Chapter 3

Modified Unaligned Permeance

3.1 Introduction

The earliest known method to determine permeance in electrical machines is the Conformal Transformation method [31]. Due to its complex algorithm, the Finite Element (FE) method is introduced. This method was found to be more accurate compared to the Conformal Transformation method, but it is computationally time consuming. In the high-tech era, where the computers are used extensively in research works, the FE method is still considered inefficient. Therefore, a new method, known as the Assumed Flux Path (AFP) is developed, to meet the requirements and is faster compared with the previous methods.

This chapter to establishes the relevance of the concept of permeance in SRMs design, and finally addresses the problem associated with the choice of suitable evaluation method applicable to a particular selected geometric type of SRM. A data-base of normalised permeance coefficients of the rectilinear geometric proportion, when subjected to an application of appropriate correction factor, can definitely form the basis of an efficient and stable algorithm in the computation of practical multi-tooth per pole SRMs.
CHAPTER 3: MODIFIED UNALIGNED PERMEANCE

The flux contours for multi-tooth per pole SRMs show a complex pattern, linking the various magnetic parts. However, in single-tooth per pole SRM, less complicated flux paths are observed when computed using the electromagnetic field computation, based on the FE method and this conveniently leads to an analysis of permeance based on the Assumed Flux Path (AFP) technique. This in turn results in a stable computation algorithm for the analysis of this type of SRM, providing an insight into added advantage of ease of mathematical manipulations.

The computational algorithm concerned is incorporated within a design package and forms an important design toolkit within the final CAD program, which is based on the analytical expressions of permeance components, corresponding to the various defined flux paths.

3.2 Fundamental relationship

Flux is generally a function of permeance and mmf. In this light, permeance can be evaluated purely in terms of geometrical dimension. The elemental air-gap permeance in a normalised form due to a cylindrical shell of flux within the strip is

$$\text{Normalised elemental permeance} = \frac{dn}{d\alpha}$$  \hspace{1cm} (3.1)
Fig. 3.1: Non-parallel plane equipotential surface, bounded by lines parallel to a common axis at two bounding radii used for basic calculation for normalised permeance.

The total permeance due to all cylindrical shells can be obtained by integrating, between two bounding parameters, \( r_1 \) and \( r_2 \), giving:

\[
\text{Total permeance} = \frac{\ln \left( \frac{r_2}{r_1} \right)}{\alpha}
\]  \hspace{1cm} (3.2)

In a linearised geometry, normalised permeance can thus be defined in terms of the ratio of width to length in the direction of flux passage, when flux traverses a cross-section of a rectangular dimensional structure. This definition implies that when the stator and rotor teeth of a simplified 2-pole singly excited SRM are aligned, the approximate aligned permeance ratio can simply be expressed as:

\[
P_1 = \frac{t_{av}}{g}
\]  \hspace{1cm} (3.3)
where

\[ t_{av} = \frac{t_s + t_r}{2} \]

and

\[ g = \text{airgap width} \]

The above definition serves as the basis for the calculation of aligned and unaligned permeance coefficients normally cited in SRM design. These coefficients refer to the equivalent smooth surfaced tooth-pitch/gap ratios, for the extreme positions. One efficient design route involves tabulation of these two extreme permeances for idealised linearly developed geometrical structures, and this is developed later. Prior to discussions on the various models used to evaluate the unaligned permeance components, it is necessary to discuss in perspective of the significance of these permeance coefficients.

### 3.3 Significance of permeance

A design strategy employing this concept of permeance has the following merits:

1) A data-base of normalised permeance derived using rectilinear air-gap dimensions or otherwise, is always useful for a particular design strategy, provided an efficient and stable interpolation method can be used.
2) Flux lines in an SRM region such as those bounded by toothed profiles of the stator and rotor, leakage paths joining adjacent poles and those linking the back-of-core can best be analysed using leakage permeance components.

3) Difference in permeance between aligned and unaligned rotor positions relative to the stator, gives a measure of the mean torque as a result of permeance variation of the doubly salient structures. The design analysis, therefore, focused solely on the two extreme permeances, normally cited as $P_1$ and $P_2$ corresponding to aligned and unaligned permeance respectively. In addition, permeance can be used to calculate the normal force, $F_n$ for a doubly salient tooth configuration.

4) Two dimensional (2D) and even three dimensional (3D) leakage effects can best be analysed in terms of permeance component corresponding to selectively defined flux tubes or paths.

3.4 Model of the analysis

Previous aligned permeance formula, $P_1 = \frac{t_{av}}{g}$ can be improved by allowing for fringing effects. The modified formula of the aligned permeance can be written as [21]
\[ P_i = \frac{t_{av}}{g} + \frac{s}{g} \left( 1 - \frac{2}{\pi} \tan^{-1} \left( \frac{s}{g} \right) \right) + \frac{1}{\pi} \ln \left( 1 + \frac{s}{g} \right) \]  \hspace{1cm} (3.4)

where

\[ t_{av} = \frac{t_s + t_r}{2} \]

\[ g = \text{airgap width} \]

and

\[ s = \text{rotor slot width} \]

A crucial aspect of the analysis of the SRM revolves around the permeance in the unaligned position where no single analytical formula can be correctly applied and where end-region fields are not usually considered directly. Several models, as subsequently described, are therefore necessary for the derivation of the unaligned permeance coefficients forming the essential foundation bricks, and for the subsequent development of robust design algorithms. These then lead to the development of stand-alone computer routines, which are incorporated in the developed CAD package.

3.4.1 Conformal Transformation (CT) method

This section deals with the transformation of fields having polygonal boundaries by numerical techniques. Modern direct-search techniques for the minimisation of multivariable functions are employed, together with a method of integration in which the step length is continuously and automatically
varied. The correction factors derivation is due to some degrees of taperedness and effect of finite slot depth configuration. It has long been recognised for their power in dealing with 2-dimensional problems, and the range of their application has greatly increased through the exploitation of numerical methods and digital computers.

3.4.2 Finite Element (FE) method

The Finite Element model analysis is only confined to the flux contours generation and flux versus mmf characteristics determination for the sole purpose of subsequent permeance calculation.

3.4.3 Assumed Flux Path (AFP) method

Nowadays, the flux contours derived using Finite Element (FE) field computation method, can form the basis for more accurate flux paths representation, thus allowing a more precise unaligned permeance calculation using the AFP method.

The method used by Corda and Stephenson [24] provides the basis of using the AFP model in the unaligned permeance calculation, applicable to single-tooth and multi-tooth per pole structure. However, they indicate some achievement of improvement in the aligned permeance calculation, when the
CHAPTER 3: MODIFIED UNALIGNED PERMEANCE

AFP method was applied to the respective identifiable and distinct flux paths. Straight lines and circular arcs were used to estimate justifiably some paths to enable deduction of relatively less complicated analytical expressions of the respective permeance components. Although the single-tooth per pole structure used is rather idealised, it paves the way to possible adoption of the method for multi-tooth per pole SRMs, as discussed later.

3.5 A Comprehensive Derivation of the Formula of permeance Components

The appropriate selected assumed flux paths or flux tubes can be used to evaluate the corresponding permeance components, within reasonable geometrical approximations. It is considered necessary to subsequently derive more accurate expressions for individual permeance components which will be suitable for CAD algorithm. Inevitably, this involves more elaborate mathematical integration of the relevant complicated flux paths, and this must be handled with extreme care.

For this reason, full derivations of the various expressions are presented, giving an insight into the design integrity, and finally showing the feasibility of the AFP technique in the SRM design procedure. First, a single-tooth per pole configuration has been analysed in great detail, in order to achieve some skills in handling the various flux paths, and finally to establish
the generality of the analytical expressions derived for this type of SRM. Subsequently, the method has also been applied to 12/10 and 24/22 multi-tooth per pole SRM types, and is again treated in great detail. This is targeted at achieving a general design procedure for these SRM types, especially for high number of teeth/pole motor type.

3.6 Application of AFP method to single-tooth per poles SRMs

3.6.1 Analytical estimation of unaligned permeance for single-tooth per pole (6/4)

This section presents the derivation of unaligned permeance for single-tooth per pole SRMs (6/4) by considering three regions of interest. A comprehensive derivation of the various permeance components is presented in Appendix 3.6.1.

![Fig 3.2: Detailed flux path for single-tooth per pole (6/4) SRM.](image-url)
3.6.1a Region 1

Fig. 3.3 shows detail of the magnetic configuration for flux path 1 where the broken line represents the approximated back of core surface. The flux-linkage of path 1 is small in comparison with the flux-linkage of paths 3 and 4, because the linked amp-turns become smaller as the field lines are nearer to the corner between the back of core and the pole. This is taken to justify the approximation used in calculating this particular inductance. The small empty space between the pole winding and the back of core may be considered filled with pole winding turns.

The elementary flux-linkage is:

\[ \varphi_x = \mu_0 H_x A \times N_i \]  \hspace{1cm} (3.5)

\[ d\varphi_x = N_i \mu_0 H_x l_t dx \]  \hspace{1cm} (3.6)

The normalised equivalent permeance for region 1 is:

\[ P_1 = \frac{L_1}{\mu_0 l_t \left( \frac{N}{2} \right)^2} \]  \hspace{1cm} (3.7)
\[ \frac{\gamma m^4}{4W^2(U + 2V)^2} \] (3.8)

3.6.1b Region 3

![Diagram of Region 3](image.png)

Fig 3.4: Detailed flux path 3 (path 3a and 3b)

Region 3a

![Diagram of Region 3a](image.png)

Fig 3.5: Detailed flux path 3a

Fig 3.5 shows details of the magnetic configuration for flux path 3a. The amp-turns linked by field lines decrease and the length of the lines increases the further they are from line BB₁. Thus the greater the radius of the
line, the smaller the contribution of flux-line to the value of inductance component \( L_x \). This reasoning supports the approximation that the rotor pole surface may be represented by a straight line at right angle to the stator pole side through the point \( C_3 \).

The elementary flux-linkage is:

\[
\varphi_x = \mu_0 H x A x N_t \quad (3.9)
\]

\[
d\varphi_x = N_t H_x \mu_0 l_x dx \quad (3.10)
\]

The normalised equivalent permeance for region 3a is:

\[
P_{3a} = \frac{L_x}{\mu_0 l_x \left( \frac{N}{2} \right)^2} \quad (3.11)
\]

\[
= \frac{2}{\pi} \left( \ln\left( \frac{n}{h} \right) + \frac{2y(n - h)}{WV} - \frac{n^2 - h^2}{4(WV)^2} (\pi WV - 2y) \right) \quad (3.12)
\]

\[
- \left( \frac{(n^3 - h^3)^3 \pi y}{6(WV)^2} + \frac{(n^4 - h^4) \pi ^2}{64(WV)^2} \right)
\]
Region 3b

Fig 3.6: Detailed flux path 3b

Fig 3.6 shows details of the magnetic configuration for flux-path 3b between the semi-broken lines B₁B and AA₁. However, the field lines B₁B and AA₁ are not concentric circular arcs with centres C₃ and C₅ and therefore lines between them cannot be simply represented by concentric circular arcs with centres A and B and parallel straight line segments which are perpendicular to the diagonal C₃C₅.

The elementary flux-linkage is:

\[
\mathrm{d}\varphi_x = \left(\frac{N}{2}\right) H_x \mu_0 l_f \, \mathrm{d}x
\]  

(3.13)

The normalised equivalent permeance for region 3b is:

\[
P_{3b} = \frac{L_x}{\mu_0 l_f \left(\frac{N}{2}\right)^2}
\]  

(3.14)
(3.15)

The total unaligned permeance for region 3 is:

\[ P_3 = P_{3a} + P_{3b} \]  

(3.16)

3.6.1c Region 4

Fig 3.7: Detailed flux path 4

Fig 3.7 shows details of the magnetic configuration for flux path 4. The field lines of path 4 consists of parallel straight line segments. The entire flux path 6 can be assumed as rectangular of \( AA_3A_4A_5 \).

The elementary flux-linkage is:

\[ d\phi_x = N_i \mu_0 H_x l_i dx \]  

(3.17)

The normalised equivalent permeance for region 4 is:
\[ P_4 = \frac{L_x}{\left(\frac{N}{2}\right)^2 \mu_0 l_r} \]  
\[ = \frac{P}{G_i} \]  
(3.18) (3.19)

The above equations (3.8), (3.16) and (3.19) are derived based on half flux paths passing through the stator pole. Thus, the total unaligned permeance for the three regions of interest must be multiplied by two to obtain the total unaligned permeances for the whole magnetic circuit. Hence, the total permeance can be written as:

\[ P_T = 2(P_1 + P_3 + P_4) \]  
(3.20)

Some of the end-effects for the single-tooth per pole (6/4) SRM contributes twenty-three percent of the total permeance [24], therefore the effective unaligned permeance is:

\[ P_{2eff} = P_T \times 1.23 \]  
(3.21)

3.7 Application of AFP to Multi-tooth per pole SRMs

Having implemented AFP method for single tooth/pole SRM in the previous section, the next logical step is to investigate the feasibility of the method, when applied to multi-tooth per pole structure. There are more leakage components of permeance in multi-tooth per pole configuration
CHAPTER 3: MODIFIED UNALIGNED PERMEANCE

compared to single-tooth per pole. They can be deduced from the path which can be estimated with the aid of the FE computation of electromagnetic fields.

In multi-tooth per pole structure, shallow slots on the stator exist and each slot is linked to rotor poles side and surface by flux lines. Due to non-linear nature of the distribution, experience shows that complexities arise when attempts are made to derive the exact analytical formulae for various permeance components.

3.7.1 Derivation equation of AFP for multi-tooth per pole SRM (12/10)

This section presents the derivation of unaligned permeance for multi-tooth per pole SRMs (12/10) by considering five regions of interest. A comprehensive derivation of the various permeance components is presented in Appendix 3.7.1.

Fig 3.8: Detailed flux path for multi-tooth 12/10
3.7.1a Region 1

![Diagram of flux path](image)

Fig 3.9: Detailed flux path 1

Fig 3.9 shows details of the magnetic configuration for flux path 1. OMDBAC can be approximated as triangle OME₁. Broken lines ME₁ represents the approximated back of core. The flux linkage of path 1 is small, because the linked amp-turns become smaller as the field lines are nearer to the corner between the back of core and the pole. Sector B₁F₁ centred at O can be approximated to be half of sector B₁F₁ centred at E₁.

The elementary flux-linkage is:

\[
\delta \varphi_x = N_1 H_x \mu_0 l_r \, dx
\]

(3.22)

\[
= \frac{\gamma x^2}{lh} \left( \frac{N}{2} \right) x \left( \frac{Ni}{2} \right) \mu_0 l_r \, dx
\]

(3.23)

The normalised equivalent permeance for region 1 is:

\[
P_1 = \frac{L_1}{\mu_0 l_f \left( \frac{N}{2} \right)^2}
\]

(3.24)
\[ = \frac{\gamma R_1^4}{4(h)^2} \]  

(3.25)

3.7.1b Region 2

Region 2a

Fig 3.10: Detailed flux path 2a

Fig 3.10 shows details of the magnetic configuration for flux path 2a. The amp-turns linked by field lines decrease and the length of the lines increases the further they are from line \( A_2A_3 \). Thus, the greater the radius of the line, the smaller the contribution of flux-line to the inductance component \( L_x \).

The elementary flux linkage is:

\[ d\phi_x = H_x N_1 \mu_0 I_1 dx \]  

(3.26)

The normalised equivalent permeance for region 2a is:

\[ P_{2a} = \frac{L_x}{\mu_0 I_1 \left( \frac{N}{2} \right)^2} \]  

(3.27)
\[ = \frac{1}{\beta} \ln \left( \frac{r_2}{r_2 - \frac{t}{8}} \right) \]  

(3.28)

Region 2b

Fig 3.11: Detailed flux path 2b

Fig 3.11 shows details of the magnetic configuration for flux path 2b between the semi-broken lines \( A_2A_3 \) and HK. However, the field lines \( A_2A_3 \) and HK are not cocentric circular arcs with centres \( K_1 \) and \( K_2 \) and lines between them cannot be simply represented by cocentric circular arcs with centres \( K_1 \) and \( K_2 \). Therefore path 2b is approximated by lines which consist of cocentric circular arcs with centres \( A_2 \) and \( K \), and parallel straight line segments which are perpendicular to the diagonal \( K_1K_2 \).

The elementary flux linkage is:

\[ d\varphi_x = N_1H_x\mu_0l_1dx \]  

(3.29)
The normalised equivalent permeance for region 2b is:

\[ P_{2b} = \frac{L_x}{\left( \frac{N}{2} \right)^2 \mu_0 I_r} \]  \hspace{1cm} (3.30)

\[ = \left( \frac{1}{\sqrt{3} + \beta} \right) \]  \hspace{1cm} (3.31)

The total unaligned permeance for region 2 is:

\[ P_2 = P_{2a} + P_{2b} \]  \hspace{1cm} (3.32)

3.7.1c Region 3

Fig 3.12: Detailed flux path 3

Fig 3.12 shows details of the magnetic configuration for flux path 3.

The field lines of path 3 consists of parallel straight line segments. The entire flux path 3 can be approximated as rectangle of KK3K4K5.

The elementary flux-linkage is:

\[ d\varphi_x = N_x H_x \mu_0 I_r \, dx \]  \hspace{1cm} (3.33)

The normalised equivalent permeance for region 3 is:
\[
P_3 = \frac{L_s}{\left(\frac{N}{2}\right) \mu_0 I_f}
\]
(3.34)

\[
= \left(\frac{T_s}{L + g}\right)
\]
(3.35)

3.7.1d Region 4

Fig 3.13: detailed flux path 4

Fig 3.13 shows details of the magnetic configuration for flux path 4 between the semi-broken lines \(R_1T_1\) and \(K_3T_2\). However, the field lines \(R_1T_1\) and \(K_3T_2\) are not cocentric circular arcs with centres \(P_3\) and \(T_3\) and therefore lines between them cannot be simply represented by cocentric circular arcs with centres \(P_3\) and \(T_3\). Therefore path 4 is approximated by the lines which consist of cocentric circular arcs with centres \(K_3\) and \(R_1\), and parallel straight line segments which are perpendicular to the diagonal \(P_3T_3\).

The elementary flux-linkage is:
\[ d\phi_x = \mu_0 N_i H_x l_i \, dx \quad (3.36) \]

The normalised permeance for region 4 is:

\[ P_4 = \frac{L_4}{(N/2)^2 \mu_0 l_i} \quad (3.37) \]

\[ = \frac{1}{(\gamma^* - \gamma^\nu)} \ln \left( \frac{\sqrt{3} + \gamma^\nu}{\sqrt{3} + \gamma^*} \right) \quad (3.38) \]

**3.7.1e Region 5**

![Fig 3.14: Detailed flux path 5](image)

Fig 3.14 shows details of the magnetic configuration for flux path 5.

The field lines of path 5 consists of parallel straight line segments form the broken lines \( y_3y_2 \) to the line \( R_1y_1 \). The entire flux path 5 can be approximated as a rectangle \( y_1y_2y_3R_1 \).

The elementary flux-linkage is:

\[ d\phi_x = N_i H_x \mu_0 l_i \, dx \quad (3.39) \]
The normalised equivalent permeance for region 5 is:

\[
P_5 = \frac{L_x}{\left(\frac{N}{2}\right)^2 \mu_0 l_t}
\]  

(3.40)

\[
= \left(\frac{1}{d + g}\right)\left(\frac{t}{2}\right)
\]  

(3.41)

\[
= \frac{t}{2(d + g)}
\]  

(3.42)

The above equations (3.25), (3.32), (3.35), (3.38), (3.42) are derived based on the whole flux paths passing through the stator pole. Therefore, the total unaligned permeance for the five regions of interest can be written as:

\[
P_T = P_1 + P_2 + P_3 + P_4 + P_5
\]  

(3.43)

Some of the end-effect for the 12/10 SRM contributes thirty percent of the total permeance, therefore the effective unaligned permeance is:

\[
P_{\text{eff}} = P_T \times 1.3
\]  

(3.44)

3.7.2 Derivation of the equation of AFP for multi-tooth per pole SRM (24/22)

This section presents the derivation of unaligned permeance for multi-tooth per pole SRM (24/22) by considering six regions of interest. A
comprehensive derivation of the various permeance components is presented in appendix 3.7.2.

Fig 3.15: Detailed flux path for multi-tooth 24/22

3.7.2a Region 1

Fig 3.16: Estimation of the winding area
Figure 3.17 shows details of the magnetic configuration for region 1a where the broken line BB represents the approximated back of core surface. The flux path 1a is small, because the amp-turns become smaller as the field lines arc get nearer to the corner between the back of core and the pole.

The elementary flux-linkage is:

$$d\varphi_x = \mu_0 N_l H_x l_f dx$$

(3.45)

The normalised equivalent permeance for region 1a is:

$$P_{la} = \frac{L_x}{\left(\frac{N}{2}\right) \mu_0 l_f}$$

(3.46)

$$= \left(\frac{\theta h_n^4}{16 S^2}\right)$$

(3.47)
Fig 3.18 shows details of the magnetic configuration for flux path 1b.

The broken line AF can be approximated as the radius of a sector centred at D.

The broken lines D,E and BF can be approximated into rectangular shape BFD,E. The field lines of path 1b consist of parallel straight line segments.

The elementary flux-linkage is:

\[ d\phi_x = \mu_0 H_x l_f N_x dx \]  \hspace{1cm} (3.48)

The normalised equivalent permeance for region 1b is:

\[ P_{1b} = \frac{L_x}{\mu_0 l_f \left( \frac{N}{2} \right)^2} \]  \hspace{1cm} (3.49)

\[ = \frac{b(a - h_n)^3}{3S^2} \]  \hspace{1cm} (3.50)

The total unaligned permeance for region 1 is:
\[ P_1 = P_{1a} + P_{1b} \]  \hspace{1cm} (3.51)

3.7.2b Region 2

Region 2a

Fig 3.19: Detailed flux path 2a

Fig 3.19 shows details of the magnetic configuration for flux path 2a. The amp-turns linked by field lines decreases and the length of the lines increases the further arc from broken line \( T_2T_3 \). Thus, the greater the radius of the line, the smaller the contribution of flux-line to value of inductance component \( L_x \).

The elementary flux-linkage is:

\[ d\varphi_x = \mu_0 H_x l_f N_i dx \]  \hspace{1cm} (3.52)

The normalised equivalent permeance for region 2a is:

\[ P_{2a} = \frac{L_x}{\left(\frac{N}{2}\right)^2 \mu_0 l_f} \]  \hspace{1cm} (3.53)
\[
\left( \frac{2}{\pi} \right) \ln \left( \frac{2w - g}{w} \right)
\]  
(3.54)

\( w \gg g \), so

\[
P_{2a} = \left( \frac{2}{\pi} \right) \ln 2
\]  
(3.55)

Region 2b

Fig 3.20: Detailed flux path 2b

Fig 3.20 shows details of the configuration for flux path 2b. The line segments 2b'\ 2b' can be divided into several sections ACBD. The sector AB centred at \( T_4 \) and sector CD centred at \( T_z \).

The elementary flux-linkage is:

\[
d\varphi_x = \mu_0 H_x l_x N_x dx
\]  
(3.56)

The normalised equivalent permeance for region 2b is:

\[
P_{2b} = \frac{L_x}{\mu_0 l_x \left( \frac{N}{2} \right)^2}
\]  
(3.57)
\[ P_{2b} = \frac{2}{\pi} \]  

(3.59)

The total unaligned permeance for region 2 is:

\[ P_2 = P_{2a} + P_{2b} \]  

(3.60)

3.7.2c Region 3

Fig 3.21: Detailed flux path 3

Fig 3.21 shows details of the configuration for flux path 3. The sector \( T_1 D_2 \) and \( S_2 D_3 \) have their centres at \( D_1 \). The rotor tooth can be approximated as straight line \( D_1 D_4 \), therefore \( D_2 \) and \( D_3 \) start from those straight line. This flux path is the same as flux path 2a for 24/22 SRM, but the angle is 90° - taper tooth angle instead of 90°. Therefore the normalised permeance of flux path 3 is:
\[ P_3 = \frac{90^\circ \left( \frac{2}{\pi} \right) \ln 2}{(90^\circ - \text{taper tooth angle})} \quad (3.61) \]

\[ = \frac{180^\circ}{\pi (90^\circ - \text{taper tooth angle})} \ln 2 \quad (3.62) \]

### 3.7.2d Region 4

Fig 3.22: Detailed flux path 4

Fig 3.22 shows details of the magnetic configuration for flux path 4. The field lines of path 4 consist of parallel straight line segments 4a'4a between the surface stator tooth \( S_1S_2 \) and rotor slot depth \( S_2S_4 \). The field lines can be approximated into the rectangular shape \( S_1S_2S_3S_4 \).

The elementary flux-linkage is:

\[ d\phi_x = N_1 H_x \mu_0 l_x \, dx \quad (3.63) \]

The normalised equivalent permeance for region 4a is:
\[ P_{4a} = \frac{L_s}{\left(\frac{N}{2}\right)^2 \mu_0 l_f} \] (3.64)

Thus, the total normalised permeance for flux path 4 is:

\[ P_4 = 2(P_{4a}) \] (3.65)

\[ = 2 \left( \frac{\left( \frac{t}{2} - w + g \right)}{d + g} \right) \] (3.66)

### 3.7.2e Region 5

Due to the symmetry of the machine structure,

Region 5 = region 3

Flux path 5 = flux path 3

Therefore, the normalised equivalent permeance for region 5 is:

\[ P_5 = P_3 \] (3.67)

\[ P_5 = \frac{180^0}{\pi (90^0 - \text{taper tooth angle})} \ln 2 \] (3.68)

### 3.7.2f Region 6

Due to the symmetry of the machine structure,

Region 6a = region 3

Flux path 6a = flux path 3
Therefore, the normalised equivalent permeance for region 6a is:

\[ P_{6a} = P_3 \]  

\[ = \frac{180^\circ}{\pi (90^\circ - \text{taper tooth angle})} \ln 2 \]  

(3.70)

Region 6b = region 2b

Flux path 6b = flux path 2b

Therefore, the normalised equivalent permeance for region 6b is:

\[ P_{6b} = P_{2b} \]  

\[ = \frac{2}{\pi} \]  

(3.72)

The total normalised equivalent permeance for region 6 (flux path 6) is:

\[ P_6 = P_{6a} + P_{6b} \]  

(3.73)

Thus, the total unaligned permeance for the six regions of interest can be written as:

\[ P_T = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 \]  

(3.74)

Some the end-effects for the 24/22 SRM contributes thirty percent of the total permeance, therefore the effective unaligned permeance is:

\[ P_{\text{eff}} = P_T \times 1.3 \]  

(3.75)
3.8 Application of permeance in SRMs design

This section explains the application of permeance in SRMs design. As mentioned before in Chapter 2, performance prediction is an important aspect of SRMs research. Before performance can be considered, the magnetisation characteristic (flux linkage vs current) must be known. Torque can be calculated simply by using this characteristic. The magnetisation characteristic is therefore indirectly related to permeance.

The air-gap permeances are related to mmf by the following equations.

\[
\text{mmf}_a \text{ (aligned)} = \frac{\phi}{i\mu_0 l_r P_1} \quad (3.76)
\]

\[
\text{mmf}_u \text{ (unaligned)} = \frac{\phi}{i\mu_0 l_r P_2} \quad (3.77)
\]

where \( P_1 \) and \( P_2 \) are the aligned and unaligned permeances respectively.

For each value of \( \phi \), the corresponding gap mmf can be calculated. In SRMs design, the mmfs at the air-gap is used in conjunction with the mmfs at other parts of the motor to find the total mmf of the magnetic circuit, yielding what is termed the magnetisation characteristics \( \varphi(\theta, i) \)
From the current and flux linkage data, flux linkage vs current can be plotted against current and the value of mean torque can be evaluated from this graph. Further discussion on this will be presented in Chapter 6.