PRICING OF AMERICAN CALL OPTIONS USING SIMULATION AND NUMERICAL ANALYSIS

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2011

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THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

INSTITUTE OF MATHEMATICAL SCIENCES
FACULTY OF SCIENCE
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KUALA LUMPUR

2011

ABSTRACT

Consider the American basket call option in the case where there are N underlying assets, the number of possible exercise times prior to maturity is finite, and the vector of asset prices is modeled using a Levy process. A numerical method based on regression and numerical integration is proposed to estimate the prices of the American options. In the proposed method, we make use of the distribution for the vector of asset prices at a given time t in the future to determine the "important" values of the vector of asset prices of which the option values should be determined. In determining the option values at time t, we first perform a numerical integration along the radial direction in the N-dimensional polar coordinate system. The value thus obtained is expressed via a regression procedure as a function of the polar angles, and another numerical integration is performed over the polar angles to obtain the continuation value. The larger value of the continuation value and the immediate exercise value will then be the option value. A method is also proposed to estimate the standard error of the computed American option price.

ABSTRAK

Pertimbangkan opsyen Amerika jenis *basket call* dalam kes ketika ada N aset yang terlibat, jumlah kali pelaksanaan yang mungkin sebelum kematangan adalah terhingga, dan vektor harga aset dimodelkan dengan menggunakan proses Levy. Suatu kaedah berangka berasaskan regresi dan pengamilan berangka dicadangkan untuk menilai harga opsyen Amerika. Dalam kaedah yang dicadangkan, kita menggunakan taburan bagi vektor harga aset pada suatu masa hadapan t untuk menentukan nilai "penting" dari vektor harga aset yang mana nilai opsyen harus ditentukan. Dalam menentukan nilai opsyen pada masa t, kita mula-mula melakukan pengamilan berangka sepanjang arah jejari dalam sistem koordinat polar N-dimensi. Nilai yang diperolehi kemudian diungkapkan dengan menggunakan tatacara regresi sebagai fungsi bagi sudut kutub, dan satu lagi pengamilan berangka dilakukan terhadap sudut kutub untuk mendapatkan nilai lanjutan opsyen. Kemudian nilai yang lebih besar antara nilai lanjutan opsyen dan nilai perlaksanaan opsyen serta-merta merupakan nilai opsyen. Suatu lagi kaedah juga dicadangkan untuk menganggar ralat piawai bagi harga opsyen Amerika.

ACKNOWLEDGEMENTS

First and foremost I offer my sincerest gratitude to my supervisors, Professor Dr. Pooi Ah Hin and Professor Dr. Goh Kim Leng, who have supported me throughout my thesis with their patience, knowledge and support. Without their encouragement and effort, this thesis would not have been completed or written. One simply could not wish for better or friendlier supervisors.

I also thank the Department of Mathematics lab staff, especially Miss Ng Lee Leng. I would like to express my gratitude to all those who have supported me in any respect during the course of my research.

My special appreciation goes to my family members, especially my husband, Cheang Tze Kin, and my son Cheang Yong En, who was born before this dissertation was completed. Thanks for their supporting and encouraging me to pursue this degree. Without my husband's encouragement, I would not have finished the degree.

Finally, I would like to express special thanks to my parents, especially my mother, for looking after my son at a moment's notice and for all her encouragement and profound understanding.

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