3.0 Theoretical Perspective

3.1 Introduction

In this chapter the theoretical outline on input-output analysis is presented. Chapter 4 discusses the methods used to analyse the input-output table. Empirical results are presented and discussed in Chapter 5. The discussion of the input-output analysis is presented in the following order, (I) the origin of input-output economics, (II) the input-output model, (III) assumption of input-output model, (IV) Rasmussen's linkage relationship and (V) multipliers.

3.2 The Origin Of Input-Output Economics

The input-output model was introduced and pioneered by the Russian-born United States economist Wassily Leontief. Leontief commenced his research on an empirical model of the United Stated economy in 1930 and published his first results in 1936. In 1936 Wassily Leontief published an input-output system of the United States economy and developed a more rigorous analytical framework, which we now call the input-output analysis. The model has long been applied to analyse the structural interrelationship within the sectors, through the relationship between the producing and consuming sectors in the economy. It also examines the degree of interrelationship or linkages between sectors.

However, the origin of input-output analysis may be traced back to Francois Quesnay's "Tableau Economique" in 1758, which depicted the workings of a farm and the interdependence between the production sectors of the economy that stressed in Walras's general equilibrium model in 1870s. As pointed out by Dorfman

(1954), Leontief's work was to simplify Walras's general equilibrium model, such that the model's equation could be estimated empirically, by using more readily available published data. Leontief's original table showed how each sector of the economy depended upon every other sector, but it was still highly aggregate. The advancement in computer technology enhances the computational methods and permitted a great deal of disaggregation in input-output analysis.

The progress in the development of national input-output models proceeded in the 1940s and 1950s. In 1950, the development of regional input-output analysis was constructed. The development of input-output model is still undergoing a process of revision and change and is expected to be the new and developing branch of economic science in future.

3.3 The Input-Output Model

The structure of the input-output model is in the form of a transaction matrix. The standard layout of input-output transactions are provided in Table 3 and 4. Table 3 is subdivided into four quadrants. Quadrant I is the value of goods and services that flow between sectors. This quadrant indicates that the outputs of sectors become inputs for other sectors. This quadrant is also known as the processing sector.

Quadrant II shows the consumption of goods and services by the final buyers. This quadrant indicates sectors from which the purchases are made. Quadrant III shows payments made for the use of "primary" inputs in each sector. The rows of this quadrant show the primary inputs used in the economy have been divided among the various productive sectors, whereas, the columns show the value of primary input used by each sector

Quadrant IV shows how the primary inputs pass directly into final. This includes imports of consumer and capital goods and direct employment of labour by households and non-trading government departments.

The accounting of input-output transactions is obtained by summing across rows and down the columns. Summing across rows indicates the output allocation from each sector or the sales of industries to other industries. For instance sector i in row i sells X_{i2}, X_{i3} and X_{i4} unit to other sectors of the economy while retained X_{ii} for its own used.

Summing down the column indicates the absorption of inputs by the sectors or the input amounts purchase by industry j. The purchases by industry j come from three major components that comprise intermediate inputs (X_{ij}), primary inputs (V_{ij}) and imports (M_{ij}). For instance sector j in column j receives X_{2j}, X_{3j} and X_{4j} units from the other sectors as inputs and X_{jj} units input from itself respectively. The final demand sectors consist of two main groups. The local final demand and exports. As simplified by Richardson (1971) the local final demand consist of household consumption (C), private investment (I) and government expenditure (G).

Assuming we have n industries in an economy. The total gross output of sector i, X_i can be written as :

$$X_i = \sum_{j=1}^{n} X_{ij} + f_i \tag{3.1}$$

Where :

X_1 = the total output or gross total sales of sector i.

X_{ij} = the total amount of sector i's output used by sector j as input.

f_i = the final demand.

The intermediate inputs refer to inputs produced by other local producing sectors, primary inputs are the value added components in the form of wages, profit, rent, interest and taxes.

In order to produce a unit of the jth commodity an amount of the ith commodity is required as input. The amount of input required could be denoted as "a_{ij}" representing input coefficients. To produce one unit of the jth commodity the industry requires inputs from other commodities given by a_{1j}, a_{2j}, until a_{nj} respectively. In the form of a mathematical expression the coefficients are derived as :

$$a_{11} = X_{11} / X_1 \qquad (3.2)$$

$$a_{12} = X_{12} / X_2 \qquad (3.3)$$

That is, by dividing each element in a column by the column total.

3.4 Assumptions Of The Input-Output Model

The three basic fundamental assumptions underlying the input-output model are as below:

a. Linear homogeneous production function for each sector. This assumption it implies that :

   i. Technical coefficients are constant.

ii. There are no economies and diseconomies of scale and externalities.

iii. No substitution among inputs. Inputs purchased by each sector are a unique function of the level of sectoral output.

iv. Constant and stable trade patterns among sectors, between sectors and the rest of the world.

b. The economy operates under conditions of spare capacity. Consequently, any increase in the final demand can be fulfilled by an increase in sectoral output.

c. Assumption about the existence of unemployment in the economy. Therefore, the additional demand for labour will be interpreted as the number of employed persons.

3.5 Structural Interdependence

To estimate "linkage" relationships and 'key sectors" Rasmussen's measure was used, because it takes into account the indirect and direct repercussion and also defines the coefficients in each linkage measure. Rasmussen's measure also distinguish the distribution of inputs or deliveries to various sectors using the coefficients for variation for each linkage estimate.

The gross output level X's required to maintain a given level of final demand f are determined by the below equation :

$$X = (I-A)^{-1} f \qquad (3.4)$$

Where :

I = is the identity matrix

A = is the technology matrix of input-output coefficients

$(I-A)^{-1}$ = is the Leontief inverse matrix

f = is the final demand vector.

By denoting the elements of the inverse matrix $(I-A)^{-1}$ as Z_{ij}'s, the sum of the column elements :

$$\sum_{i=1}^{n} Z_{ij} = Z_j \qquad (j = 1.2\ldots\ldots n) \qquad (3.5)$$

The above equation indicates the total input requirement for a unit increase in the final demand of the jth sectors. Summing across the row elements yields the following equaiton :

$$\sum_{j=1}^{n} z_{ij} = z_i \qquad (i = 1.2\ldots\ldots n) \qquad (3.6)$$

The above indicates the increase in the output of the ith sector needed to satisfy the intermediate input demand of all jth sectors induced from a unit increase in the final demand of all these jth sectors.

The set of averages as below :

$$\frac{1}{n} z_j \qquad (j = 1.2\ldots\ldots n) \qquad (3.7)$$

The above equation estimated increase in output (direct and indirect) to be supplied by the ith sectors if the final demand of the product of sector j (where j=1,2 ....n) increases by one unit.

The row's averages set :

$$\frac{1}{n} z_{i.} \qquad (i =1.2 \ldots\ldots n) \qquad (3.8)$$

The equation can be interpreted as the increase in output to be supplied by sector i (where i=1,2.....n), for a one unit increase in final demand of the sector's product.

For inter-sectoral comparison the above set of averages are normalized by relating them to the overall average defined as below :

$$\frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n} z_{ij} = \frac{1}{n^2}\sum_{j=1}^{n} z_{.j} = \frac{1}{n^2}\sum_{i=1}^{n} z_{i.} \qquad (3.9)$$

The sector's degree of interdependence with its economy can be known through the interdependence in transactions. The relationship can be obtained by the well known sectoral linkages : backward linkages and forward linkages

The backward and forward linkages as define by Rasmussen (1956) are U_j and Ui respectively. The two indices are defined as below :

Backward linkages :

$$U_j = \frac{\frac{1}{n} Z_{.j}}{\frac{1}{n^2}\sum_{j=1}^{n} Z_{.j}} \qquad (3.10)$$

Forward linkages :

$$U_i = \frac{\frac{1}{n} Z_{i.}}{\frac{1}{n^2}\sum_{j=1}^{n} Z_{i.}} \qquad (3.11)$$

U_j and U_i have been termed the "index of power of dispersion: and the "index of sensitivity of dispersion". U_j and U_i are also measures of backward and forward linkages respectively. The index of power of dispersion indicates to which extent a change in the final demand of sector j affects the whole system of sectors. Alternatively, U_j shows how much a change in sector j will cause changes in the rest of the system of sectors. The backward linkages refer to activities that use significant amounts of intermediate input from other activities for the production function. The forward linkage is defined as that which caters for final demand but also induces attempts to utilize its outputs to be used as inputs in other new activities (Hirschman, 1958).

The backward and forward linkages according to Hirschman (1958) can be interpreted as below :

Where $U_j > 1$, sector j draws heavily on economy. Alternatively, it indicate the impact of a unit increase in the final demand of sector j on the system of sectors will be big as compared to the sectors in general.

Where $U_i > 1$, sector i has to increase its output more than other sectors for each unit increase in final demand. Alternatively, indicate the impact of a unit increases in final demand of sector i on the system of sectors will have great repercussions compared to the sectors in general.

From equation 3.4 letting the Leontif inverse matrix $(I\text{-}A)^{-1}$as R, equation will be as below :

$$X = Rf \tag{3.12}$$

According to Richardson (1972) each entry in R is the interdependency coefficient (r_{ij}), which indicates the direct, and indirect requirement for sector i per unit of final demand for the output of sector j. The gross output X for each industry is obtained by multiplying the inverse matrix R by the size of final demand f. Hence, indirectly the above equation can be used to estimate the total impact on the economy and the individual sub sectors which is due to changes in final demand.

As described previously, the final demand includes the consumption of households, private investment, government expenditure and exports. If the entire final demand components are exogenous, then this kind of analysis is refered as type I analysis. The type II analysis refers to the inclusion of one or more final demand components in the Leontief inverse matrix, R. By transforming the equation in 3.9 the type II analysis will be as shown below :

$$X = R^{*}f \tag{3.13}$$

The matrix of R^{*} is an (n+1) by (n+1) matrix of direct, indirect and induced requirement coefficients r_{ij}.

3.6 Variation

The value of the coefficient of variation was introduced in order to overcome the problem of exaggerated values due to the sensitivity of U_j and U_i . Rasmussen (1958) developed the measures of variability for backward and forward linkages as bellow :

Backward Variation :

$$V_j = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(z_{ij} - \frac{1}{n}\sum_{i=1}^{n} z_{ij}\right)^2}}{\frac{1}{n}\sum_{i=1}^{n} z_{ij}}$$

(3.14)

Forward Variation:

$$V_i = \frac{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(z_{ij} - \frac{1}{n}\sum_{j=1}^{n} z_{ij}\right)^2}}{\frac{1}{n}\sum_{j=1}^{n} z_{ij}} \qquad (3.15)$$

3.7 Indices of Employment Generations

The measures of employment generation based upon backward as well as forward linkages can be obtained by extending the method suggested by Rasmussen for calculating linkage measures for output. The extension involves consideration of the matrix, E given by :

$$\begin{aligned} E &= b(I-AX)^{-1} \\ &= bz \end{aligned} \qquad (3.16)$$

b = sectoral employment output coefficients

z = elements of the inverse matrix

The above equation shows the direct and indirect employment requirements per unit of final demand. The elements of matrix, E, can be denoted as E_{ij} (i, j = 1,2 ....n).

Therefore, the sum of column elements :

$$\sum_{i=1}^{n} E_{ij} = \sum_{i=1}^{n} b_{i} z_{ij}$$
$$= E_{.j} \qquad (3.17)$$

indicates the total employed which is required by jth sector per unit increase of jth final demand.

The sum of row elements :

$$\sum_{j=1}^{n} E_{ij} = \sum_{j=1}^{n} b_{i} z_{ij}$$
$$= E_{i.} \qquad (3.18)$$

indicates the increase in employment in sector i per unit increase in the final demand in all sectors

The column and row averages :

$$\frac{1}{n} E_{.j}$$

$$\frac{1}{n} E_{i.}$$

$$(3.19)$$

The overall averages :

$$\frac{1}{n^2} \sum_{j=1}^{n} \sum_{i=1}^{n} E_{ij} = \frac{1}{n^2} \sum_{j=1}^{n} E_{.j} = \frac{1}{n^2} \sum_{i=1}^{n} E_{i.} \qquad (3.20)$$

Therefore, the employment indices will be as bellow :

Backward Employment Generation :

$$EU_{j} = \frac{\frac{1}{n}E_{.j}}{\frac{1}{n^2}\sum_{j=1}^{n} E_{.j}} \tag{3.21}$$

Forward Employment Generation :

$$EU_{i} = \frac{\frac{1}{n}E_{i.}}{\frac{1}{n^2}\sum_{j=1}^{n} E_{i.}} \tag{3.22}$$

The EU$_{.j}$ indicates the extent of employment generated in other sectos by the jth sector due to a change in the jth final demand, while EU$_{i}$ indicate the extent to which employment in the ith sector is affected by the increase in the final demand of all sectors. The corresponding measures of relative variability will be given by :

Backward variability :

$$EV_{j} = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(E_{ij} - \frac{1}{n}\sum_{i=1}^{n} E_{ij}\right)^2}}{\frac{1}{n}\sum_{i=1}^{n} E_{ij}} \tag{3.23}$$

Forward variability :

$$EV_i = \frac{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(E_{ij} - \frac{1}{n}\sum_{j=1}^{n}E_{ij}\right)^2}}{\frac{1}{n}\sum_{j=1}^{n}E_{ij}} \qquad (3.24)$$

3.8 Determination of Key Sactor

The forward and backward linkages enables the determination of key sectors through the process of ranking. There are two steps in identifying the key sectors. Firstly, sectors are ranked in descending order by the size of linkages indices U_j and U_i or EU_j and EU_i. Then, ranking is carried out based on the coefficient of variation with the smallest deviation considered first. At the end of the process of ranking, the sectors with high linkage indices and low coefficient of variation will come first.

These are the sectors with high backward or forward linkages with other intermediate sectors. A key sector is defined as a sector that has both strong backward and forward linkages and relatively low variability. Therefore, the development priority should focus on this sector in order to accelerate economic growth.

3.9 The Multipliers

The concept of multipliers play an important role in the input-output model. The uniqueness of multipliers is that they are disaggregated and the total impact depends on the initial final demand change. According to Richardson (1972) there are three common types of multipliers : output multipliers, income multipliers and employment multipliers.

3.9.1 Output Multipliers

Output multipliers for industry i measures the sum of direct and indirect requirement (output) from all sectors that is needed to deliver an additional dollar of output of i to final demand. Indirectly, output multipliers indicate the degree of interdependence between a sector and the rest of the economy. The larger the multiplier, the greater the interdependence of the sector to the rest of economy. To obtain the output multiplier of a sector, simply sum up the entries in the column of a particular sector in Leontief inverse matrix ,R. There are two types of output multipliers: type I and II.

3.9.1.1 Type I Output Multipliers

Type I (direct and indirect) is the output multiplier for sector j obtained by summing the jth column in the matrix R. The type I output multipliers indicate the direct and indirect local requirement per unit of sales from sector j to final demand.

3.9.1.2 Type II output Multipliers

The type II direct, indirect and induced output multipliers is the sum of the jth column in matrix R^*. The second type of output multipliers can be interpreted as the local requirements for unit sales from sector j to final demand and the resultant impact of local consumption spending associated with changing wages and income.

3.9.2 Income Multipliers

Compared to output multipliers, the income multipliers is more useful in the analysis of input output model. The income multipliers can also be divided into two types : type I and type II.

3.9.2.1 Type I Income Multipliers

The type I income multiplier is the ratio of direct and indirect income change relative to the direct income change, resulting from a unit increase in final demand for a sector. The direct income change for a sector is the household row entry of their regional input output table that is expressed in an input coefficient. The direct and indirect income change can be derived by summing up the product of the ith column entry of the R matrix (r_{ij}) and their corresponding direct income coefficient.

The type I income multiplier is as below :

$$\frac{\sum_{j=1}^{n} (a_i)(r_{ij})}{a_j} \qquad (3.25)$$

3.9.2.2 Type II Income Multiplier

The type II income multiplier is the ratio of the combined direct indirect and induced income change to the direct income change due to a unit increase in final demand for a given sector. This type of income multipliers includes the effect of secondary consumer spending. Therefore, the income expansion will be larger which is caused by endogenizing the household's sector in the economy. The direct and indirect and induces income change per unit of final demand is given by the household (row) coefficient of matrix, R^*. Denoting the household coefficient as (r^*_{ij}) in the matrix, R^*, the type II income multiplier will be as below :

$$\frac{\dot{r}_{ij}}{a_i} \tag{3.26}$$

3.9.3 The Employment Multiplier

To forecast the changes in jobs in a particular area the employment multipliers will be needed. He employment multiplier is one of the important multiplier besides income and output multiplier. A description of the tow type of employment multipliers is given below.

3.9.3.1 Type I Employment Multipliers

The type I employment multiplier is the ratio of direct and indirect employment change to direct employment change. The direct employment for sector j, e_j, can be derived using the slope of employment production functions (Richardson, 1972).

$$E_i = a + bX_i \tag{3.27}$$

Where E_i = is the employment for sector i

X_i = is the output for sector i

a and b are constant

The changes of direct employment for i :

$$e_i = \frac{E_i}{X_i} \tag{3.28}$$

The above equation measures the employment per dollar worth of output of sector i. The direct and indirect change in employment is derived by summing up the products of the jth column in the R matrix and the corresponding direct employment changes (Miller and Blair 1985, and Ceperly 1978; Richardson,1972) as below:

$$\sum_{i=1}^{n} r_{ij} e_i \qquad (3.29)$$

The type I employment multiplier :

$$\frac{\sum_{i=1}^{n} r_{ij} e_i}{e_i} \qquad (3.30)$$

3.9.3.2 Type II Employment

Type II employment multiplier is derived by the ratio of the direct indirect and induced employment changes to the direct employment change. The direct indirect and induced employment changes for sector j is shown below :

$$\sum_{i=1}^{n} \dot{r}_{ij} e_i \qquad (3.31)$$

The $\dot{r}_{ij}$ is the entry from the expanded inverse matrix with household endogenous, R^*. Therefore, the type II employment multiplier is :

$$\frac{\sum_{i=1}^{n} \dot{r}_{ij} e_i}{e_i} \qquad (3.32)$$

The above equation is the multiplier of total jobs created in all sectors throughout the local economy as a result of each new job created by sector i (Millere and Blair,1985).

3.10 Conclusion

In this chapter, we outlined the origin of input output economics, discussed the calculation of output and employment linkages. This chapter also touched on the concept of multipliers. However, it is beyond the scope of this study to discuss the economic impact based on the concept of multipliers.

Table 3 : Transaction Table

From / To	Purchasing sectors 1 2 3 4…j…n	Local Final Demand			Exports	Total Gross Output
		Household	Private Investment	Government		
Producing sectors 1 2 3 4 I . . . . n	Quadrant I	Quadrant II				
Labour (L) Other Value Added (V)	Quadrant III	Quadrant IV				
Import (m)						
Total Gross Outlay						