CHAPTER 2

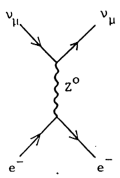
NEUTRINO-ELECTRON SCATTERINGS

2.1 Introduction

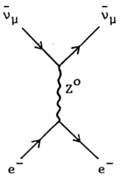
Processes involving neutrinos play an important role in astrophysics. A neutrino interacts very weakly with matter with cross section ~ 10^{-43} cm^2 and hence the neutrino carry away all its energy when it escapes from a star. Among the important neutrino processes are URCA process, neutrino bremsstrahlung, photo-neutrino process, pair annihilation and neutrino emission from plasma [5]. In this section we will discuss the neutrino-electron scattering processes using the electroweak theory.

2.2 Neutrino-Electron Elastic Scatterings

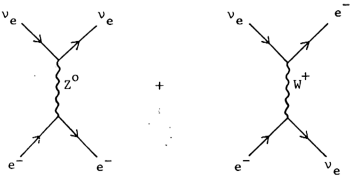
In this section, we will derive the invariant amplitude for the $\nu - e$ scatterings within the framework of the standard electroweak theory and hence calculate the cross-sections. The neutrino electron elastic scattering can be of the neutral or charged current type [6]. The $\nu_e - e$ process involves both the neutral and charged current interactions while for other types of neutrinos only via neutral current interaction (Fig 2.1).



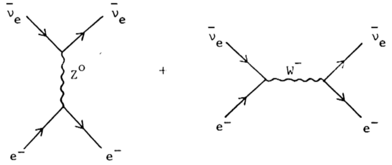
(i) $\nu_\mu e^- \rightarrow \nu_\mu e^-$



(ii) $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$



(iii) $\nu_e e^- \rightarrow \nu_e e^-$



(iv) $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$

Fig 2.1 Feynman diagrams for the $\nu(\bar{\nu})+e^- \rightarrow \nu(\bar{\nu})+e^-$ elastic scattering processes.

In the $\nu e \rightarrow \nu e$ neutral current interaction, the leptons are coupled to the Z^o boson. The general neutral current interaction for the coupling $Z^O \rightarrow f\bar{f}$ (Fig 2.2) is written as

$$\varphi_f = -i \frac{g}{\cos\theta_W}\left[\overline{\varphi_f}\gamma_\mu \frac{1}{2}\left(g_V^f - g_A^f\gamma_5\right)\right]\varphi_f Z^\mu \tag{2.1}$$

with the vertex factor written as

$$-i \frac{g}{\cos\theta_W}\gamma_\mu \frac{1}{2}\left(g_V^f - g_A^f\gamma_5\right) \tag{2.2}$$

where the vector and axial-vector couplings are given in Table 2.1.

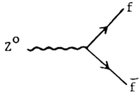
The invariant amplitude for a neutral current process M^{NC} of a general $\nu e \rightarrow \nu e$ scattering described by the Feynman diagrams with vertex factor (2.2) can be calculated using the Feynman's rule, i.e.

$$M^{NC} = \left(\frac{g}{\cos\theta_W}\bar{\nu}\gamma^\mu \frac{1}{2}\left(g_V^\nu - g_A^\nu\gamma_5\right)\nu\right)\left[-\frac{\left(g_{\mu\sigma} - q_\mu \frac{q_\sigma}{M_Z^2}\right)}{q^2 - M_Z^2}\right]$$

$$\times \left(\frac{g}{\cos\theta_W}\bar{e}\gamma^\sigma \frac{1}{2}\left(g_V^e - g_A^e\gamma_5\right)e\right). \tag{2.3}$$

f	Q_f	g_A^f	g_V^f
ν_e, ν_μ, ν_τ	0	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	$-\frac{1}{2} + 2\sin^2\theta_W$
u, c, t	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$

Table 2.1 The g_A^f and g_V^f couplings for $Z^0 \to f\bar{f}$ vertex factor in the standard electroweak theory.



$$-i \frac{g}{\cos\theta_W} \gamma^\mu \frac{1}{2}(g_V^f - g_A^f \gamma_5)$$

Fig. 2.2 $Z^0 \to f\bar{f}$ neutral current coupling.

where the terms in the square bracket are the Z^0 propagator and q is the four-momentum transfer in the scattering process. At low momentum, $|q|^2 \ll M_z^2$ and M^{NC} becomes

$$M^{NC} = \frac{g}{8M_Z^2 \cos\theta_W}\left[\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\right]\left[\bar{e}\gamma_\mu(g_v^e - g_A^e\gamma_5)e\right]. \tag{2.4}$$

M_Z can be expressed in terms of M_W from the equation $M_W/M_Z = \cos\theta_W$ so that the coupling constant g can be written in terms of the Fermi constant G_F. Finally the neutral current invariant amplitude takes this form:

$$M^{NC} = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\right]\left[\bar{e}\gamma_\mu(g_v^e - g_A^e\gamma_5)e\right]. \tag{2.5}$$

In the $\nu_e e^- \rightarrow \nu_e e^-$ charged current interaction, the leptons are coupled to the W boson. In the standard electroweak theory, the left-handed fermions are grouped as doublets and thus the charged current interaction expression for the $\nu_e e^- \rightarrow \nu_e e^-$ scattering is written as

$$-i\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^\mu e_L W_\mu^+ + \overline{e_L}\gamma^\mu \nu_{eL} W_\mu^-\right). \tag{2.6}$$

For the coupling $W^+ \rightarrow \nu_e e^+$, depicted in Fig 2.3 (a), the interaction term is

$$-i\frac{g}{\sqrt{2}}\ \overline{\nu_{eL}}\gamma^{\mu}e_L W_{\mu}^{+} \tag{2.7}$$

with vertex factor

$$-i\frac{g}{\sqrt{2}}\ \gamma^{\mu}\frac{1}{2}(1-\gamma_5). \tag{2.8}$$

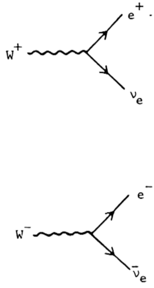
For the coupling $W^- \rightarrow \overline{\nu_e} e^-$, (Fig. 2.3 (b)), the interaction is described by

$$-i\frac{g}{\sqrt{2}}\ \overline{e_L}\gamma^{\mu}\nu_{eL} W_{\mu}^{-} \tag{2.9}$$

and the vertex factor is the same as in the $W^+ \rightarrow \nu_e e^+$ coupling.

Using the vertex factor (2.8), the amplitude for the charged current M^{CC} at low momentum transfer $|q|^2 << M_W^2$ is

$$M^{CC} = -\frac{G_F}{\sqrt{2}}\left[\overline{\nu_e}\gamma^{\mu}(1-\gamma_5)e\right]\left[\overline{e}\gamma_{\mu}(1-\gamma_5)\nu_e\right]. \tag{2.10}$$



(a) $-i\,\frac{g}{\sqrt{2}}\,\gamma^{\mu}\,\frac{1}{2}(1-\gamma_5)$

(b) $-i\,\frac{g}{\sqrt{2}}\,\gamma^{\mu}\,\frac{1}{2}(1-\gamma_5)$

Fig 2.3 Charged current interaction for the couplings (a) $W^+ \rightarrow \nu_e e^+$ and also (b) $W^- \rightarrow \overline{\nu_e} e^-$.

The negative sign arises from the interchange of the outgoing leptons. Using Fiertz transformation in which the states ν_e and e are interchanged with a corresponding change of sign, the charged current amplitude acquires a positive sign,

$$M^{CC} = +\frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right]\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]. \tag{2.11}$$

To calculate the cross-sections of the $\nu e \to \nu e$ scattering processes in the laboratory frame it is necessary to consider scattering in all possible spin configurations. This can be done by averaging $|M|^2$ over initial and summing over final electron spins. Let k_1 and k_2 be the four-momenta of the initial neutrino (ν_1) and final neutrino (ν_2) respectively, and p_1 and p_2 be the four-momenta of the initial electron (e_1) and final electron (e_2) respectively.

(i) $\nu_\mu(\bar{\nu}_\mu) + e^- \to \nu_\mu(\bar{\nu}_\mu) + e^-$

The spin averaged amplitude $\overline{|M|^2}$ for these processes are evaluated by using the neutral current amplitude (2.5) and the spin summations are carried out by using trace theorems.

Thus, we obtain

$$\overline{\left|M\left(\nu_\mu e^- \to \nu_\mu e^-\right)\right|^2}$$

$$= \frac{G_F^2}{m_{\nu\mu}{}^2 m_e{}^2} \left[(k_1 \cdot p_1)(k_2 \cdot p_2)(g_v + g_A)^2 + (k_1 \cdot p_2)(k_2 \cdot p_1)(g_v - g_A)^2\right.$$

$$\left. + (g_A - g_v)^2 m_e{}^2 k_1 \cdot k_2\right]. \tag{2.12}$$

In the above expression and for all subsequent $\overline{|M|^2}$, we use the approximation, $m_\nu \approx 0$. The neutrino mass in the denominator will be cancelled when evaluating the cross-section. The couplings $g_v (\equiv g_v^e)$ and $g_A (\equiv g_A^e)$ refer to the electron couplings.

For the $\overline{\nu_\mu} e^- \to \overline{\nu_\mu} e^-$ scattering, $\overline{|M|^2}$ is obtained by making the replacement $k_1 \leftrightarrow k_2$, i.e.

$$\overline{\left|M\left(\overline{\nu_\mu} e^- \to \overline{\nu_\mu} e^-\right)\right|^2}$$

$$= \frac{G_F^2}{m_{\nu\mu}{}^2 m_e{}^2} \left[(k_2 \cdot p_1)(k_1 \cdot p_2)(g_v + g_A)^2 + (k_2 \cdot p_2)(k_1 \cdot p_1)(g_v - g_A)^2\right.$$

$$\left. + (g_A - g_v)^2 m_e{}^2 k_1 \cdot k_2\right]. \tag{2.13}$$

(ii) $\nu_e(\bar{\nu}_e) + e^- \rightarrow \nu_e(\bar{\nu}_e) + e^-$

The invariant amplitude is the sum of the neutral and charged currents:

$$M\left(\nu_e e^- \rightarrow \nu_e e^-\right) = \frac{G_F}{\sqrt{2}}\left[\overline{\nu_e}\gamma^\mu\left(1-\gamma_5\right)\nu_e\right]\left[\bar{e}\gamma_\mu\left(g'_v - g'_A\gamma_5\right)e\right] \tag{2.14}$$

where $g'_V = g_V + 1$ and $g'_A = g_A + 1$. Hence,

$$\overline{\left|M\left(\nu_e e^- \rightarrow \nu_e e^-\right)\right|^2}$$

$$= \frac{G_F^2}{m_{\nu e}^{\ 2} m_e^{\ 2}}\left[\left(k_2 \cdot p_2\right)\left(k_1 \cdot p_1\right)\left(g'_v + g'_A\right)^2 + \left(k_2 \cdot p_1\right)\left(k_1 \cdot p_2\right)\left(g'_v - g'_A\right)^2\right.$$

$$\left. + \left(g'_A - g'_v\right)^2 m_e^{\ 2} k_1 \cdot k_2\right]. \tag{2.15}$$

The $\overline{|M|^2}$ for $\overline{\nu_e}e^- \rightarrow \overline{\nu_e}e^-$ is obtained by interchanging $k_1 \leftrightarrow k_2$:

$$\overline{\left|M\left(\overline{\nu_e}e^- \rightarrow \overline{\nu_e}e\right)\right|^2}$$

$$= \frac{G_F^2}{m_{\nu e}^{\ 2} m_e^{\ 2}}\left[\left(k_1 \cdot p_2\right)\left(k_2 \cdot p_1\right)\left(g'_v + g'_A\right)^2 + \left(k_2 \cdot p_1\right)\left(k_2 \cdot p_2\right)\left(g'_v - g'_A\right)^2\right.$$

$$\left. + \left(g'_A - g'_v\right)^2 m_e^{\ 2} k_1 \cdot k_2\right]. \tag{2.16}$$

We can combine the amplitudes $\overline{|M|^2}$ into a single general form. In the laboratory frame where the initial electron is stationary, target with $\underline{p}_1 = 0$, the elastic scattering kinematics relations in $\overline{|M|^2}$ can be approximated using the four-momentum $k_1^\mu = (E_{\nu 1}, \underline{k}_1)$, $k_2^\mu = (E_{\nu 2}, \underline{k}_2)$ together with $p_1^\mu = (m_e, 0)$ and also $p_2^\mu = (E_{e2}, \underline{p}_2)$:

$$k_1 \cdot p_1 \;=\; k_2 \cdot p_2 \;=\; m_e E_{\nu 1} \;;$$

$$k_1 \cdot p_2 \;=\; k_2 \cdot p_1 \;=\; m_e E_{\nu 2} \;=\; m_e\left(E_{\nu 1} + m_e - E_{\nu 2}\right) \;=\; m_e E_{\nu 1}\ (1 - y), \tag{2.17}$$

where $y = \dfrac{E_{e2} - m_e}{E_{\nu 1}}$

$$k_1 \cdot k_2 \;=\; m_e E_{\nu 1} y .$$

Therefore, the general form of $\overline{|M|^2}$ is written as

$$\overline{\left|M\left(\nu e^- \to \nu e^-\right)\right|^2}$$

$$= \frac{G_F^2}{m_\nu^{\,2} m_e^{\,2}} \left[A m_e^{\,2} E_{\nu 1}^{\,2} + B m_e^{\,2} E_{\nu 1}^{\,2} (1-y)^2 + C m_e^{\,2} E_{\nu 1} y\right]. \tag{2.18}$$

The coefficients in Eq (2.18) A, B and C are given in Table 2.2. Substituting (2.18) into the formula for the differential cross-section $d\sigma$ we have

$$\frac{d\sigma}{dy} = \frac{G_F^2 m_e E_{\nu 1}}{2\pi}\left[A + B(1-y)^2 + Cy\frac{m_e}{E_{\nu 1}}\right]. \tag{2.19}$$

The differential cross-section (2.19) is integrated from $y = 0$, to $y = 1$ and thus yielding the total cross-section,

$$\sigma(\nu e \rightarrow \nu e) = \frac{G_F^2}{2\pi} m_e E_{\nu 1}\left[A + \frac{1}{3}B + C\frac{m_e}{2E_{\nu 1}}\right]. \tag{2.20}$$

Neutrinos are relativistic particles and hence $m_e \ll E_{\nu 1}$. The cross-section is reduced to

$$\sigma(\nu e \rightarrow \nu e) = \frac{G_F^2}{2\pi} m_e E_{\nu 1}\left[A + \frac{1}{3}B\right] \sim 10^{-42}\ E_{\nu 1}\ (\text{cm}^2) \tag{2.21}$$

where $\frac{G_F^2}{2\pi} m_e = 4.31 \times 10^{-42}$ cm^2 GeV^{-1} and $E_{\nu 1}$ is in GeV.

Processes	A	B	C
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	$(g_v + g_A)^2$	$(g_v - g_A)^2$	$(g_A{}^2 - g_V{}^2)$
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$	$(g_v - g_A)^2$	$(g_v + g_A)^2$	$(g_A + g_V)^2$
$\nu_e + e^- \rightarrow \nu_e + e^-$	$(g_v' + g_A')^2$	$(g_v' - g_A')^2$	$(g_v' - g_A')^2$
$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$	$(g_v' - g_A')^2$	$(g_v' + g_A')^2$	$(g_v' + g_A')^2$

Table 2.2 The coefficients A and B, where $g_v' = g_v^e + 1$ and $g_A' = g_A^e + 1$.

The CM total cross-section for $\nu e \rightarrow \nu e$ elastic scattering is defined as

$$\sigma_{CM}(\nu e \rightarrow \nu e) = \frac{G_F^2}{4\pi} S\left[A + \frac{1}{3}B\right] \tag{2.22}$$

with $S \approx 2m_e E_{\nu_l}$.

2.3 The Seesaw Mechanism

If the right-handed neutrino is assumed to exist. Dirac mass terms for the neutrino similar to the charged-leptons can be constructed as

$$m_\nu \overline{\nu_L} \nu_R + m_\nu \overline{\nu_R} \nu_L = m_\nu \bar{\nu} \nu . \tag{2.23}$$

In some GUTs, in which neutrinos can acquire a non-vanishing mass, a majorana mass term is introduced by the coupling of two fermions or two antifermions and thus violates the fermion number conservation. Dirac mass term does not violate the fermion number conservation.

In the SO(10) models, right-handed neutrinos are arranged as SU(5) singlets and the (B-L) number is usually not conserved. The fermions of each generation are grouped into a 16-dimensional irreducible spinor representation of SO(10), while the SU(5) content is $16 = \bar{5} + 10 + 1$.

A neutrino mass of the Dirac type comparable to the quark or charged lepton mass can be generated using the coupling of the left- and right-handed neutrinos with the Higgs scalars (with non-zero vacuum expectation value).

Gell-Mann, Ramond and Slansky [7] suggested the seesaw mechanism where neutrinos acquire a small non-vanishing mass. In their scheme, the neutrino mass matrix for one generation [8] in the Lagrangian is written in the form

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}. \tag{2.24}$$

The Dirac mass term of the order of charge $-\frac{2}{3}$ or up-type quark mass of the same generation is m_D. M_R is the singlet Majorana mass of the order of the grand unification scale. By diagonalising the neutrino mass matrix and since $m_D << M_R$, the mass eigenvalues of the light neutrino mass is given as

$$m_1 \approx \frac{{m_D}^2}{M_R} \qquad (2.25)$$

and the heavy neutrino mass by giving it the mass of the order of the grand unification scale is

$$m_2 \approx M_R \, . \qquad (2.26)$$

Eq (2.25) can be generalised to the three-generation neutrinos by writing it as

$$m_i \approx \frac{{M_{Di}}^2}{M_R} \quad (i = 1,2,3\,) \qquad (2.27)$$

with the assumption that $M_{D1} < M_{D2} < M_{D3}$. The values of the neutrino masses predicted by the seesaw model is

$$m_1 \sim 10^{-10}\ \text{eV}\ , \quad m_2 \sim 10^{-5}\ \text{eV}\ , \quad m_3 \sim 10^{-2}\ \text{eV} \qquad (2.28)$$

where we have used the masses of u, c and t quarks and $M_R \sim 10^{14}$ GeV. Hence the seesaw mechanism gives a natural way of generating small neutrino masses.