

CHAPTER III

THEORETICAL FRAMEWORK

CHAPTER 3

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3.1 INTRODUCTION

This chapter will discuss monetary policy in relation to real activity. The effect of monetary policy (interest rate) shock to inflation expectations and real interest rate is also explained out. Meanwhile, in this chapter also reviews a benchmark of the theory by Lucas.

3.2 A General Framework

This section views the general framework of monetary policy and macroeconomic variables

3.2.1 Money Growth and Inflation

When it comes to understanding inflation over the longer term, economists look to one factor which is growth of the money supply. This is because repeated increases in prices require either repeated falls in aggregate supply or repeated rises in aggregate demand. Lets say technological

progress rather than money supply where repeated falls in aggregate supply are unlikely. This means most of other factors are limited scope. On the other hand, money supply can grow at almost any rate and observe huge variations in money growth (Romer, 1996)

To see why money is crucial to inflation, consider money market:

Condition equilibrium in the money market is

$$M/P = L(i, y) \quad (3.1)$$

Where M is the money stock, P is the level of price, i is the nominal interest rate, y is real income and $L(.)$ is the demand for the real money balances.

This condition implies that P level is

$$P = M/L(i, y) \quad (3.2)$$

Conventional estimates of money demand suggest that income elasticity of money demand is about 1 and the interest elasticity is about -0.2. Thus for the price level to double over some period of time without a change in the money supply, income must fall roughly in half or the interest rate must rise by a factor of about 32. Alternatively, the demand for real balance at a given interest rate, income must fall in half. In contrast, a doubling of the

money supply either over several years in a moderate inflation or over a few days at the height of a hyperinflation is not uncommon.

Thus, money growth plays a special role in determining inflation not because money affects prices more directly than other factors do, but because empirically variations in money growth account for most of the variation in money growth of aggregate demand.

3.2.2 Money Growth and Interest Rate

There are interesting links between the growth of the nominal money stock and the behavior of inflation, real and nominal interest rates and real balances. With the assumption that prices are completely flexible, money supply does not affect real output or real interest rates.

The relation of nominal interest rate, inflation expectation and real interest rate can be explain such that

$$I = E + R$$

Or

$$R = I - E \quad (3.3)$$

I = nominal interest rate

E = expectation inflation

R = real interest rate

In the medium run, nominal interest rates increase one for one with inflation known as the Fisher effect, or the Fisher hypothesis, after Irving Fisher an economist at Yale University who first stated it at the beginning of the twentieth century. Normally, in the medium run, nominal interest rate expected inflation is equal to actual inflation. Meanwhile, money growth does not affect the real interest rate, but effects both inflation and the nominal interest rate.

Using 3.3 and our assumption that r and y are constant, rewrite 3.2 as

$$P = M/L(r + \pi^e, y) \quad (3.4)$$

Assume that initially M and P are growing together at some steady rate (M/P) and that the inflation expectation equals actual inflation. Now suppose that at some time, time t there is a permanent rise in the money growth. The resulting path of the money stock is shown in the top panel figure 3.1. After the change, since M is growing at a new steady rate and the r and y are constant by assumption, m/p is constant that is (3.4) is satisfied with inflation expectation equal to the new rate of money growth.

If the price level rises faster after the change than before, expectation inflation jumps up when the change occurs. Thus the nominal interest rate jump up and so quantity of real balances demanded falls discontinuously. It

follows that price must jump up at the time of the change. There are 2 points here, first is change in inflation resulting from the change in money growth is reflected 1 for 1 in the nominal interest rate. In the medium run, nominal interest rates increase one for one with inflation is known as the Fisher effect, or the Fisher hypothesis, after Irving Fisher an economist at Yale University who first stated it at the beginning of the twentieth century. Normally, in the medium run, nominal interest rate expected inflation is equal to actual inflation. Meanwhile, money growth does not affect the real interest rate, but effects both inflation and the nominal interest rate. Thus, the assumption is inflation does not affect the real rate.

Secondly, a higher growth rate of the nominal money stock reduces the real money stock. The rise in the money growth increases expectation inflation, thereby increasing the nominal interest rate. This increase in the opportunity cost of holding money reduces the quantity of real balances that individuals want to hold. This equilibrium requires that price raises more than money does.

That is, there must be a period when inflation exceeds the rate of money growth. In our model this occurs at the moment that money growth increases. Suppose the policy makers want to reduce expectation inflation and that they do not want the price level to change discontinuously. A decline in inflation will reduce the inflation expectations and thus lower the nominal interest rate and raise the quantity of real balances demanded. For

sure money must jump up. To keep inflation low, the money stock must then grow slowly from this higher level.

Following Sargent (1982) declines in inflation - the ends of hyperinflations - area accompanied by spurts of very high money growth that continues for a time after prices have stabilized.

3.2.3 Price Rigidity

Theoretically, an increase in money growth increases nominal interest rates. In practice, the immediate effect of monetary expansion is to lower short term nominal rates. This negative effect of a monetary expansion on nominal rates is known as liquidity effects. Following conventional explanation means is that monetary expansions reduce real rates. If prices are not completely flexible, an increase in the money stock raises output, which requires a decline in the real interest rate, in terms of the IS LM, LM curve shifts to the right along the downward sloping IS curve. If the decline in the real rate is large enough, it more than offsets the effects of the rise in expectation inflation. If prices fully flexible in the long run then the real rate eventually returns to normal following a shift to higher money growth. Thus, if the real rate effects dominate the expectation inflation effect in the short term, the shift depresses the nominal rate in the short term but increases in the long run. As Friedman (1969) viewed, this appears to provide an accurate description of the effects of monetary policy in practice.

3.2.4 Monetary Policy and Inflation

Money affects output through an increase or decrease in the money supply. There are several views on this aspect (Sim's, 1992)

Hicks's IS LM model (1937) for the Keynesian view, described that an increase in the nominal money supply will cause disequilibrium in the monetary sector as there will be an excess supply of real balances because prices are assumed to remain unchanged. Hence, the increase in nominal money supply leads to equivalent changes in real money balances.

Monetarist argue that changes in the money supply will only affect the price level and have no impact on the real variables, interest rate or income. With effect of each economy, there is natural level of real output. When the nominal money supply increase, this will cause excess supply of money. Individuals will try to moving into bonds from holding money. This will reduce the interest rate and stimulate demand and also direct impact to higher money supply on expenditure. Further, individuals will substitute real assets for money balances. That is why output increase and spending on the real assets will increase.

The neoclassical macroeconomists led by Robert Lucas and Thomas Sargent had risen in 1970s. Monetary policy from their view is impotent.

Framework of neoclassical is based on 2 assumptions, rational expectation and complete wage and price flexibility.

For the first assumption, when monetary authority unexpectedly decides to raise money supply; the nominal aggregate demand will increase. The term “unexpectedly” means, prices remain unchanged while there is a rise in real demand. Thus, it’s pushing the economy over its natural rate. To deflate economy to its natural level prices will begin to increase. In the second assumption, when private sector anticipates the rise in money supply, they will demand higher prices to offset the rise of prices later. This, when prices and nominal demand rise together, real demand would be no impact.

New Keynesian such as Stanley Fischer, Edmund Phelps and John Taylor do not agree with the assumptions from the New Classical - complete wage and price flexibility. This is because from their view some wages and prices will not be full or complete due to long term contracts in the long term. When wages do not adjust fully, a rise in money supply will have an impact on the real sector even when it is anticipated. This describes the situation when prices do not offset completely the rise in nominal demand as above. If the assumption is denied, as new Keynesians said monetary policy can still influence the economy even with rational expectations.

Keynesians have their own view in using monetary policy to influence output or inflation. The interest rate may not be sensitive to changes in the money supply. Thus, the changes in the money supply will

only affect the interest rates slightly. The small changes in interest rates means that output will only be affected marginally. While investment and consumption maybe unresponsive to interest rate changes. When money supply increases, the reductions in interest rate would have little impact on investment and consumption and same goes to output. So, effectiveness of monetary policy will depend on how sensitive the changes in interest rates are to the demand for money and investment. On the other hand monetarist believes that monetary policy is affective in influencing output and controlling inflation.

3.3 A Lucas (1990) Analysis on Liquidity and Interest Rates

Preferences of the typical household is

$$E \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\} \quad (3.5)$$

Where (c_t) is a stochastic stream of consumption of a single good, β is a discount factor between zero and one, and U is a bounded twice differentiable function with $U'(c) > 0$, $U'(0) = \infty$, and $U''(c) < 0$. The household has a constant, non-storable goods endowment, y and in equilibrium, $c_t = y$ for all t and realization of shocks will be the next specify.

The assumption for trading is that there are three members of household who go their own way during a period. Then, the three will regroup at the end of a day to pool goods, assets and information.

- A: collect endowment, y and must sell to other households. A cannot consume the endowment
- B: takes an amount $N_t - Z_t > 0$ of the households initial cash balances N_t and it to purchase goods from other households
- C: C takes the remaining cash balances, $Z_t > 0$ and engages in securities trading

Now, considered the only security is a one period, dollar denominated government bond that entitles its purchaser to one dollar at the beginning of the following period, prior to any trading. The auctioned price of these bonds is q_t . So with N_t dollar, a household chooses the division Z_t if this balance can acquire $B_t < Z_t / q_t$

For the following period, initial cash balances given by

$$N_{t+1} = P_t y + Z_t + (1 - q_t) B_t \quad (3.6)$$

Followed Lucas(1990), “ the size of the government bond issue, expressed relative to the economy’s beginning-of-period money stock will be taken to be an i.i.d random variable x_t , with a probability distribution λ on a compact set $X(0, \infty)$ ”. The price of 1 period bonds λ will be the only variable responding to these shocks.

Lucas employed normalization for the $m_t = N_t / M_t$ and same to z_t, b_t, p_t . So it becomes

$$m_{t+1} = [1 + (1 - q_t) x_t]^{-1} [p_t y + z_t (1 - q_t) b_t] \quad (3.7)$$

Next, is define a stationary equilibrium consisting of a constant (normalized) price level $p > 0$, a constant division of money balances $0 \leq z < 1$ and a bond price $q(x)$ consistent with utility maximizing behavior and market clearing.

The Bellman equation, with $v(m)$ maximized objective functions for household beginning a period with (normalized) balances m

$$v(m) = \max_{0 \leq z \leq m} \left\{ U[(m - z) / p] + \beta \int \max_{x: 0 \leq q(x)b \leq z} [v(m')] \lambda(dx) \right\} \quad (3.8)$$

where m' is defined by

$$m' = [1 + (1 - q(x)) x]^{-1} [py + z + (1 - q(x)) b] \quad (3.9)$$

Then an equilibrium is defined as a value function $V: \mathbb{R} \rightarrow \mathbb{R}$, a number $Z \in [0, 1]$, a number $p > 0$, a bond purchase function $b: X \rightarrow \mathbb{R}$ bond price function $q: X \rightarrow (0, 1)$ such that

- i. Given p and $q(x)$, $v(m)$ satisfies (3.8)
- ii. z and b attain the right side of (3.8) at $m=1$
- iii. $1-z = py$
- iv. $b(x) = x$ for all x ,

and the conditions: (1) and (ii) describe utility maximization at equilibrium prices(iii) and (iv) require cleared goods and bonds market.

Next, is characterize equilibrium behavior. The assumption is the value function is exists and are increasing, differentiable and concave. The possibilities are $q(x) = 1$, and any feasible value of $q(x) < 1$ and $=z/q(x)$ or $q(x) > 1$ and $b=0$ for the inner maximization in (3.8)

For any m , the unique maximizing value of z in the outer maximization in (3.8) satisfies the first order condition

$$\frac{1}{p} U' \left(\frac{m-z}{p} \right) = \beta \int_x v'(m) \frac{1}{q(x)[1-xq(x)+x]} \lambda(dx) \quad (3.10)$$

Since $q(x) = \min [1, z/x]$, eliminating $q(x)$ from (3.10) to obtain

$$z = \beta \int_x \max \left\{ z, \frac{x}{1+x-z} \right\} \lambda(dx) \quad (3.11)$$

the (3.11) has a unique solution $z^* (0, 1)$. This is due to the right side of (3.11) is positive at $z=0$ and equal to β at $z=1$. it is a continuous and strictly increasing

function of z , with a scope strictly increasing function of z , with a scope strictly less than 1 if $z < 1$

Since $z^* \in (0, 1)$, then $p = (1 - z^*)/y$ is the equilibrium price level and $q(x) = \min [1, z^*/x]$ is the equilibrium bond price function.

Then, we use the above example to illustrate the potential force of the liquidity effect on the stochastic behavior of interest rates. The case when $q(x) < 1$ for all x so that $q(x) = z^*/x$.

With the example and the case as mentioned we can see that the interest rate is the i.i.d random variable.

$$r_t \cong -\ln[q(x_t)] = -\ln(z^*) + \ln(x_t) \quad (3.12)$$

By using Fisherian fundamental for the interest rate, the real rate is constant $[-\ln(\beta)]$ plus the expected inflation $(\ln p_{t+1} / p_t)$. So

$$\begin{aligned} &= -\ln(\beta) + \ln(p_{t+1} / p_t) \\ &= \ln(M_{t+1} / M_t) \\ &= -x_t q(x_t) + x_t \\ &= -z^* + x_t \end{aligned}$$

The interest rate in this case is much more variable than fisherian grounds because of the x_t is a small fraction. In this case, real interest rates are constant due to the endowments and real consumptions are constant. Changes in interest rates result from a mix of inflation expectation effects and liquidity effects, both driven by the same random variables x_t

3.3.1 Pure Liquidity Effects: A General Framework

This part is to specify the motion of the state more abstractly and to us functions defined on the state space to define various aspects of securities

Follows a Markov process with the transition function p

$$P(s, A) = \Pr\{s_{t+1} \in A | s_t = s\}, s \in S, A \in S \quad (3.13)$$

The end of t and beginning of $t+1$ holdings are linearly related ($s+1$ as being realized after cash is divided in period t , but before securities are traded) so an $n \times n$ matrix is B will be

$$a(s_{t+1}) = B \theta(s_t, s_{t+1}) \text{ for all } (s_t, s_{t+1}) \quad (3.14)$$

This notation is important to capture at the same time fixed maturity and infinite maturity securities. Let $q(s_t, s_{t+1})$ be the vector of securities prices when the current and next period states are (s, s')

The liquidity constraint for a household that carries z units of cash into securities trading and traded from the portfolio a to the portfolio u takes the form

$$z \geq q(s, s').(u - a) \quad (3.15)$$

This will begin next period with cash balances of

$$m' = \pi_0(s, s') + -q(s, s').(u - a) + \pi(s, s').u \quad (3.16)$$

Where $\pi(s, s)$ = net cash flow inflow from sources other than securities.

The household functional equation is

$$v(m, a, s) = \max_{0 \leq z \leq m} \left\{ U\left(\frac{m - z}{p(s)}\right) + \beta \int_s [\max v(m', Bu, s')] P(s, ds') \right\} \quad (3.17)$$

m is defined by (3.16) and inner maximization is subject to the constraint (3.15) then, the first order is us to exchange condition to characterize equilibrium

The conditions are

$$v_m(m, a, s) = U' \left(\frac{m - z}{p(s)} \right) \frac{1}{p(s)} \quad (3.18)$$

and

$$v_{ai}(m, a, s) = \beta \int_s [v_m(m', Bu, s') + \mu(s, s')] g_i(s, s') P(s, ds') \quad (3.19)$$

$$v_m(1, a(s), s) = y U'(y) \frac{1}{1 - z(s)} \quad (3.20)$$

We will define the function $\varphi_i : S \rightarrow R$, and define $\theta : S \times S \rightarrow R$ to get the Equilibrium

The constraint becomes, in equilibrium

$$z(s) + q(s, s') \cdot (a(s) - \phi(s, s')) \geq 0 \quad (3.21)$$

with equality if $\theta(s, s') > 0$

Can be written in more compactly as

$$\frac{1}{1 - z(s')} [q(s, s') - \pi(s, s')] + \theta(s, s') q(s, s') = B^T \varphi(s') \quad (3.22)$$

and

$$\varphi(s) = \beta \int_s \left[\frac{1}{1-z(s')} \pi(s, s') + B^T \varphi(s') \right] P(s, s') \quad (3.23)$$

Solved equation to get equilibrium asset prices

$$q(s, s') = \left[\frac{1}{1-z(s')} + \phi(s, s') \right]^{-1} \left[B^T \varphi(s') + \frac{1}{1-z(s')} \right], \quad (3.24)$$

where

$$\frac{1}{1-z(s')} + \phi(s, s') = \max \left\{ \frac{1}{1-z(s')}, \frac{1}{z(s)} \left[B^T \varphi(s') + \frac{1}{1-z(s')} \right] [\theta(s, s') - a(s)] \right\} \quad (3.25)$$

3.3.2 The Case of independent shocks

The assumption from Modigliani-Miller-Ricardian-equivalence, the outstanding stocks of securities should not matter unless they help to predict. So, if the conjecture of a constant solution is correct, equation (3.22) and (3.21) become

$$\varphi = \beta \int_x \left[\frac{1}{1-z} \pi + B^T \varphi \right] \lambda(dx) \quad (3.26)$$

and

$$\frac{z}{1-z} = \beta \int_x \left\{ \max \left[\frac{z}{1-z'} \left[\frac{\pi}{1-z} + B^T \varphi \right] x \right] \right\} \lambda(dx) \quad (3.26.1)$$

assumes the matrix $[I - \beta B]$ exists. Then solve (3.26)

$$\varphi = \beta \frac{1}{1-z} [I - \beta B^T]^{-1} \pi \quad (3.27)$$

We obtain

$$z = \beta \int_x \left\{ \max \left[z, [(I - \beta B^T)^{-1} \pi] x \right] \right\} \lambda(dx) \quad (3.28)$$

Rewrite(3.28)

$$z = \beta \int \max(z, \omega) \mu(d\omega) \quad (3.29)$$

Specific cases

A) Consol

In this case we assume that that only one security in the system; a consol with the coupon payment $\pi=1$. x_t is the issue of new consols at t and β is the matrix; w_t is the random variable which is equal to $(1 - \beta) x_t$. The equilibrium price of consols is $q^* = (1 - \beta) \beta$.

Define the rate of time preference ρ by $\beta = (1 + \rho)$. Then

$$q^* = 1/\rho \quad (3.30)$$

The equilibrium bond price function $q(x)$ can be obtained from (3.22) and (3.24).

So by using this example, these imply

$$q(x) = \left[\frac{1}{1-z} + \theta(x) \right]^{-1} \left(\frac{1}{1-z} \right) \left(\frac{1}{1-\beta} \right) \quad (3.31)$$

Where

$$\frac{1}{1-z} + \theta(x) = \max \left\{ \frac{1}{1-z}, \left(\frac{1}{1-z} \right) \left(\frac{1}{1-\beta} \right) \frac{x}{z} \right\}$$

Can be conclude that

$$q(x) = \begin{cases} z/x & \text{if} \\ \frac{1}{1-\beta} & \text{if} \end{cases} \quad (3.32)$$

If the liquidity constraint is slack the consol price is $(1+\rho)^{-1}$ not ρ^{-1} as if the current period is zero and all forward rates are ρ .

B) Fixed Maturity Bonds

For this case, we assume that bonds are issued maturing in 1, 2, ..., n periods. So let $x_t = (x_{1t}, \dots, x_{nt})$ β is an $n \times n$ matrix which is for this example and the payout function π is the vector $(1, 0, \dots, 0)$

To be relevant to this case, equation () becomes $\phi = \beta [I - z]$ and $\phi = \beta \phi$, $I=1 \dots n-1 \dots$ Solved the equation and obtain $\phi = \beta (1-z)$.

So the right of (4.2) implies

$$\left[\frac{1}{1-z} \pi + B^T \varphi \right] .x = \frac{1}{1-z} \sum_{i=1}^n \beta^{i-1} x_i = \frac{1}{1-z} \omega \quad 3.28$$

and the second equality defines the random variable $\omega = \eta(x)$. $(1-z)$ cancelled, so (3.26.1) implies

$$z = \beta \int_x \max(z, \omega) \mu(d\omega) \quad (3.29)$$

Thus, when are specialized to these, equilibrium bond prices must satisfy

$$q_i(x) = \left[\frac{1}{1-z} + \theta(x) \right]^{-1} \left(\frac{1}{1-z} \right) \beta^{i-1}, \quad i=1 \dots n, \quad (3.30)$$

where

$$\frac{1}{1-z} + \theta(x) = \max \left\{ \frac{1}{1-z}, \left(\frac{1}{1-z} \right) \frac{1}{z} \sum_{i=1}^n \beta^{i-1} x_i \right\}$$

These equations can be simplified as

$$q_i(\omega) = \begin{cases} \beta^{i-1(z/\omega)} & \text{if } z < \omega \\ \beta^{i-1} & \text{if } z \geq \omega \end{cases} \quad (3.31)$$

From the mathematical approach as stated above, I present the conclusion for this theory. There are two constraints for cash in advance, one for goods market and one for asset markets. Thus, the household has to decide how much money will be use to buy goods and to buy assets. The assumption as earlier stated is that no further portfolio adjustment is allowed in the beginning of the period. So, these monetary disturbances will affect the price of bonds and the nominal and real interest rats. If bond supply decrease, bond prices will going up and interest rates will going down to clear the bond market. Due to the disturbances that do not affect the growth rate of money and expected inflation, only the real asset prices are affected. Therefore, the situation of an “excess volatility” for the asset prices is exists. This is different from this theory with the traditional fundamentals.

3.4 Conclusion

This chapter presents the theoretical review for this study.