0.9794	0.9663
Ele2	0.9996	0.9989	0.9975	0.9950	0.9919	0.9868	0.9823
Ele3	0.9995	0.9987	0.9965	0.9921	0.9845	0.9685	0.9528
Ele4	0.9993	0.9977	0.9936	0.9848	0.9690	0.9394	0.9169
Ele5	0.9988	0.9963	0.9900	0.9768	0.9558	0.9251	0.9069
Ele6	0.9985	0.9953	0.9878	0.9732	0.9530	0.9280	0.9149
Ele7	0.9985	0.9954	0.9883	0.9754	0.9592	0.9406	0.9311
Ele8	0.9988	0.9965	0.9912	0.9819	0.9706	0.9577	0.9509
Ele9	0.9993	0.9981	0.9952	0.9900	0.9835	0.9757	0.9712
Ele10	0.9998	0.9994	0.9985	0.9967	0.9942	0.9906	0.9880
Ele11	0.9999	0.9997	0.9993	0.9985	0.9971	0.9946	0.9923
Ele12	0.9997	0.9989	0.9972	0.9936	0.9880	0.9789	0.9722
Ele13	0.9991	0.9973	0.9930	0.9843	0.9712	0.9516	0.9389
Ele14	0.9987	0.9959	0.9894	0.9766	0.9586	0.9344	0.9200
Ele15	0.9985	0.9952	0.9882	0.9748	0.9570	0.9350	0.9228
Ele16	0.9987	0.9961	0.9900	0.9789	0.9649	0.9481	0.9390
Ele17	0.9992	0.9975	0.9936	0.9867	0.9778	0.9668	0.9606
Ele18	0.9997	0.9990	0.9975	0.9945	0.9906	0.9851	0.9815
Ele19	0.9999	0.9997	0.9993	0.9985	0.9973	0.9949	0.9929
Ele20	0.9998	0.9993	0.9983	0.9961	0.9928	0.9877	0.9839
Ele21	0.9993	0.9979	0.9946	0.9882	0.9790	0.9658	0.9575
Ele22	0.9988	0.9964	0.9906	0.9796	0.9641	0.9434	0.9310
Ele23	0.9985	0.9954	0.9884	0.9748	0.9566	0.9327	0.9189
Ele24	0.9986	0.9957	0.9890	0.9764	0.9596	0.9382	0.9261
Ele25	0.9990	0.9969	0.9921	0.9831	0.9711	0.9556	0.9467
Ele26	0.9995	0.9985	0.9962	0.9917	0.9855	0.9770	0.9717
Ele27	0.9999	0.9996	0.9990	0.9977	0.9959	0.9928	0.9902