### POLYELECTROLYTE MODELS AND MOLECULAR DYNAMICS STUDIES OF THE DNA POLYMER AND DNA-NCP COMPLEXES IN SALT SOLUTIONS

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#### Abstract

The effects of salt concentration on the flexibility and charge distribution of a DNA chain are probed using MD simulation. For this purpose, the DNA is modeled as 360 negatively charged monomer spheres with radius 10 Å and charge -12 (all in reduced units) linked linearly by a harmonic bonding potential. Each of these spheres represent 6 base pairs. The harmonic force constant  $k_{\rm h}$  is determined by balancing the intramolecular and intermolecular forces between 2 monomers separated by an equilibrium bond length of 20.4 Å. This method resulted in good agreement with the experimental contour length and therefore guarantees the reasonableness of the  $k_{\rm h}$  parameter. The bending angle force constant  $k_{\theta}$  is determined by relating the chain bending modulus with the experimental DNA persistence length. In a cell, the DNA strand wraps around the so called nucleosome core particle (NCP). In this study, the NCP is represented by a large sphere with radius 35 Å and charge +150. The simulations were performed both with and without the NCP, which when present were 12 in number. The NaCl salt is represented as a charged sphere of radius 2 Å of either +1 or -1 charge modeling a single Na<sup>+</sup> or Cl<sup>-</sup> ion respectively. The salt concentration was chosen to be in the 0.0-0.25 mM range for a DNA concentration of 0.005 mg/ml and in the 0.0-100 mM range for DNA concentration of 2.0 mg/ml. The upper limit corresponded to the maximum computational resources available here. The excluded volume effect was made possible by the application of a purely repulsive 6-12 Lennard-Jones potential and the interaction between charges was strictly Coulombic, where the Ewald summation was computed by the P3M method. The results show that the NaCl salt concentration within the cell play a significant role in determining the DNA conformation, as well as the strength of the DNA-NCP and DNA-ions interaction. Some theoretical equations for calculating the polyelectrolyte persistence length seem to disagree with the simulation results. A new concept that leads to the "ionic bridging effect" is introduced and shown to be important in determining the DNA topology. By the RDF data, it is shown that the classical PB cylindrical cell model neglects the counterion condensation at monomer neighbors. The above results all imply the importance of molecular simulations in obtaining information which cannot so readily be made available from experiments.

#### Abstrak

Simulasi dinamika molekular telah digunakan untuk menganalisa kesan kepekatan garam ke atas kelenturan and distribusi cas dalam rantai DNA. Untuk tujuan kajian ini, DNA telah dimodelkan sebagai gabungan 360 sfera monomer yang bercas negatif dengan radius 10 Å dan cas -12 dengan ikatan harmonik potential. Setiap sfera terdiri dari 6 pasangan bes. Pemalar tenaga harmonik,  $k_{\rm h}$ , telah ditentukan dengan menyeimbangkan tenaga intramolekular dan intermolecular di antara 2 monomer dengan jarak ikatan keseimbangan 20.4 Å. Kaedah ini memberikan bacaan parameter  $k_{\rm h}$  yang baik apabila dibandingkan dengan nilai eksperimentasi contour length. Pemalar tenaga pembengkokan sudut  $k_{\theta}$  ditentukan dengan menghubungkan modulus pembengkokan rantai dengan data percubaan persistence length rantai DNA. Dalam sel, rantai DNA membungkus nucleosome core particle (NCP). Dalam kajian ini, NCP diwakili dengan sebuah sfera besar dengan radius 35 Å dan cas +150. Simulasi dilakukan dengan dan tanpa NCP, yang jika ada berjumlah 12. Garam NaCl diwakili sebagai sebuah sfera bercas dengan radius 2 Å dengan cas salah satu +1 atau -1 yang masing-masing memodelkan sebuah ion Na<sup>+</sup> and Cl<sup>-</sup>. Kepekatan garam dipilih dalam jarak 0.0–0.25 mM untuk kepekatan DNA 0.005 mg/ml and jarak 0.0-100 mM untuk kepekatan DNA 2.0 mg/ml. Batas atas kepekatan garam sesuai dengan maksimum sumber pengkomputeran yang tersedia disini. Kesan pengecualian ruang dibuat mungkin dengan aplikasi penolakan Lennard-Jones potensi dan interaksi antara cas adalah Coulombic dimana penjumlahan Ewald dihitung dengan kaedah P3M. Keputusan menunjukkan bahawa kepekatan garam NaCl dalam sel bermain peranan penting dalam menentukan penglarasan DNA, maupun kekuatan interaksi DNA-NCP dan DNA-ion. Beberapa teori untuk menghitung persistence length polielektrolit kelihatan tidak bersetuju dengan keputusan simulasi. Sebuah konsep baru yang membawa kepada "kesan jambatan ionis" diperkenalkan dan ditunjukkan menjadi penting dalam menentukan topologi DNA. Dengan data RDF, ditunjukkan bahawa model klasik sel silinder PB mengabaikan kondensasi counterion pada monomer-monomer tetangga. Keputusan-keputusan diatas bererti pentingnya simulasi molekular dalam mendapat maklumat yang tidak mudah didapat dari percubaan.

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### List of Symbols and Abbreviations

DNA	=	Deoxyribonucleic acid
DHA	=	Debye-Huckel Approximation
FFA	=	Far-Field Approximation
NCP	=	Nucleosome Core Particle
OSF	=	Odijk-Skolnick-Fixman
PBC	=	Periodic Boundary Condition
PB	=	Poisson-Boltzmann
PBCCM	=	Poisson-Boltzmann Cylindrical Cell Model
PLDNA	=	Particle-like DNA monomer
RDF	=	radial distribution function
WLC	=	worm-like chain
a	=	acceleration
b	=	monomer bond length
В	=	bending modulus
Ci	=	concentration of ion <i>i</i>
$c_{\rm shift}$	=	constant defining $U_{LJ}(r_{cut}) = 0$
δx	=	standard deviation of the variable x
$\epsilon_{\rm LJ}$	=	Lennard-Jones energy unit
<i>e</i> <sub>0</sub> , <i>e</i>	=	elementary proton charge
ε	=	dielectric constant of the medium
$E_{\rm el}$	=	electrostatic energy
f'	=	random force
f(r)	=	conservative force
g(r)	=	radial distribution function
κ	=	ionic screening parameter (inverse of Debye length)
$k_{ m B}$	=	Boltzmann constant
<i>k</i> <sub>h</sub>	=	harmonic bond constant
$k_{\Theta}$	=	bending constant
$l_0$	=	additional parameter for our persistence length theory ( $l_0 = 1 \text{ Å}$ )
$l_{\mathrm{B}}$	=	Bjerrum length
L	=	chain length
$L_{\mathrm{P}}$	=	persistence length
L <sub>OSF</sub>	=	electrostatic persistence length from OSF theory
$L_{ m P}^0$	=	neutral/uncharged polymer persistence length

$L_{ m P}^{ m el}$	=	electrostatic persistence length
m	=	mass
n	=	total number of bonds
$n_M^{lpha}$	=	coordination number of $\alpha$ type ions about M macroion
Ν	=	total number of monomers
N <sub>A</sub>	=	Avogadro number
$\phi_{\mathbf{M}}$	=	reduced surface potential
$\Psi_M$	=	surface potential
$\psi(r)$	=	potential at position $r$ from a macroion surface
$q_i$	=	charge of particle <i>i</i>
r	=	distance
$\bar{r}$	=	average equilibrium bond distance from Boltzmann integration
$r_{\rm off}$	=	sum of the hard core radii
<b>r</b> <sub>cg</sub>	=	center of gravity
r <sub>D</sub>	=	Debye (screening) length
r <sub>h</sub>	=	harmonic equilibrium bond distance
$R_{\rm c}$	=	cylinder radius
R <sub>e</sub>	=	end-to-end distance
R <sub>g</sub>	=	radii of gyration
R <sub>n</sub>	=	radius of the normalized RDF
ρ	=	density, charge density
S	=	segment length
$S_{\rm el}$	=	electrostatic entropy
σ	=	sum of the soft core radii
$\sigma_M$	=	surface charge density
θ	=	bending angle
$\theta_0$	=	equilibrium bending angle
Т	=	Temperature
$U_{\text{C-P3M}}(r)$	=	Coulombic potential energy calculated by P3M method
$U_{\rm h}(r)$	=	harmonic stretching potential energy
$U_{\rm LJ}(r)$	=	Lennard-Jones potential energy
$U_{ heta}$	=	bending angle potential energy
ξ	=	frictional constant, line charge density
$z_i, Z_i$	=	valence