

PERPUSTAKAAN UNIVERSITI MALAYA

PERKHIDMATAN REPROGRAFI

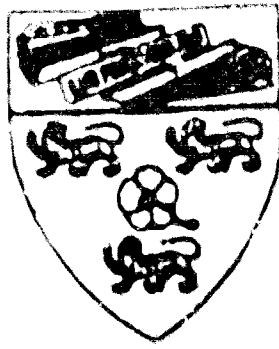
UNIVERSITY OF MALAYA LIBRARY

REPROGRAPHIC SERVICE



UNIVERSITY OF MALAYA LIBRARY

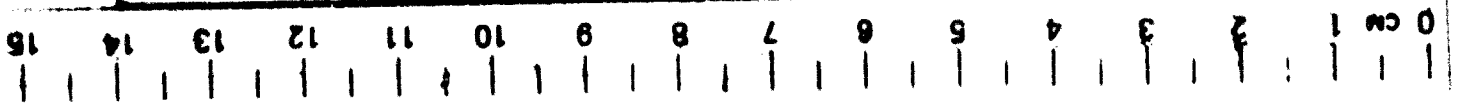
MICROFILM



MULA



UNIVERSITY OF MALAYA LIBRARY . MICROFILM



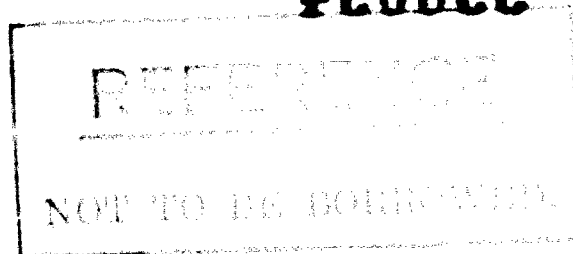


THE BINOMIAL DISTRIBUTION
ITS DERIVATION AND APPLICATIONS

by

CHIAM TAH WEN

12032E



[St 8]

A Graduation Exercise submitted as
part fulfilment towards the
Degree of Bachelor of Arts
in Economics (Statistics)

Division of Statistics,
Faculty of Economics and Administration,
University of Malaya.

August 1966

TABLE OF CONTENTS

	Page
SYNOPSIS	ii
ACKNOWLEDGMENTS	iv
LIST OF TABLES	v
CHAPTER ONE: INTRODUCTION	1
CHAPTER TWO: SAMPLE SPACES	4
1. Sample space, outcomes, events	
2. Probabilities in sample spaces	
3. Conditional probability; Independence	
4. Trials repeated under identical conditions	
CHAPTER THREE: RANDOM VARIABLES	12
1. Random variables and probability functions	
2. The Mean of a random variable	
3. The Variance of a random variable	
CHAPTER FOUR: THE BINOMIAL DISTRIBUTION	19
1. Bernoulli trials and the binomial distribution	
2. Some properties of the binomial distribution	
(i) The Central Term	
(ii) Theorem on Tails	
(iii) The Mean of the binomial distribution	
(iv) The Variance of the binomial distribution	
3. The Law of Large Numbers	
CHAPTER FIVE: THEORETICAL RESULTS CONCERNING THE BINOMIAL DISTRIBUTION	28
1. The Poisson approximation; Derivation of Poisson approximation to the binomial distribution	

2. The Normal approximation
Proof of the DeMoivre-Laplace
Theorem

CHAPTER SIX: CALCULATIONS WITH THE BINOMIAL
DISTRIBUTION -- TABLES 40

1. The binomial distribution
function
2. Tables

CHAPTER SEVEN: APPLICATIONS OF THE BINOMIAL
DISTRIBUTION 49

1. True-false test
2. Winning in a series of games
3. Operation on patients
4. Production of metal parts
5. Industrial quality control
6. Acceptance and rejection:
Operating Characteristic Curve
7. Testing a statistical hypothesis
8. Tests of significance for dif-
ferences in samples
9. Power supply
10. Testing sera or vaccines
11. A rocket designer's problem
12. One- and two-engine planes
13. Random walk problem
14. Application-genetics: the
Mendelian Hereditary Theory
15. A parking problem

APPENDICES 86

1. Mathematical derivation of
variance of a random variable
2. (A) Theorem on central term
(B) Theorem on Tails
3. On Hereditary laws
4. Area under Normal distribution
curve

BIBLIOGRAPHY 95

SYNOPSIS

This exercise gives an account of the derivation of the Binomial Distribution from first principles and the applications of the distribution to various fields of human activities.

As the Binomial Distribution is based on the theory of probability, which is the corner-stone of many physical, biological and social sciences, we first discuss some fundamental concepts and the basic calculus of probability for experiments with a number of possible outcomes. A probability measure is first introduced over the events of a sample space; independence of events and trials repeated under identical conditions are then discussed.

Next we introduce the analytic theory of probability in the finite case. Random variables are defined as functions on sample spaces, and probability distributions, means, variance and standard deviation are dealt with.

In the chapter on the Binomial Distribution, which is the most important probability function defined on a finite sample space, we derive the basic properties of a Bernoulli process and a binomially distributed random variable, and discuss some of the important properties of the distribution. We include in this chapter a discussion on the law of large numbers, which serves as a basis for the intuitive notion of probability as a measure of relative frequencies. Without this law, the whole probability theory would lose its intuitive foundation.

In many practical problems, the values of n and k in the Binomial Distribution formula $b(k;n,p)$ are very large; a direct use of the formula becomes almost impossible as the binomial coefficients are difficult to evaluate. Two approximate methods of calculation, based on the Poisson distribution and the Normal distribution, are discussed. We also illustrate, with examples, the computation of the Binomial Distribution for very large n , ($n \geq 100$) by means of a short method.

The Binomial Distribution is applied, to a large extent, in economics, engineering, medicine and genetics. Included in this exercise are the applications of this distribution in industrial quality control, decision-making, testing of a statistical hypothesis, testing of

significance for differences in samples, in power supply, in sera or vaccine testing, in random walk, in parking problem and in the Mendelian hereditary theory.

Appendices in this exercise explain the mathematical derivation of some formulae and approximations.

ACKNOWLEDGEMENTS

The writer wishes to express his most sincere gratitude to Mr. Lim Kai Leng, Lecturer, Division of Statistics, Faculty of Economics and Administration, University of Malaya, Kuala Lumpur, for the valuable help and the unflagging interest he has taken in the production of this graduation exercise.

The writer also wishes to thank Mr. Lim Swee Hock, Head of Division of Statistics, Faculty of Economics and Administration of the University for his advice which is greatly appreciated.

LIST OF TABLES

Table		Page
1.	Distributing three distinguishable balls in three cells	6
2.	Distributing three indistinguishable balls in three cells	7
3.	The Binomial Distribution $b(k;n,p)$ for $n=5, 10, 20, 50$ and 100 and $p = \frac{1}{n}$ together with the Poisson Distribution	32
4.	The Binomial Distribution $b(k;n,p)$ for $n=5, 10, 20, 50$ and 100 and $p=0.1$ together with the Normal Distribution for mean=10 and Standard Deviation 3	39
5.	Computation of the distribution function f_k and the Cumulative distribution fraction $P\{k\}$ for the Binomial Distribution with $n=100$ and $p=0.1$	42
6.	Cumulative Binomial Probabilities	44
7	Probabilities of Inheritance of a Single Pair of genes	73