CHAPTER 4

COVERAGE PROBABILITIES OF REGION ESTIMATES IN NONLINEAR MODELS WITH FOUR OBSERVATIONS

4.1 Introduction

In Chapter 2, the quartic approximation of the coverage probability of the region estimates based on likelihood ratio has been derived for a two-parameter nonlinear model. Chapter 3 shows that to the extent that the quartic approximation of the coverage probability is adequate, the adjusted region estimates based on likelihood ratio has a coverage probability of $1 - \alpha$.

In Section 4.2, we give the basic idea of computing the exact coverage probability for a nonlinear model with $n = 4$ observations and $p = 2$ parameters by numerical integration implemented on a computer.

In Section 4.3, numerical integration is used to obtain the exact coverage probabilities for fifteen simple nonlinear models with $n = 4$ and $p = 2$. The exact coverage probabilities are compared with the theoretical coverage probabilities based on quadratic and quartic approximation for a selected range of σ. It is found that the coverage probabilities based on the quartic approximations are closer to the corresponding exact coverage probabilities than those based on the quadratic approximations.

In the same section, the exact coverage probabilities of the stage one and stage two adjusted confidence regions are computed by numerical integration for some nonlinear models. The coverage probabilities obtained for the stage two adjusted confidence regions are found to be distinctively closer to the desired value $1 - \alpha$ than those obtained for the stage one adjusted confidence regions.

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In Section 4.2, we give the basic idea of computing the exact coverage probability for a nonlinear model with $n = 4$ observations and $p = 2$ parameters by numerical integration implemented on a computer.

In Section 4.3, numerical integration is used to obtain the exact coverage probabilities for fifteen simple nonlinear models with $n = 4$ and $p = 2$. The exact coverage probabilities are compared with the theoretical coverage probabilities based on quadratic and quartic approximation for a selected range of σ. It is found that the coverage probabilities based on the quartic approximations are closer to the corresponding exact coverage probabilities than those based on the quadratic approximations.

In the same section, the exact coverage probabilities of the stage one and stage two adjusted confidence regions are computed by numerical integration for some nonlinear models. The coverage probabilities obtained for the stage two adjusted confidence regions are found to be distinctively closer to the desired value $1 - \alpha$ than those obtained for the stage one adjusted confidence regions.

4.2 Coverage Probability Calculated by Numerical Integration

Numerical integration carried out on a computer can be used to calculate the exact value of the coverage probability of the unadjusted region estimates for a two-parameter nonlinear model with small value of n.

For a model with $n = 4$, we apply a transformation of z_1', z_2', z_3' and z_4' given by

$$z_1' = \rho_1 \cos \gamma_1$$

$$z_2' = \rho_1 \sin \gamma_1$$

$$z_3' = \rho_2 \cos \gamma_2$$

$$z_4' = \rho_2 \sin \gamma_2$$

where $\rho_1, \rho_2 \geq 0$ and $0 \leq \gamma_1, \gamma_2 \leq 2\pi$. The exact coverage probability I can then be written as

$$I = \left(\frac{1}{\sqrt{2\pi}}\right)^4 \int_{\gamma_2=0}^{2\pi} \int_{\rho_2=0}^{\infty} \rho_2 e^{-\rho_2^2/2} \int_{\gamma_1=0}^{2\pi} \int_{\rho_1=0}^{\rho_1^*} \rho_1 e^{-\rho_1^2/2} d\rho_1 d\gamma_1 d\rho_2 d\gamma_2 \tag{4.2.1}$$

where for a given value of $(\gamma_1, \rho_2, \gamma_2)$, $\rho_1^* = \rho_1^*(\gamma_1, \rho_2, \gamma_2)$ is the largest value of ρ_1 which satisfies (2.3.6). When the F_{α^*} in (2.3.6) is replaced by F_{α^+} or $F_{\alpha^{++}}$, (4.2.1) gives the coverage probability of the stage one or stage two adjusted confidence regions for a two-parameter nonlinear model. It is computationally feasible to evaluate the multiple integral in (4.2.1) by using numerical integration.

4.3 Comparison of Approximate and Exact Values of Coverage Probability

Now, let us consider the nonlinear model with $n = 4$, $p = 2$ and $\mathbf{z}^*$ (see (2.2.5)) given by

$$z_i^* = \begin{cases} \phi_i, & i = 1, 2 \\ \boldsymbol{\phi}^2 \mathbf{A}_i \boldsymbol{\phi} + \sum_{j=1}^{2} [\boldsymbol{\phi}^T \mathbf{A}_{ij} \boldsymbol{\phi}] \phi_j + \sum_{j=1}^{2} \sum_{k=1}^{2} [\boldsymbol{\phi}^T \mathbf{A}_{ijk} \boldsymbol{\phi}] \phi_j \phi_k & \\ \quad + \sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{t=1}^{2} [\boldsymbol{\phi}^T \mathbf{A}_{ijkt} \boldsymbol{\phi}] \phi_j \phi_k \phi_t, & i = 3, 4. \end{cases} \tag{4.3.1}$$

A simplified version of the above nonlinear model is obtained by setting certain a_{ijk}, $a_{ijk\ell}$, $a_{ijk\ell m}$ or $a_{ijk\ell mv}$ to be nonzero while the others to be zero. As an example, if only a_{311} is set to be nonzero, then (4.3.1) can be reduced to

$$z_1^* = \phi_1$$

$$z_2^* = \phi_2$$

$$z_3^* = a_{311}\phi_1^2$$

$$z_4^* = 0.$$

This particular model together with other simple nonlinear models are summarized in the table below.

Model number	The only nonzero term(s)
1	a_{311}
2	a_{312}
3	a_{311}, a_{322}
4	a_{311}, a_{312}
5	a_{311}, a_{312}, a_{322}
6	a_{311}, a_{411}
7	a_{312}, a_{412}
8	a_{311}, a_{422}
9	a_{312}, a_{411}
10	a_{311}, a_{322}, a_{411}
11	a_{312}, a_{411}, a_{422}
12	a_{311}, a_{322}, a_{411}, a_{422}
13	a_{311}, a_{312}, a_{411}, a_{412}
14	a_{311}, a_{312}, a_{412}, a_{422}
15	a_{3111}, a_{3122}, a_{3112}, a_{3222}

The multiple integral in (4.2.1) is used to compute the exact coverage probability for each of the fifteen simplified models.

The exact values (unadj (exact)) of the coverage probabilities together with the corresponding quadratic (unadj (quad)) and quartic (unadj (quart)) values given by (2.3.13) are presented in Fig. 4.k.1, $k = 1, 2, \ldots, 14$ for the first fourteen models. The exact coverage probabilities of the stage one adjusted (adj (stage 1)) confidence regions are also presented in these figures for the respective models.

These figures show that for the considered ranges of σ, the exact, quadratic and quartic coverage probabilities are close to one another. This means that both the quadratic and quartic approximations are adequate. Therefore in a more general two-parameter nonlinear model with a larger value of n, the closeness of the values based on quadratic and quartic approximations may be used as an indication that both the approximations are adequate.

As the improvement of the quartic values over the quadratic values cannot be detected in these figures, the numerical values of the exact, quadratic and quartic coverage probabilities are presented in Tables 4.1–4.14. These tables show that the coverage probabilities based on quartic approximation are much closer to the corresponding exact values than those based on quadratic approximation. This gives us a check on the validity of the quartic approximation of the coverage probability.

Furthermore, Figs. 4.1.1–4.14.1 show that the exact coverage probabilities of the stage one adjusted confidence regions are much closer to the required value of $1-\alpha$ than those of the unadjusted confidence regions.

As for model 15, the a_{ijkt} will not appear in the expression for the coverage probability based on quadratic approximation for the unadjusted regions. This means the regions after the stage one adjustment are exactly the same as the unadjusted regions. In Fig. 4.15, we present the exact and quartic coverage probabilities of the unadjusted regions together with the exact coverage probability of the stage two adjusted (adj (stage 2)) regions. The figure shows that for the considered range of σ, the quartic values of the coverage probabilities agree well with the corresponding exact values. Furthermore, the exact coverage probabilities of the stage two adjusted regions are much closer to the desired value of $1-\alpha$ than those of the unadjusted regions.

For models 1–14, the exact values of the coverage probabilities of the stage one and stage two adjusted confidence regions are presented in Figs. 4.k.2, $k = 1, 2, \ldots, 14$.

These figures show that for the considered ranges of σ, the coverage probabilities of the stage two adjusted regions are closer to $1-\alpha$ than those of the stage one adjusted regions.

Fig. 4.1.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

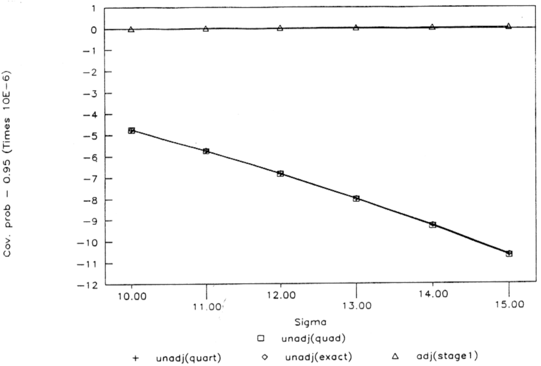

Fig. 4.1.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

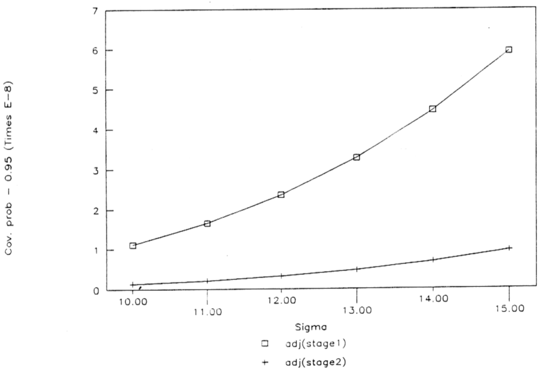

Fig. 4.2.1 Coverage Probability of Confidence Regions ($a_{312} = a_{321} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

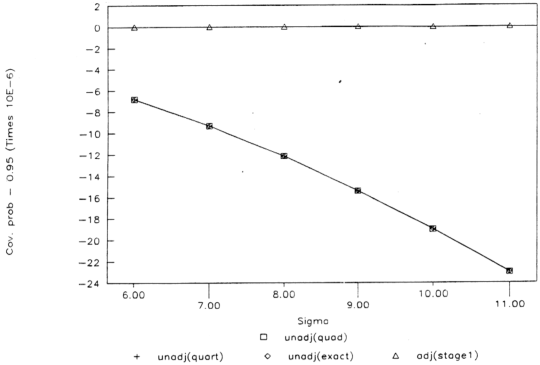

Fig. 4.2.2 Coverage Probability of Confidence Regions ($a_{312} = a_{321} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

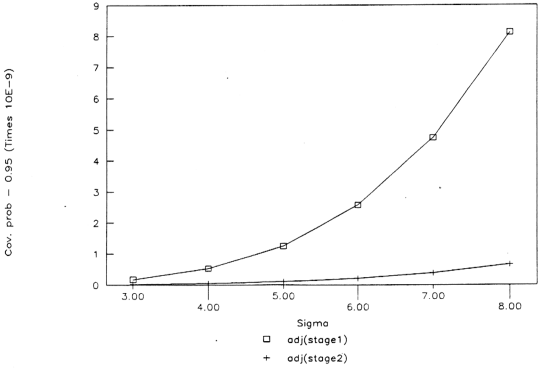

Fig. 4.3.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{322} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

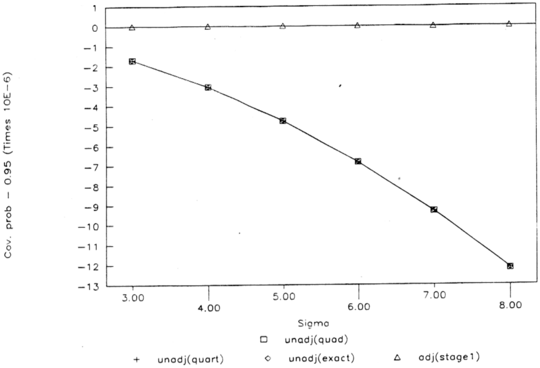

Fig. 4.3.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{322} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

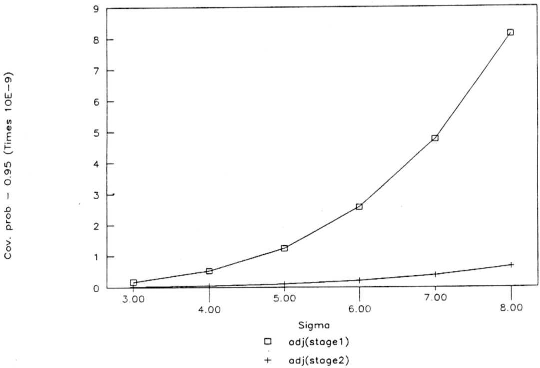

Fig. 4.4.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{312} = a_{321} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

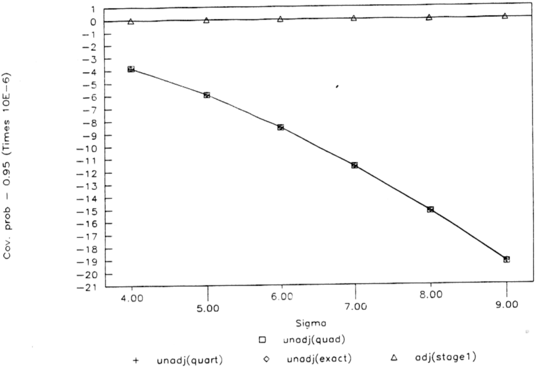

Fig. 4.4.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{312} = a_{321} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

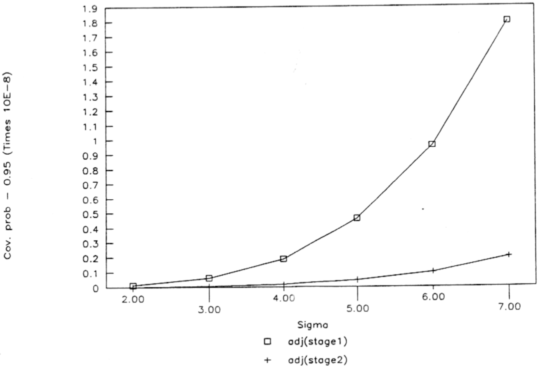

Fig. 4.5.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{312} = a_{321} = 0.001$, $a_{322} = 0.001$,
$n = 4$, $p = 2$, $\alpha = 0.05$)

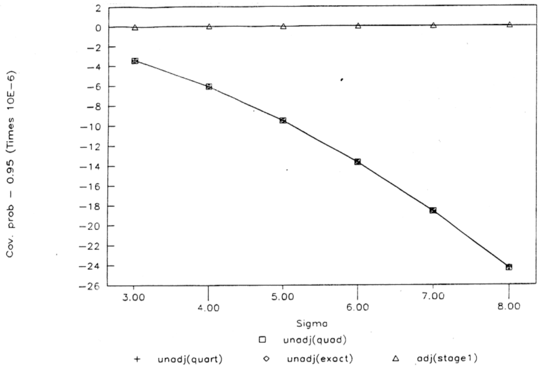

Fig. 4.5.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{312} = a_{321} = 0.001$, $a_{322} = 0.001$,
$n = 4$, $p = 2$, $\alpha = 0.05$)

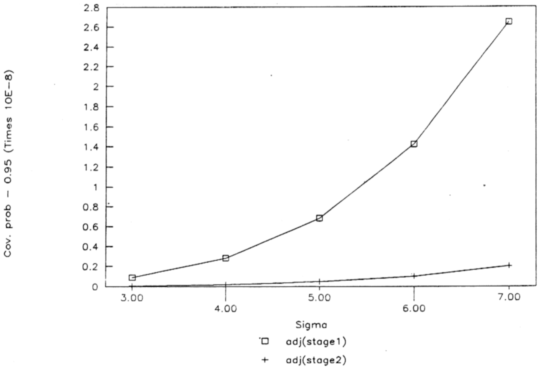

Fig. 4.6.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{411} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

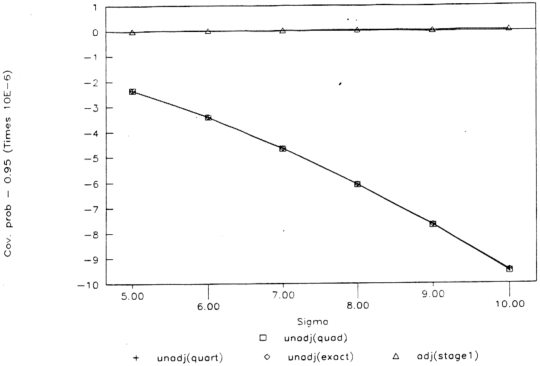

Fig. 4.6.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{411} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

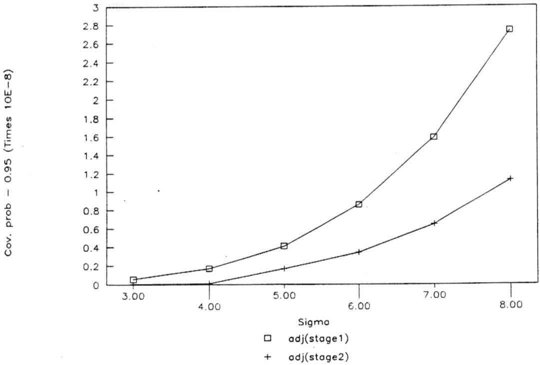

Fig. 4.7.1 Coverage Probability of Confidence Regions
($a_{312} = a_{321} = 0.001,\ a_{412} = a_{421} = 0.002,\ n = 4,\ p = 2,\ \alpha = 0.05$)

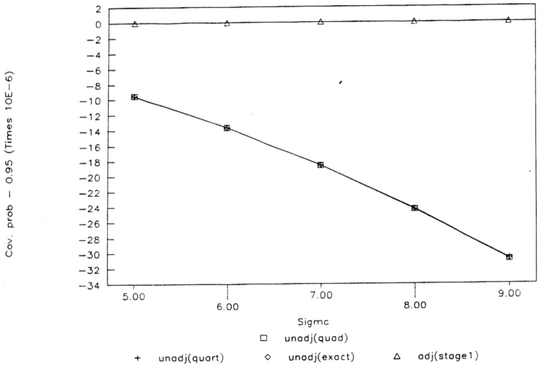

Fig. 4.7.2 Coverage Probability of Confidence Regions
($a_{312} = a_{321} = 0.001,\ a_{412} = a_{421} = 0.002,\ n = 4,\ p = 2,\ \alpha = 0.05$)

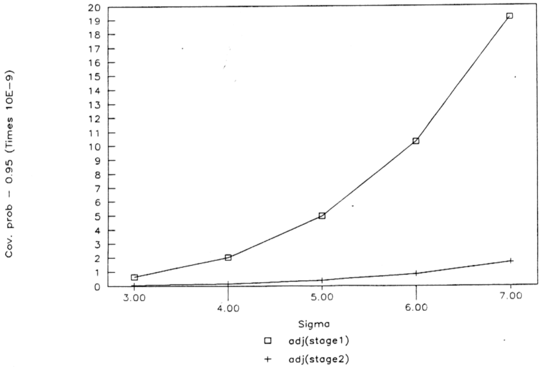

Fig. 4.8.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{422} = 0.002$, $n = 4$, $p = 2$, $\alpha = 0.05$)

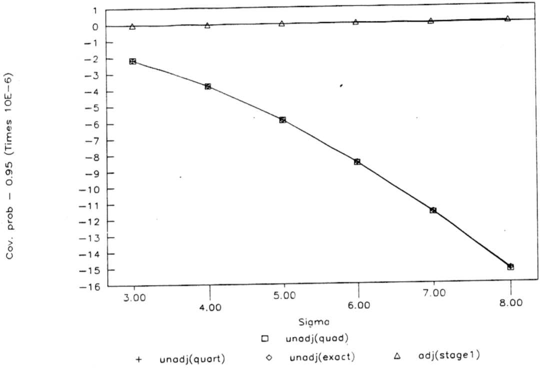

Fig. 4.8.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{422} = 0.002$, $n = 4$, $p = 2$, $\alpha = 0.05$)

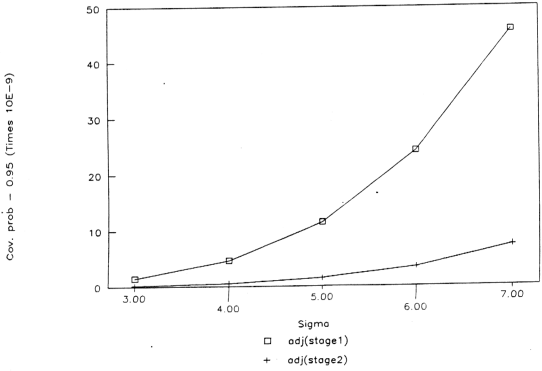

Fig. 4.9.1 Coverage Probability of Confidence Regions
($a_{312} = a_{321} = 0.001$, $a_{411} = 0.001$, $n = 4$, $p = 2$, $\alpha = 0.05$)

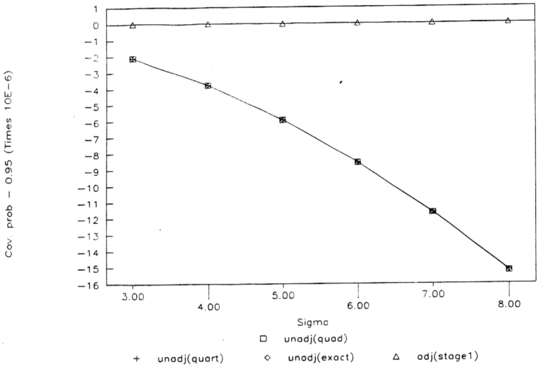

Fig. 4.9.2 Coverage Probability of Confidence Regions
($a_{312} = a_{321} = 0.001$, $a_{411} = 0.001$, $n = 4$, $p = 2$, $\alpha = 0.05$)

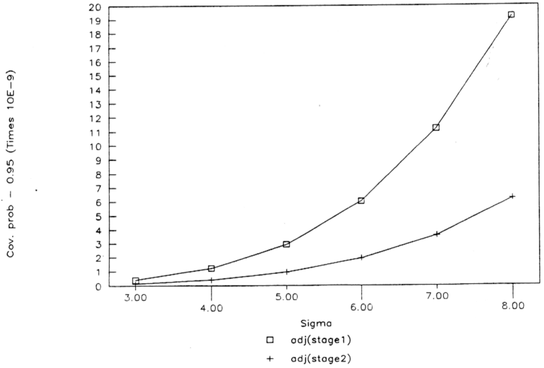

Fig. 4.10.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{322} = 0.001,\ a_{411} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

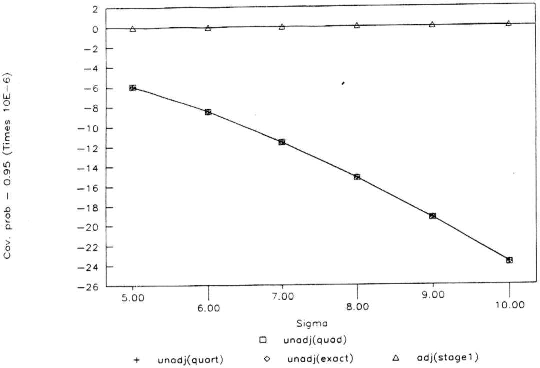

Fig. 4.10.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001,\ a_{322} = 0.001,\ a_{411} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05$)

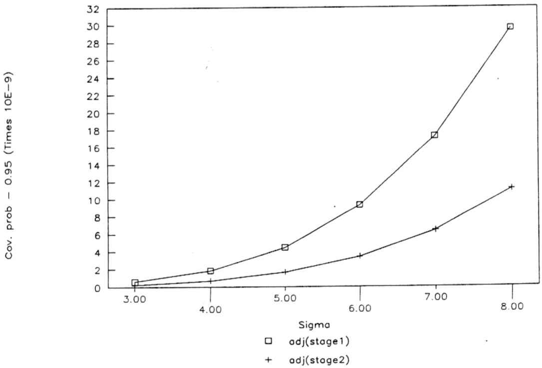

Fig. 4.11.1 Coverage Probability of Confidence Regions
$(a_{312} = a_{321} = 0.001,\ a_{411} = 0.001,\ a_{422} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05)$

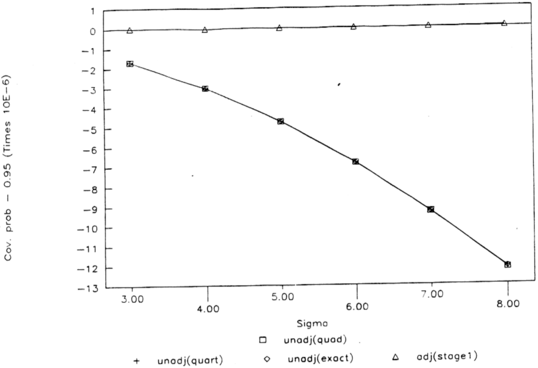

Fig. 4.11.2 Coverage Probability of Confidence Regions
$(a_{312} = a_{321} = 0.001,\ a_{411} = 0.001,\ a_{422} = 0.001,\ n = 4,\ p = 2,\ \alpha = 0.05)$

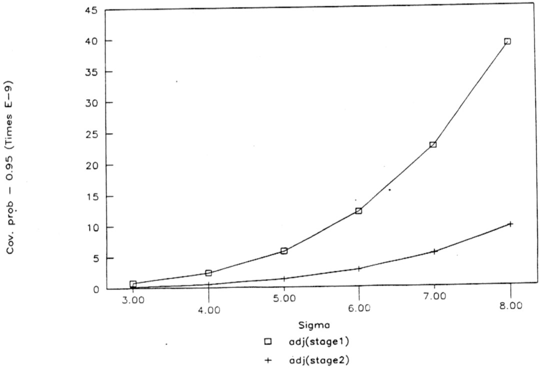

Fig. 4.12.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{322} = 0.001$, $a_{411} = 0.002$, $a_{422} = 0.001$,
$n = 4$, $p = 2$, $\alpha = 0.05$)

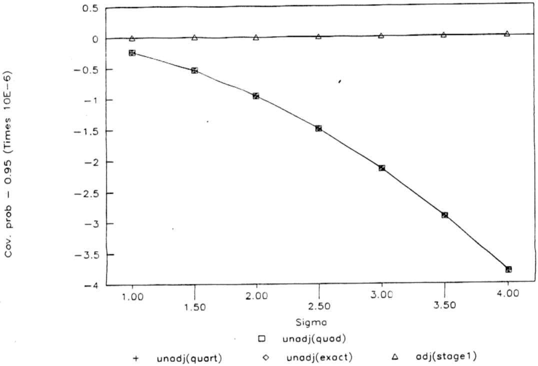

Fig. 4.12.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{322} = 0.001$, $a_{411} = 0.002$, $a_{422} = 0.001$,
$n = 4$, $p = 2$, $\alpha = 0.05$)

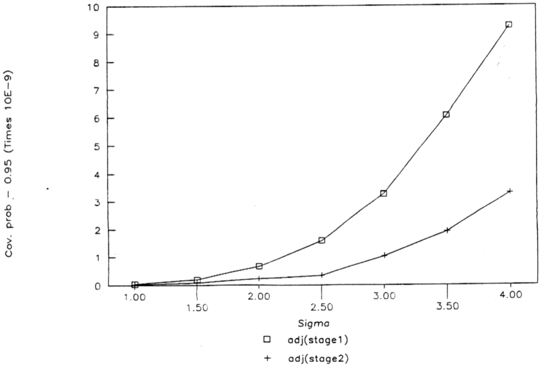

Fig. 4.13.1 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{312} = a_{321} = 0.001$, $a_{411} = 0.001$, $a_{412} = a_{421} = 0.002$, $n = 4$, $p = 2$, $\alpha = 0.05$)

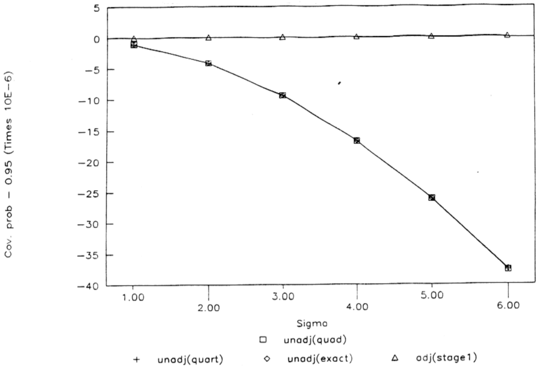

Fig. 4.13.2 Coverage Probability of Confidence Regions
($a_{311} = -0.001$, $a_{312} = a_{321} = 0.001$, $a_{411} = 0.001$, $a_{412} = a_{421} = 0.002$, $n = 4$, $p = 2$, $\alpha = 0.05$)

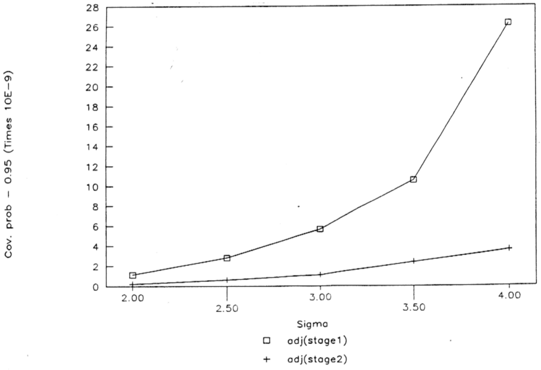

Fig. 4.14.1 Coverage Probability of Confidence Regions
($a_{311} = 0.001$, $a_{312} = a_{321} = 0.0015$, $a_{412} = a_{421} = 0.001$, $a_{422} = 0.0015$, $n = 4$, $p = 2$, $\alpha = 0.05$)

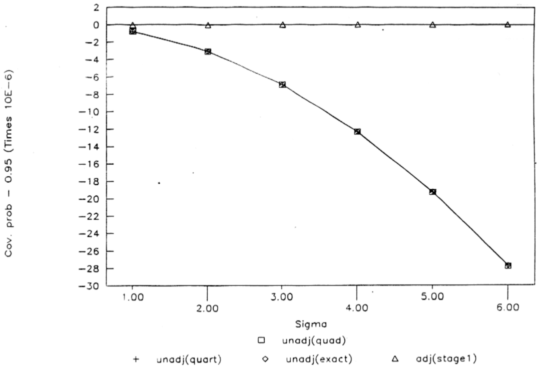

Fig. 4.14.2 Coverage Probability of Confidence Regions
($a_{311} = 0.001$, $a_{312} = a_{321} = 0.0015$, $a_{412} = a_{421} = 0.001$, $a_{422} = 0.0015$, $n = 4$, $p = 2$, $\alpha = 0.05$)

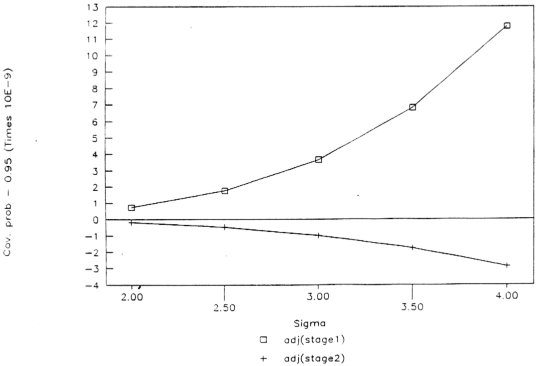

Fig. 4.15 Coverage Probability of Confidence Regions
($a_{3111} = 0.0001$, $a_{3122} = 0.00015$, $a_{3112} = 0.0001$, $a_{3222} = 0.00016$, $n = 4$, $p = 2$, $\alpha = 0.05$)

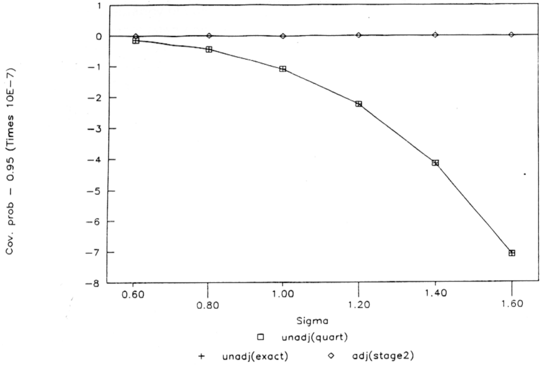

Table 4.1 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a311= −0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
10.00	−4.7500000000	−4.7394728124	−4.7381522542
11.00	−5.7475000000	−5.7320871446	−5.7300148109
12.00	−6.8400000000	−6.8181708238	−6.8150060948
13.00	−8.0275000000	−7.9974332995	−7.9927188795
14.00	−9.3100000000	−9.2695587561	−9.2627016034
15.00	−10.6875000000	−10.6342061127	−10.6248989336

Table 4.2 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a312=a321= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
6.00	−6.8400000000	−6.8361955920	−6.8359850299
7.00	−9.3100000000	−9.3029518644	−9.3025684162
8.00	−12.1600000000	−12.1479761919	−12.1473087841
9.00	−15.3900000000	−15.3707401844	−15.3696221320
10.00	−19.0000000000	−18.9706449998	−18.9688374177
11.00	−22.9900000000	−22.9470213443	−22.9441855711

Table 4.3 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a311= −0.001 , a322= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
3.00	−1.7100000000	−1.7097622245	−1.7097391060
4.00	−3.0400000000	−3.0392485120	−3.0391954271
5.00	−4.7500000000	−4.7481653125	−4.7480559620
6.00	−6.8400000000	−6.8361955920	−6.8359850835
7.00	−9.3100000000	−9.3029518645	−9.3025685121
8.00	−12.1600000000	−12.1479761919	−12.1473088375

Table 4.4 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a312=a321= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
4.00	−3.8000000000	−3.7978517840	−3.7976751074
5.00	−5.9375000000	−5.9322553320	−5.9318185467
6.00	−8.5500000000	−8.5391246564	−8.5381459825
7.00	−11.6375000000	−11.6173520834	−11.6153295257
8.00	−15.2000000000	−15.1656285438	−15.1617251835
9.00	−19.2375000000	−19.1824435731	−19.1753824801

Table 4.5 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a312=a321= 0.001 , a322= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
3.00	−3.4200000000	−3.4188134310	−3.4190892718
4.00	−6.0800000000	−6.0762498560	−6.0771537005
5.00	−9.5000000000	−9.4908443750	−9.4930645227
6.00	−13.6800000000	−13.6610148959	−13.6622860429
7.00	−18.6200000000	−18.5848277509	−18.5931655526
8.00	−24.3200000000	−24.2599976958	−24.2739121995

Table 4.6 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a411= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
5.00	−2.3750000000	−2.3723682031	−2.3720760884
6.00	−3.4200000000	−3.4145427059	−3.4139137889
7.00	−4.6550000000	−4.6448896890	−4.6436296583
8.00	−6.0800000000	−6.0627522558	−6.0603788012
9.00	−7.6950000000	−7.6673724488	−7.6631327505
10.00	−9.5000000000	−9.4578912496	−9.4506932892

Table 4.7 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a312=a321= 0.001 , a412=a421= 0.002)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
5.00	−9.5000000000	−9.4926612500	−9.4922618118
6.00	−13.6800000000	−13.6647823679	−13.6639214566
7.00	−18.6200000000	−18.5918074579	−18.5900827404
8.00	−24.3200000000	−24.2719047677	−24.2686542434
9.00	−30.7800000000	−30.7029607376	−30.6971570202

Table 4.8 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a311= −0.001 , a422= 0.002)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
3.00	−2.1375000000	−2.1360607945	−2.1358852267
4.00	−3.8000000000	−3.7954514000	−3.7949092270
5.00	−5.9375000000	−5.9263950194	−5.9249504319
6.00	−8.5500000000	−8.5269727123	−8.5235480889
7.00	−11.6375000000	−11.5948391066	−11.5874730916
8.00	−15.2000000000	−15.1272223992	−15.1301896621

Table 4.9 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions (a312=a321= 0.001 , a411= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
3.00	−2.1375000000	−2.1371019280	−2.1370547684
4.00	−3.8000000000	−3.7987418960	−3.7986253595
5.00	−5.9375000000	−5.9344284570	−5.9341704395
6.00	−8.5500000000	−8.5436308484	−8.5431038282
7.00	−11.6375000000	−11.6257003604	−11.6246899004
8.00	−15.2000000000	−15.1798703358	−15.1780334058

Table 4.10 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a322= 0.001 , a411= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
5.00	−5.9375000000	−5.9335912695	−5.9332983965
6.00	−8.5500000000	−8.5418948564	−8.5412723251
7.00	−11.6375000000	−11.6224842209	−11.6212586780
8.00	−15.2000000000	−15.1743837438	−15.1721052257
9.00	−19.2375000000	−19.1964677108	−19.1924382308
10.00	−23.7500000000	−23.6874603121	−23.6806735172

Table 4.11 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a312=a321= 0.001 , a411= 0.001 , a422= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
3.00	−1.7100000000	−1.7093282265	−1.7092304262
4.00	−3.0400000000	−3.0378768640	−3.0376142239
5.00	−4.7500000000	−4.7448165624	−4.7441921741
6.00	−6.8400000000	−6.8292516239	−6.8278976216
7.00	−9.3100000000	−9.2900873062	−9.2873626359
8.00	−12.1600000000	−12.1260298236	−12.1208812487

Table 4.12 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a322= 0.001 , a411= 0.002 , a422= 0.001)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
1.00	−0.2375000000	−0.2374706610	−0.2374583805
1.50	−0.5343750000	−0.5342264715	−0.5341920563
2.00	−0.9500000000	−0.9495305765	−0.9494501129
2.50	−1.4843750000	−1.4832289465	−1.4830591828
3.00	−2.1375000000	−2.1351235435	−2.1347904541
3.50	−2.9093750000	−2.9049723209	−2.9043546353
4.00	−3.8000000000	−3.7924892239	−3.7913968040

Table 4.13 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= −0.001 , a312=a321= 0.001 ,
a411= 0.001 , a412=a421= 0.002)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
1.00	−1.0450000000	−1.0449096621	−1.0448950108
2.00	−4.1800000000	−4.1785545940	−4.1784458539
3.00	−9.4050000000	−9.3976826321	−9.3971869856
4.00	−16.7200000000	−16.6968735038	−16.6951321110
5.00	−26.1250000000	−26.0685388278	−26.0634660254
6.00	−37.6200000000	−37.5029221132	−37.4948887727

Table 4.14 Discrepancy of coverage probability from 0.95 of the unadjusted confidence regions
(a311= 0.001 , a312=a321= 0.0015 ,
a412=a421= 0.001 , a422= 0.0015)

SIGMA	QUADRATIC (Times 10E−6)	QUARTIC (Times 10E−6)	EXACT (Times 10E−6)
1.00	−0.7718750000	−0.7718086628	−0.7717955285
2.00	−3.0875000000	−3.0864386048	−3.0863370949
3.00	−6.9468750000	−6.9415016867	−6.9410193668
4.00	−12.3500000000	−12.3330176764	−12.3312694605
5.00	−19.2968750000	−19.2554142489	−19.2502115520
6.00	−27.7875000000	−27.7015269866	−27.6988265238