CHAPTER 1

INTRODUCTION

1.1 Introduction

Let us consider the following nonlinear regression model

$$y_u = \eta(\boldsymbol{\xi}_u, \boldsymbol{\theta}) + \varepsilon_u, \qquad u = 1, 2, \ldots, n$$

where y_u is a dependent variable with mean $\eta(\boldsymbol{\xi}_u, \boldsymbol{\theta})$,

$\boldsymbol{\xi}_u$ is a vector of independent variables,

$\boldsymbol{\theta}$ is a $(p \times 1)$ vector of unknown parameters belonging to the parameter space Ω which is a subset of the p-dimensional Euclidean space $(p < n)$,
and the ε_u are random errors assumed to be independent and normally distributed with mean zero and unknown variance σ^2.

The nominally-$100(1-\alpha)\%$ confidence regions based on likelihood ratio for $\boldsymbol{\theta}$ are of the following form

$$R(\mathbf{y}) = \left\{\boldsymbol{\theta} : S(\boldsymbol{\theta}) - S(\hat{\boldsymbol{\theta}}) \le \frac{pF_\alpha}{n-p} S(\hat{\boldsymbol{\theta}})\right\} \tag{1.1.1}$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$,

$\hat{\boldsymbol{\theta}}$ is the least squares estimate of $\boldsymbol{\theta}$,

$$S(\hat{\boldsymbol{\theta}}) = \sum_{u=1}^{n} \{y_u - \eta(\boldsymbol{\xi}_u, \hat{\boldsymbol{\theta}})\}^2$$

and F_α is the $100(1-\alpha)$ percentage point of an F-distribution with p and $(n-p)$ degrees of freedom.

For a given model, the probability that the regions given by (1.1.1) will cover the true value $\boldsymbol{\theta}_T$ of $\boldsymbol{\theta}$ may be treated as a function of $\boldsymbol{\theta}_T$ and the true value σ_T of σ as follows

$$I(\boldsymbol{\theta}_T, \sigma_T) = P(\boldsymbol{\theta}_T \in R(\mathbf{y}) \mid \boldsymbol{\theta}_T, \sigma_T).$$

Let $\Omega_f \subseteq \Omega$ and $\sigma_f > 0$ be large enough such that it is plausible to believe that the true values of $\boldsymbol{\theta}$ and σ lie in Ω_f and $(0, \sigma_f)$ respectively. Let $\boldsymbol{\theta}_f \in \Omega_f$ and $\sigma \in (0, \sigma_f)$. If the extreme values of $I(\boldsymbol{\theta}_f, \sigma)$ over Ω_f and $(0, \sigma_f)$ differ only slightly from $1 - \alpha$, then the actual coverage probability of the regions given by (1.1.1) may be taken to be approximately $1 - \alpha$.

The present work is concerned with the derivation of the quartic approximation of the coverage probability of the regions given by (1.1.1), and the adjustment of the regions such that their coverage probability has approximately the desired value of $1 - \alpha$.

1.2 A Survey of Works on Nonlinear Models

Much has been written on the numerical methods of finding the least squares estimate $\hat{\boldsymbol{\theta}}$ [cf. Draper and Smith (1981) and Stol (1975) for the list of references]. The asymptotic properties of $\hat{\boldsymbol{\theta}}$ have been studied by Jenrich (1969), Malinvaud (1970) and Wu (1981). The asymptotic distributions of the statistics based on likelihood ratio for testing hypotheses regarding $\boldsymbol{\theta}$ have been investigated by Wald (1943), Gallant (1975a), Gallant (1975b) and Milliken and DeBruin (1978).

Hartley (1964) derived an exact confidence region for the parameter vector $\boldsymbol{\theta}$. Williams (1962) constructed an interval estimate for the only nonlinear parameter θ_3 in the model

$$y_u = \theta_1 + \theta_2 f(\xi_u, \theta_3) + \varepsilon_u, \qquad u = 1, 2, \ldots, n.$$

Later, Halperin (1963) extended Williams' results to a larger class of nonlinear models.

When $\eta(\boldsymbol{\xi}_u, \boldsymbol{\theta})$ can be expressed as a series expansion in $\boldsymbol{\theta}$, it is possible to investigate the deviation from the linear theory results in a nonlinear model.

Box (1971) derived the bias of $\hat{\boldsymbol{\theta}}$ while Clarke (1980) obtained the variance and covarinace of $\hat{\boldsymbol{\theta}}$. Beale (1960), Linssen (1975), Bates and Watts (1980) together with Cook and Goldberg (1986) proposed measures of nonlinearity for assessing whether the linear approximation inference is adequate in a nonlinear model.

Guttman and Meeter (1965) have investigated Beale's measures of nonlinearity, while Donaldson and Schnabel (1986) have used Monte Carlo simulation study to obtain the coverage probabilities of confidence regions and tried to relate them to Bates and Watts' curvature measures.

Beale (1960) and Johansen (1983) derived the quadratic approximation of the coverage probability of the confidence regions based on likelihood ratio for the parameter vector $\boldsymbol{\theta}$. Hamilton and Wiens (1987) and Pooi (1989, 1991b) proposed adjustments of the confidence regions based on likelihood ratio for a subset of components of the parameter $\boldsymbol{\theta}$.

Pooi (1992) derived the quartic approximation of the coverage probability of the confidence intervals based on likelihood ratio in a two-parameter nonlinear model.

In Pooi and Loke (1989), approximations for powers of the likelihood ratio tests for hypotheses regarding subsets of the parameters were derived. Research Report (R.R.) No. 15/80 describes the derivation of powers in the case of testing hypotheses concerning one or more nonlinear functions of the parameter vector $\boldsymbol{\theta}$.

Wu (1989) derived the quadratic approximation of the coverage probability of the confidence regions based on local linearization for the parameter vector $\boldsymbol{\theta}$. The corresponding result for coverage probability of the confidence intervals was derived in Pooi and Khoo (1991). Adjustments of the confidence regions and intervals based on local linearization were proposed respectively in Wu (1989) and Khoo and Pooi (1994).

In Khoo and Pooi R.R. No. 7/88, the coverage probabilities of the interval estimates based on likelihood ratio and local linearization were compared. It was found that in a two-parameter nonlinear model, the coverage probability of the interval estimates based on likelihood ratio is more stable when the nonlinear terms in the model are subjected to perturbation.

In Pooi (1991a) a method of deriving the quadratic approximation of the expected coverage of the prediction intervals for the future observation was presented. In R.R. No. 8/86 the explicit expression for the quadratic approximation of the expected coverage for a two-parameter nonlinear model was found. The same method was later used in Goh and Pooi (*) to derive an explicit expression for the quadratic approximation of the expected coverage for a p-parameter nonlinear model.

1.3 Layout of the Dissertation

Chapter 2 derives the quartic approximation of the coverage probability of the nominally-$100(1-\alpha)\%$ confidence regions based on likelihood ratio for a two-parameter nonlinear model.

In Chapter 3, the nominally-$100(1-\alpha)\%$ confidence regions based on likelihood ratio are adjusted in two stages. It is found that to the extent that the quartic approximation of the coverage probability is adequate, the actual coverage probability of the adjusted confidence regions is equal to $1-\alpha$.

Finally in Chapter 4, numerical integration is used to compute the exact coverage probabilities of the unadjusted and adjusted confidence regions in some simple nonlinear models with $n = 4$ and $p = 2$. The exact values are then compared with the corresponding theoretical results based on the quadratic and quartic approximations.