CHAPTER 3 INTRODUCTION OF MODAL ANALYSIS

3.1 INTRODUCTION

Vibration or dynamic motion has a significant effect on elastic components such as machines and structures. In today's world, machines and structures are almost everywhere. Therefore reliable vibration and dynamic analysis tools are a basic need to measure the response of machines and structures when subject to external and internal forces, which will cause deformations and overall motions. Modal analysis is one of the tools which provides an understanding of structural characteristics, operating conditions and performance criteria.

As an engineering tool, modal analysis is the process of characterizing the dynamic properties of a structure in terms of its modal parameters, that is natural frequency, mode shape and damping. Generally, the understanding of modal analysis can be divided into two namely experimental modal analysis and analytical modal analysis. Although both experimental and analytical approaches can be used to obtain modal parameters, they differ totally in the process of characterizing dynamic properties.

3.2 EXPERIMENTAL MODAL ANALYSIS

Experimental modal analysis is based on the use of experimentally determined data which is obtained from modal testing [21]. Today, Transfer Function Method (TFM) is commonly used as the basic of modal testing. This method involves the acquisition of point to point frequency response functions (FRFs) at a set of points defined as a dynamic model. The time data are collected and converted into the frequency domain as frequency response functions in the Fast Fourier Transfer (FFT) analyzer. In addition, different excitation techniques can be used in the estimation of frequency response functions. Modal parameter estimation methods are used to obtain modal parameters of the structure from measured frequency response functions. Figure 3.1 illustrates the process of a modal test.

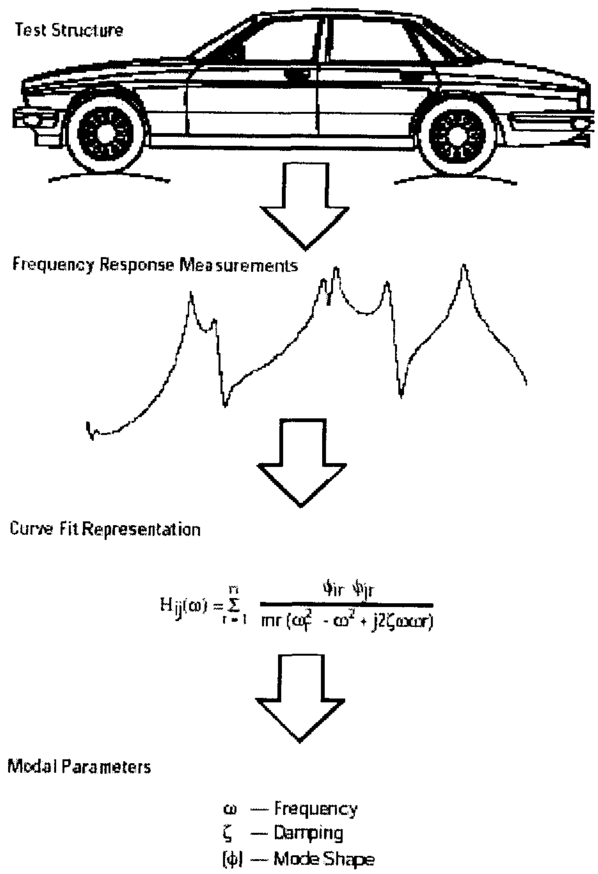
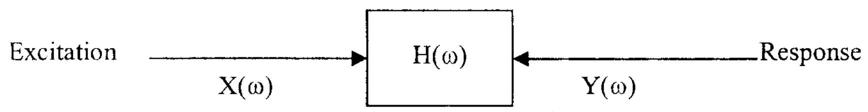


Figure 3.1 Modal test [20]

3.2.1 FREQUENCY RESPONSE FUNCTION (FRF)

The frequency response function (FRF) is an inherent measurement in the modal testing, which is defined as Fourier transform of the output (acceleration response) divided by Fourier transform of the input (excitation force) [20]. The measurement of input excitation and output response is measured simultaneously, as shown in the block diagram in Figure 3.2, to obtain the frequency response function.



X(ω) – Fourier transform of the input

Y(ω) – Fourier transform of the output

$H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$ – Frequency response function

Figure 3.2 System block diagram [20]

In order to obtain good frequency response functions, the test setup and the measurement acquisition process of modal testing are critical. It is important to understand the purpose of the test and results that are expected during the test setup. Knowledge of anticipated mode shapes and excitation techniques in a structure are also required in order to define the test so that its frequency range, measurement points and other relevant features can be identified. A proper test setup avoids obtaining poor quality or unnecessary frequency response measurements, thus enabling the estimation of modal parameters to be carried out more easily during the curve fitting process, which will be discussed in Section 3.2.3.

The acquisition of measurement is a time-consuming process and careful documentation is important during the test to help uncover problems. Basic understanding of concepts associated with digital signal processing such as leakage, windows, time and frequency relationships, fast fourier transform, transfer function formulation, excitation techniques are also important in assessing measurements all the way through the measureming process. This knowledge helps the user to ensure quality frequency response measurements.

3.2.2 EXCITATION TECHNIQUE

Impact testing is a common excitation technique available in modal testing. An understanding of this excitation technique is important in order to carry out the modal test successfully. Impact testing applies an impulsive excitation which is very short in the time window usually lasting only less than 5% of the sample interval. The response of the system is made up of the sum of exponentially decaying sine waves depending on the number of modes excited. This technique requires very little hardware and provides shorter measurement times but it is only suitable for testing a simple and linear structure [22].

Generally, the understanding of functionality of the hammer tip, pre-trigger delay and window functions is important to perform the impact testing. The former is used to determine the frequency spectrum of the input force falling in the desired frequency range. For a lower frequency response function, a softer tip should be used. Pre-trigger delay is most effective to eliminate an incorrect input force spectrum due to an input trigger from the hammer impact. Window is a weighting function that is applied to a measured signal to minimize the effects of noise and leakage. The setup of impact testing is detailed in Section 4.3.

3.2.3 MODAL PARAMETER ESTIMATION

Once a complete set of frequency response functions has been collected from a structure, modal parameter estimation method namely "curve fitting" is performed to identify modal parameters. Figure 3.3 shows the fundamental concept of modal parameters from frequency response functions. As shown in Figure 3.3, frequency and damping of any mode in a structure can be identified from any frequency response function. Mode shape is assembled from the identified modal coefficients from each measurement at the same modal frequency.

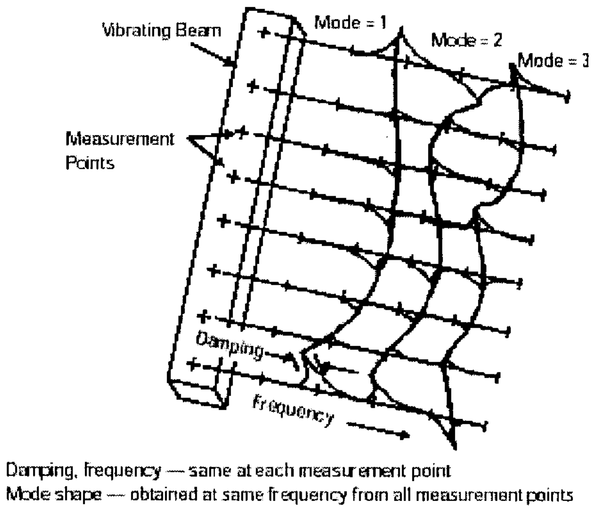


Figure 3.3 Concept of modal parameters [21]

The identification of modal frequency is illustrated in Figure 3.4 which can be identified by the following observations.

- The magnitude of the frequency response is a maximum.
- The imaginary part of the frequency response is a maximum or minimum.
- The real part of the frequency response is zero.
- The response lags the input by 90° phase.

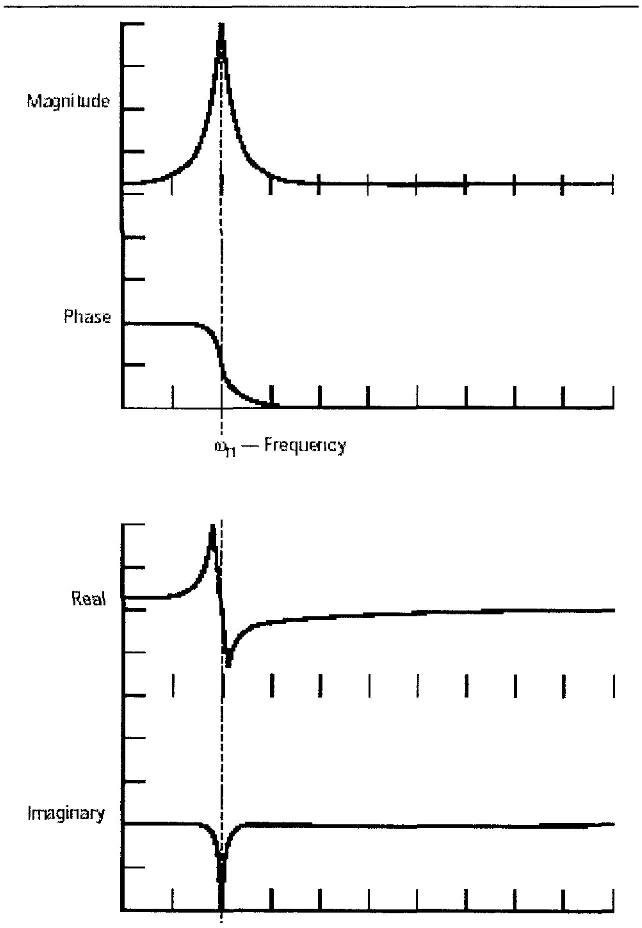


Figure 3.4 Frequency response [20]

The damping can be obtained by measuring the width of the modal peak between the half-power points or at 70.7% of its peak value, as illustrated in Figure 3.5. This method is known as half-power point method since 70.7% of the magnitude is the same as 50% of the magnitude squared. Finally, the modal coefficient can be identified from the peak value of the imaginary part of the frequency response, as illustrated in Figure 3.6.

Commonly, curve fitting method can be divided into single mode and multiple mode methods. Single mode method involves only curve fitting a single mode form of a modal resonance peak to identify the modal parameters. Multiple mode method on the other hand involves curve fitting multiple mode forms of a frequency response function to identify all the modal parameters simultaneously. The modal parameters obtained from the multiple mode method are comparative especially when closely spaced modes occur.

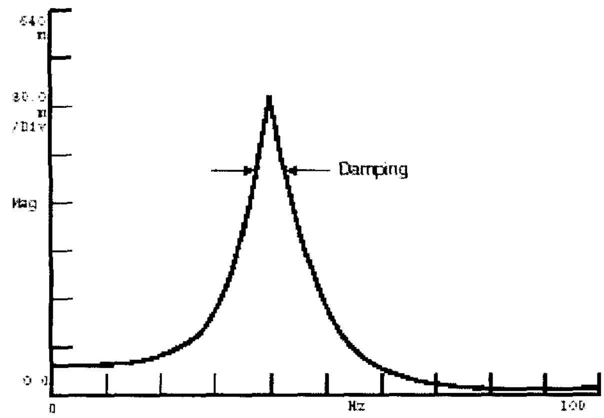


Figure 3.5 Damping from half power [20]

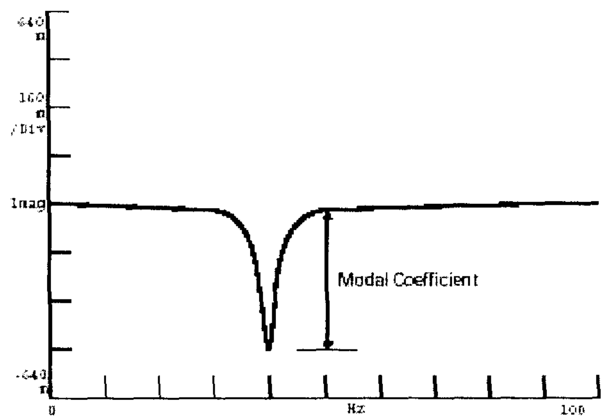


Figure 3.6 Modal coefficient [20]

3.3 ANALYTICAL MODAL ANALYSIS

Analytical modal analysis is based on the use of differential equations of motion of a structure which are generated using theoretical and finite element (FE) modeling technique. The resulting equations are then decomposed into eigenvalues (frequencies) and eigenvectors (mode shapes).

3.3.1 TRANSVERSE FREE VIBRATION OF A SIMPLE BEAM

The differential equations of motion of a long, thin beam undergoing transverse free vibration may be derived using Newton's second law. Figure 3.7(a) shows a portion of beam in the X-Y plane undergoing transverse motion, which is assumed to be a plane of symmetry for any cross section. Figure 3.7(b) shows the free body diagram of an element of length dx with internal and inertial actions upon it. The symbol y represents the transverse motion of a particular segment on the neutral axis of the beam. The bending moment is M and the transverse shear force is V.

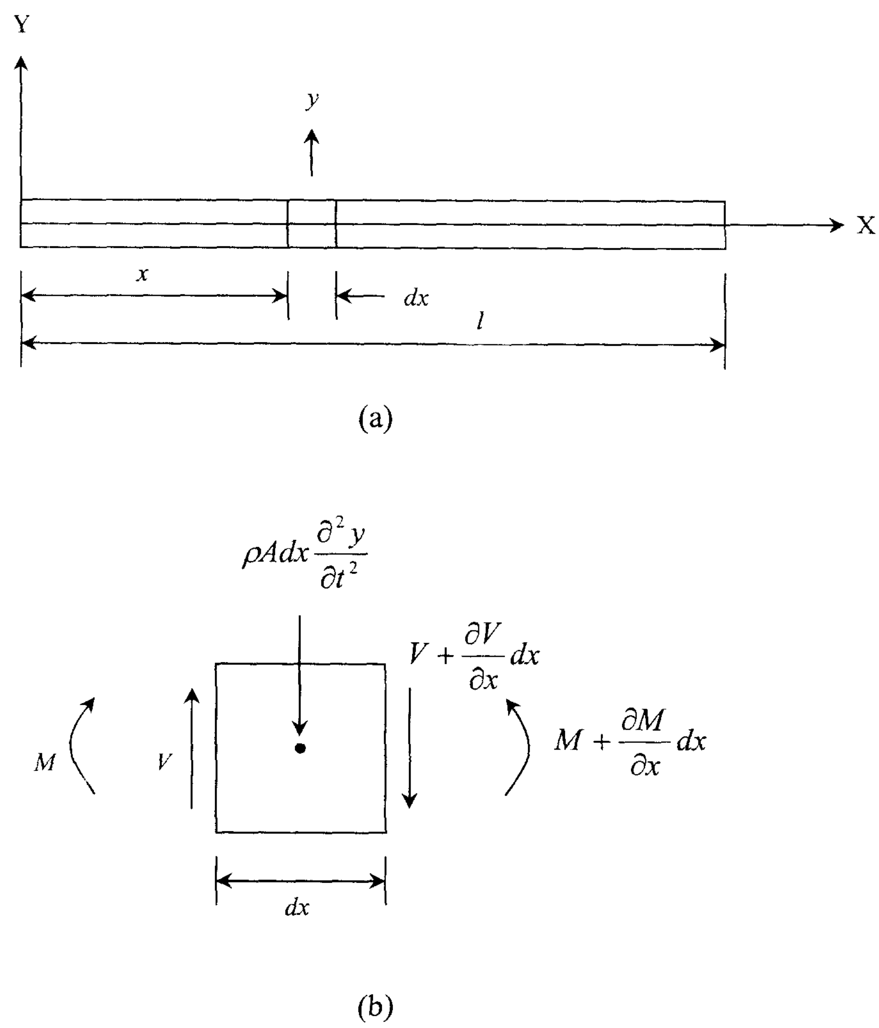


Figure 3.7 Beam undergoing transverse free vibration [1]

From kinematics the bending strain can be related to the curvature, 1/R, of the beam by

$$\varepsilon = \frac{-y}{R} \qquad \text{Equation 3.1}$$

where y is the distance in the cross section measured from the neutral axis and R is radius of curvature. Then, for a linearly elastic beam whose properties are independent of the position in the cross section, the bending moment can be related to the curvature by

$$M = \frac{EI}{R} \qquad \text{Equation 3.2}$$

where I is the moment of inertia of the cross section. If the slope, $\partial y/\partial x$, of the beam remains small, then the curvature can be approximated by $\partial y^2/\partial x^2$, so Equation 3.2 becomes

$$M = EI\frac{\partial^2 y}{\partial x^2} \qquad \text{Equation 3.3}$$

When the beam is undergoing transverse free vibration, the dynamic equilibrium condition for forces in y direction is

$$V - V - \frac{\partial V}{\partial x}dx - \rho A dx \frac{\partial^2 y}{\partial x^2} = 0 \qquad \text{Equation 3.4}$$

and the moment equilibrium condition is

$$-Vdx - \frac{\partial V}{\partial x}\frac{dx^2}{2} + \frac{\partial M}{\partial x}dx = 0$$

where the equation can be written as

$$-Vdx + \frac{\partial M}{\partial x}dx \approx 0 \qquad \text{Equation 3.5}$$

by neglecting the higher order dx^2.

Substitution of V from equation 3.5 into Equation 3.4 produces

$$\frac{\partial^2 M}{\partial x^2}dx + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{Equation 3.6}$$

and substitute M from Equation 3.3 into Equation 3.6, we obtain

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\rho A\frac{\partial^2 y}{\partial t^2}=0 \qquad \text{Equation 3.7}$$

which is the equation of motion for transverse free vibration of a beam. In the particular case of a simple beam the flexural rigidity EI does not vary with x, so Equation 3.7 can be written as

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0 \qquad \text{Equation 3.8}$$

In harmonic motion, the deflection in y direction at any location varies harmonically with time, as follows

$$y = X\cos(\omega t-\alpha) \qquad \text{Equation 3.9}$$

and substitute this into Equation 3.8 to get the fourth-order ordinary differential equation

$$EI\frac{\partial^4 X}{\partial x^4}+\rho A\omega^2 X=0 \qquad \text{Equation 3.10}$$

As an aid in solving this equation, Equation 3.10 is reduced to

$$\frac{\partial^4 X}{\partial x^4}+\lambda^4 X=0 \qquad \text{Equation 3.11}$$

where

$$\lambda^4=\frac{\rho A\omega^2}{EI} \qquad \text{Equation 3.12}$$

To satisfy Equation 3.11, we let $X = e^{nx}$ and obtain

$$e^{nx}\left(n^4-\lambda^4\right)=0 \qquad \text{Equation 3.13}$$

Thus, the values of n are found to be $n_1 = \lambda$, $n_2 = -\lambda$, $n_3 = j\lambda$, and $n_4 = -j\lambda$, where $j = \sqrt{-1}$. The general form of the solution for Equation 3.13 becomes

$$X = Ce^{\lambda x}+De^{-\lambda x}+Ee^{j\lambda x}+Fe^{-j\lambda x} \qquad \text{Equation 3.14a}$$

which may also be written in the equivalent form

$$X = C_1 \sin \lambda x + C_2 \cos \lambda x + C_3 \sinh \lambda x + C_4 \cosh \lambda x \qquad \text{Equation 3.14b}$$

This expression represents a typical normal function for transverse free vibrations of a simple beam.

The four amplitude constants C_1, C_2, C_3 and C_4, and the eigenvalue λ are determined in each particular case from the boundary conditions at the ends of the beam. For example, the end conditions of simply support are

$$(X) = 0 \qquad \text{Equation 3.15a}$$

$$(d^2X/dx^2) = 0 \qquad \text{Equation 3.15b}$$

and the end conditions of free-free are

$$(dX/dx) = 0 \qquad \text{Equation 3.16a}$$

$$(d^2X/dx^2) = 0 \qquad \text{Equation 3.16b}$$

Utilization of the end restraint permits one to draw conclusions that lead to the evaluation of the values of λ and the corresponding mode shapes of the beam. With known λ's, the natural frequencies ω can be obtained from

$$\omega_i = \lambda_i^2 \sqrt{\frac{EI}{\rho A}} \qquad i = 1,2,3\ldots\ldots\infty \qquad \text{Equation 3.17}$$

where i is the corresponding mode shapes.

3.3.2 FINITE ELEMENT MODELLING TECHNIQUE

The finite element modeling technique is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Basically finite element modeling subdivides a structure up into very small region called finite elements where the displacement of the element is analytically obtainable. The selection of element types is determined from the basic theory of elasticity and strength of materials. Nodes are used to define each region of an element. All of the finite elements are assembled into one large model taking care of the balancing of forces and compatibility at each interface. Then boundary conditions are applied and this model with a large set of simultaneous equations is solved using numerical procedures.

The knowledge or skills of defining nodes, elements, boundary conditions and solving equations are needed in order to model correctly. Nodes are defined according to the geometry of a structure, anticipated mode shapes and type of elements. Elements are selected based on the type of characteristic deformations anticipated which gives some ideas of mode shape patterns that are expected. Thus, the knowledge of expected mode shapes is important for identifying the type of elements and node density.

The boundary conditions are defined in the model to reflect the appropriate support conditions on a real structure. The physical restrain of the support needs to be known so that the actual boundary conditions can be modelled. In order to perform a correct analysis, types of solution schemes should be known which must adhere with the required analysis. The detail of finite element modeling of beam is further discussed in Chapter 4, Section 4.5.