

RESIDUALLY FINITE GROUPS

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ABSTRACT

In this thesis, we will study a stronger residually finite property called weak potency. More precisely, we aim to study the weak potency of HNN extensions and generalised free products of weakly potent groups and the main tools we used are filters.

First we study the weak potency of HNN extensions by introducing the concept of h -filters and then use it to prove the main criterion. Then we prove several characterisations for the weak potency of certain HNN extensions with cyclic associated subgroups as well as a characterisation for the Baumslag-Solitar groups. Next, we will also apply our results to HNN extensions of finitely generated nilpotent groups. We shall give characterisations for certain HNN extensions of characteristically weakly potent groups with finitely generated central associated subgroups and HNN extensions of free abelian groups of finite rank to be weakly potent.

In the last part we study the weak potency of generalised free products. We first introduce w -filter and prove a criterion for generalised free products to be weakly potent. By using it, we then give characterisations for the weak potency of generalised free products with cyclic amalgamated subgroups and with central amalgamated subgroups. Then we extend the results to tree products of finitely many groups. Finally we show that certain one-relator groups with torsion are weakly potent.

ABSTRAK

Dalam tesis ini, kami mengkaji sifat terhingga tersisa yang dikenali sebagai sifat poten lemah. Kajian kami lebih tertumpu kepada sifat poten lemah atas perluasan HNN dan hasil darab teritlak dengan alat utama kami, penapis.

Kami mula dengan kajian ke atas sifat poten lemah perluasan HNN dengan memperkenalkan konsep penapis- h . Kami menggunakan konsep ini untuk membuktikan kriteria utama. Lepas itu, kami memberi beberapa pencirian yang menunjukkan sifat poten lemah perluasan HNN tertentu dengan kumpulan-kumpulan sekutu kitaran serta kumpulan Baumslag-Solitar. Dengan ini, kami memperluaskan keputusan kami ke atas perluasan HNN dengan kumpulan asas nilpoten dijana terhingga. Kemudian, kami membuktikan ciri-ciri perluasan HNN tertentu dengan kumpulan asas yang mempunyai sifat cirian poten lemah dan kumpulan-kumpulan sekutu pusat yang dijana terhingga. Kami juga membuktikan sifat poten lemah perluasan HNN dengan kumpulan asas bebas abelian yang dijana terhingga.

Dalam bahagian terakhir, kami mengkaji sifat poten lemah ke atas hasil darab teritlak. Kami mula dengan memperkenalkan konsep penapis- w dan membuktikan kriteria utama yang membawa sifat poten lemah kepada hasil darab teritlak. Dengan penapis ini, kami dapat membuktikan ciri-ciri hasil darab teritlak dengan penggabungan subkumpulan-subkumpulan kitaran atau pusat yang bersifat poten lemah. Lepas ini, kami memperluaskan keputusan kami ke atas hasil darab pokok dengan nombor kumpulan yang terhingga. Akhirnya, kami membuktikan bahawa kumpulan satu-penghubung berkilas yang tertentu adalah bersifat poten lemah.

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