This research considered two combinatorial problems, Message Scheduling Problem (MSP) specifically Point to Multipoint Routing Problem (PMRP) and Integrated Inventory Routing Problem (IRP). MSP was solved by both the Genetic Algorithm (GA) and Variable Neighborhood Search (VNS). In IRP, the problem was solved by VNS and enhanced VNS and the results were compared with the GA (Aziz and Moin, 2007) and the results obtained from CPLEX.

The overview of each chapter and future research suggestions are presented in the next sections.

5.1 Chapter Overview

In Chapter 2, metaheuristic methods were introduced by giving some definitions. The metaheuristic methods were classified into single solution (trajectory) metaheuristic and population based metaheuristic. Some examples were given, based on the respective categories, of several well known metaheuristic methods, such as Ant Colony Optimization (ACO), Tabu Search (TS), Genetic Algorithm (GA), Variable Neighborhood Search (VNS) and Simulated Annealing (SA). Some local search heuristics focusing on the ones used in this research were presented.

In Chapter 3, the case study that is the Message Scheduling Problem (MSP), specifically the Point to Multipoint Routing Problem (PMRP) was described. The problem was to schedule a set of requests, where finding the optimal path for each request consisted of a single source with multiple destinations. The problem description was given and the steps used illustrated
how to solve the problem. A step by step illustration on the implementation of GA and VNS was presented. The results obtained were compared to the work of Christensen et al. (1997) and better results were obtained. Alternative solutions were also identified using the GA and VNS. The algorithm was further tested on medium and large data sets taken from the OR library and modified to suit the problems. The VNS produced better results compared to the GA with a good underlying heuristics as the local search.

In Chapter 4, the IRP was studied where the problem addressed was a single depot, with $N$ geographically dispersed suppliers and an assembly plant where a fleet of capacitated vehicles transported products from the suppliers to meet the demand specified by the assembly plant. The problem was based on a finite horizon, multi-product, multi-period and multi-supplier. Two algorithms were proposed: VNS and Enhanced VNS (EVNS), where the algorithms differed in embedded the local search. The VNS was compared to the GA developed by Aziz and Moin (2007) and the results showed the superiority of the VNS especially in large data sets. The EVNS was compared to the results obtained from CPLEX and again it showed that the EVNS performed better in large data sets.

5.2 Contribution to Knowledge

This section summarizes the contributions of the research to knowledge. The contributions are presented as follows:

- Point to Multipoint Routing Problem (PMRP)
  - This research presented two metaheuristic methods, GA and VNS in solving the PMRP where three cases were considered: small case (4 requests), medium case (8 requests) and large case (20 requests). The
network for small, medium and large cases consisted 8 nodes and 11 edges, 50 nodes and 83 edges, and 61 nodes and 133 edges respectively.

- In case 1: the GA algorithm developed produced better quality solutions compared to the solutions by hybrid GA from the previous work done by Christensen et al. (1997). This algorithm even produced variation of order to route for the 4 requests with the same cost (alternative solutions).

- VNS produced results at a better time compared to the GA in the small case (4 Requests). And for the medium case, VNS produced the same optimal results in lesser CPU times. VNS and GA produced comparable results for the large case (20 requests).

- VNS programme were tested to PMRP with different underlying local search: swap, invert, or-opt and restricted or-opt. It is shown that results obtain using restricted or-opt performs better in term of quality solutions with a longer cpu time.

- It is proven that VNS performed better (good solution) with a good and suitable underlying local search method.

**Inventory Routing Problem (IRP)**

- This research successfully developed two VNS methods namely VNSIRP and EVNS in solving IRP. The case considered was based on a finite horizon, multi-period, multi-product, multi-supplier, single assembly plant, where a fleet of homogenous capacitated vehicles, housed at a depot, transported products from the suppliers to meet the demand specified by the assembly plant. The distribution network consisting of a depot, an assembly plant and \( N \) geographically
dispersed suppliers where each supplier supplied a distinct product to the assembly plant to meet the demand in each period.

- Neighbourhood structure was defined as a distance function, that is the cardinality of the symmetric difference between any two solutions $v_1$ and $v_2$ written as $\rho(v_1, v_2) = |v_1 \setminus v_2|$. The symmetric difference of two solutions was defined as the number of different suppliers that were visited in a period.

- EVNS was suitable to be applied to a bigger case of IRP problem without any large modifications.

- Two different EVNS were developed, EVNS1 with 2-opt as pre-optimization and EVNS2 with 3-opt as pre-optimization.

- Both algorithms, VNS and EVNS was applied to the original data sets taken from Lee et al. (2003) consisting of S12T14, S20T21, S50T21 and S98T14 that comprised of (12 suppliers, 14 periods), (20 suppliers, 21 periods), (50 suppliers, 21 periods) and (98 suppliers, 14 periods), respectively.

- Other data sets were also created by varying the number of periods in each planning horizon. The new data sets were: S12T5, S12T10, S20T5, S20T10, S20T14, S50T5, S50T10, S50T14, S98T5 and S98T10.

- Results obtained using EVNS were compared with the lower bound (LB) found by using CPLEX. Some results were even better than the Best Integer solution found using CPLEX. CPLEX failed to find solutions for the larger test cases (S50T21, S98T10 and S98T14).
5.3 Suggestions for Future Research

This section presents a few suggestions on how to extend and improve this research. It is noted that the algorithm coded generated good quality solution but it was not always optimal and sometimes (very rarely) produced very little improved solutions. To overcome this, both the metaheuristics considered in this research: GA and VNS, can be extended or modified to improve the solutions obtained. In GA, other promising crossover operators could be used such as in Ortiz-Boyer et al. (2007) where virtual parents are used to ensure that the new offspring born are in a promising region.

The GA or VNS can also be hybridized with other suitable heuristics or metaheuristics to improve the solution. This has been done by many researchers. The discussion on the hybridization of metaheuristics or hybridized metaheuristics is discussed in great length by Raidl et al. (2010).

There are also other options known as hyper heuristics. Hyper heuristics is a term that describes ‘heuristics to choose heuristics’ in the context of combinatorial optimization (Burke et al., 2010). Hyper heuristics is a high level approach that has the capabilities to intelligently select and apply an appropriate low level heuristics according to the situation.

As for VNS, the number of neighbours generated should be controlled in each neighbourhood, as this maybe help in diversifying the methods. Alternatively, a different neighbourhood structure can be explored to improve the solutions. Different local search algorithms such as the \( \lambda \) - interchange can be applied.

The algorithm applied is suggested to be extended to Stochastic Inventory Routing Problem (SIRP) where the future demand is uncertain. SIRP closely mimics the real-life inventory routing in which the demands for many products are uncertain and the model can run out of stock.
Other variations of the IRP problem can be applied, for example IRP with backordering and allowing direct deliveries. However, applying these problems will need some modifications to the algorithms.

For PMRP, it was seen that the algorithm could act as a fundamental to other problems in the telecommunications network. These algorithms could be applied to other similar problems in this area. An example of such problem is multicasting in wavelength division multiplexing (WDM) optical network (Znati et al., 2002). The multicast can be viewed to find an optimal route to route a set of requests subject to certain criteria such as bandwidth.

Another example of the application of PMRP, is the extension of Generalized Multiprotocol Label Switching (GMPLS) network (Wei et al., 2005). This is the multicasting of the Internet Protocol (IP), where traditionally it is multicast by best-effort forwarding. The extension of GMPLS for dynamic P2MP (or PMRP) trees in the control plane is handled as finding a minimum Steiner tree problem as they multicast group from one source and a set of destinations nodes. The devices or chips for duplicating data in the electronic domain are commercially available (Wei et al., 2005).