CHAPTER 5

STANDARD ERROR OF THE COMPUTED PRICE OF AN AMERICAN OPTION

5.1 Introduction

The price of an American option computed in Chapter 4 is not exact because in the process of computing the price, we use a set of n_v selected values of $\tilde{\boldsymbol{\Theta}}^{(k)} = (\tilde{\theta}_1^{(k)}, \tilde{\theta}_2^{(k)}, ..., \tilde{\theta}_{N-1}^{(k)})$ and n_r+1 points along the radial direction (these points may be referred to as radial points) for each k, and also a set of n_v selected values of $\boldsymbol{\Theta}^{(k)} = (\theta_1^{(k)}, \theta_2^{(k)}, ..., \theta_{N-1}^{(k)})$ and n_r+1 radial points for each k (see Section 4.2). The variation of the computed price when the selected values of $\tilde{\boldsymbol{\Theta}}^{(k)}$, $\boldsymbol{\Theta}^{(k)}$ and radial points vary may be measured by using the concept of standard errors.

In Section 5.2, we describe a procedure to estimate the relevant standard errors. The effect of the size of n_v and the number n_r of points chosen along the radial direction will be investigated in Sections 5.3-5.4.

5.2 Estimation of the standard error

We notice that for each of the 2^N quadrants in the coordinate system for $(\tilde{e}_1^{(k*)}, \tilde{e}_2^{(k*)}, ..., \tilde{e}_N^{(k*)})$, we compute the coefficients $\tilde{d}_{gi}^{(k*)}$ and $\tilde{d}_{gij}^{(k*)}$ (see Eq.(4.2.24)), which can be used to find $Q(t_{k*}, \mathbf{x}^{(k*)})$ approximately. We also notice that in the computation of $\tilde{d}_{gi}^{(k*)}$ and $\tilde{d}_{gij}^{(k*)}$, we have chosen randomly for each quadrant, a set of n_v values of $\tilde{\Theta}^{(k*)}$, and for each chosen value of $\tilde{\Theta}^{(k*)}$, we have considered n_r+1 values of $\tilde{\rho}^{(k*)}$ (see Eq. (4.2.21)), which represent n_r+1 radial points.

It is obvious that for a different set of selected values of $\tilde{\Theta}^{(k^*)}$ and radial points, the computed values of $\tilde{d}_{gi}^{(k^*)}$ and $\tilde{d}_{gij}^{(k^*)}$ in a given quadrant would be different. Let $D_j^{(k^*)}$ be the set of all values of the $\tilde{d}_{gi}^{(k^*)}$ and $\tilde{d}_{gij}^{(k^*)}$ based on the j-th selected set of 2^N x n_v x (n_r+1) values of $(\tilde{\Theta}^{(k^*)}, \tilde{\rho}^{(k^*)})$. (Note that in each selection, the number of values of $\tilde{\Theta}^{(k^*)}$ selected for each quadrant is n_v.)

A way to measure the extent of variation of $D_j^{(k^*)}$ as we vary j is by examining the following estimated standard error of $Q(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)})$ where $\mathbf{x}_0^{(k^*-1)}$ is an (Nx1) vector of which the i-th component is $E^*[S_i^{(k^*-1)} \mid \mathbf{S}(0)]$:

$$S_Q(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)}) = \sqrt{\frac{1}{n_f}\sum_{j=1}^{n_f}[Q_j(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)}) - \overline{Q}(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)})]^2} \qquad (5.2.1)$$

where

n_f is the number of selected sets of 2^N x n_v x (n_r+1) values of $(\tilde{\Theta}^{(k^*)}, \tilde{\rho}^{(k^*)})$,

$Q_j(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)})$ is the value of $Q(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)})$ based on $D_j^{(k^*)}$

and

$$\overline{Q}(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)}) = \sum_{j=1}^{n_f} Q_j(t_{k^*-1}, \mathbf{x}_0^{(k^*-1)}) / n_f \, . \qquad (5.2.2)$$

As we move backward in time from $t_{k^*-1} = (k^*-1)\Delta t$ (i.e. when k assumes the value k*-1, k*-2,…, or 1 in the indicated order), we compute the following estimated standard error of $Q(t_{k-1}, \mathbf{x}_0^{(k-1)})$ where $\mathbf{x}_0^{(k-1)}$ is an (Nx1) vector of which the i-th component is $E^*[S_i^{(k-1)} \mid \mathbf{S}(0)]$:

$$S_Q(t_{k-1}, \mathbf{x}_0^{(k-1)}) = \sqrt{\frac{1}{n_f}\sum_{j=1}^{n_f}[Q_j(t_{k-1}, \mathbf{x}_0^{(k-1)}) - \overline{Q}(t_{k-1}, \mathbf{x}_0^{(k-1)})]^2} \tag{5.2.3}$$

where

n_f now is the number of selected sets of 2^N x n_v x (n_r+1) values of $(\tilde{\Theta}^{(k-1)}, \tilde{\rho}^{(k-1)})$,

$Q_j(t_{k-1}, \mathbf{x}_0^{(k-1)})$ is the value of $Q(t_{k-1}, \mathbf{x}_0^{(k-1)})$ based on

(i) the j-th selected set of 2^N x n_v x (n_r+1) values of $(\tilde{\Theta}^{(k)}, \tilde{\rho}^{(k)})$,

(ii) a randomly selected set of 2^N x n_v x (n_r+1) values of $(\Theta^{(k+1)}, \rho^{(k+1)})$ where $\Theta^{(k+1)} = (\theta_1^{(k+1)}, \theta_2^{(k+1)}, ..., \theta_{N-1}^{(k+1)})$ and $\theta_1^{(k+1)}, \theta_2^{(k+1)}, \ldots, \theta_{N-1}^{(k+1)}$ are the (N-1) angular coordinates in the polar coordinate system for $(e_1^{(k+1)}, e_2^{(k+1)}, ..., e_N^{(k+1)})$,

and

(iii) a randomly selected member from $\{D_1^{(k+1)}, D_2^{(k+1)}, ..., D_{n_f}^{(k+1)}\}$,

and

$$\overline{Q}(t_{k-1}, \mathbf{x}_0^{(k-1)}) = \sum_{j=1}^{n_f} Q_j(t_{k-1}, \mathbf{x}_0^{(k-1)}) / n_f . \tag{5.2.4}$$

The value of $S_Q(0, \mathbf{S}(0))$ will now be an estimate of the standard error of the computed American option price.

5.3 Estimation of the standard error of the price of an American option when N = 3

Consider the case when N=3, T=10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, and $a_3 = 0.4$.

Suppose $\mathbf{P} = \{\text{corr}(w_i^{(k)}, w_j^{(k)})\} = \begin{bmatrix} 1 & 0.1 & 0.15 \\ 0.1 & 1 & 0.05 \\ 0.15 & 0.05 & 1 \end{bmatrix}$. Let $\bar{m}_3^{(i)} = E[v_i^{(k)}]^3$ and $\bar{m}_4^{(i)} = E[v_i^{(k)}]^4$ (see Eq.(2.4.2) for the definition of $v_i^{(k)}$). The values of μ_i, σ_i and $\mathbf{S}^{(0)}$ are given by Table 5.3.1 for i = 1, 2, 3.

Table 5.3.1: Values of μ_i, σ_i and $\mathbf{S}^{(0)}$

i	μ_i	σ_i	$\mathbf{S}^{(0)}$
1	0.05	0.15	50
2	0.05	0.10	60
3	0.05	0.20	35

In this section, we will show numerical results for the standard errors of the prices of the American options for four cases which correspond to four different degrees of non-normality of the distributions of $v_i^{(k)}$, i=1,2,...,N.

Case 1: Standard error of American option price when $\overline{\mathbf{m}}_3^{(i)} = 0$ and $\overline{\mathbf{m}}_4^{(i)} = 3.0$, for i = 1, 2, 3.

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 0 for different (n_v, n_r) are shown in Table 5.3.2, and the plots of the $S_Q(t_k, \mathbf{x}_0^{(k)})$ against n_v (or n_r) when n_r (or n_v) is fixed are shown in Figures 5.3.1-5.3.2 and Appendix (refer to Figures A1-A8).

Table 5.3.2: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for different values of (n_v, n_r)

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1,2,3, other parameters are as given in the beginning part of Section 5.3]

(n_v,n_r)	k									
	9	8	7	6	5	4	3	2	1	0
(20,5)	0.0003	0.0006	0.0010	0.0013	0.0016	0.0019	0.0022	0.0023	0.0027	0.0029
(20,10)	8.18E-05	0.0005	0.0009	0.0012	0.0015	0.0017	0.0021	0.0025	0.0025	0.0028
(20,15)	0.0002	0.0006	0.0008	0.0010	0.0013	0.0015	0.0018	0.0020	0.0023	0.0025
(20,20)	0.0003	0.0004	0.0008	0.0009	0.0012	0.0014	0.0015	0.0018	0.0020	0.0024
(20,25)	0.0003	0.0004	0.0008	0.0009	0.0011	0.0013	0.0015	0.0017	0.0019	0.0023
(20,30)	0.0003	0.0004	0.0007	0.0008	0.0010	0.0012	0.0015	0.0017	0.0018	0.0021
(25,5)	0.0003	0.0004	0.0007	0.0011	0.0014	0.0016	0.0019	0.0022	0.0023	0.0025
(25,10)	0.0002	0.0005	0.0008	0.0011	0.0013	0.0015	0.0018	0.0020	0.0023	0.0025
(25,15)	0.0002	0.0005	0.0007	0.0010	0.0011	0.0013	0.0015	0.0018	0.0020	0.0023
(25,20)	0.0003	0.0003	0.0007	0.0008	0.0011	0.0012	0.0015	0.0018	0.0020	0.0023
(25,25)	0.0003	0.0004	0.0008	0.0008	0.0010	0.0012	0.0014	0.0016	0.0019	0.0022
(25,30)	0.0003	0.0004	0.0006	0.0007	0.0010	0.0011	0.0013	0.0015	0.0017	0.0020
(30,5)	0.0003	0.0004	0.0007	0.0010	0.0012	0.0014	0.0016	0.0018	0.0020	0.0022
(30,10)	0.0002	0.0004	0.0007	0.0009	0.0012	0.0013	0.0016	0.0017	0.0020	0.0023
(30,15)	0.0002	0.0004	0.0005	0.0008	0.0011	0.0011	0.0013	0.0016	0.0020	0.0021
(30,20)	0.0003	0.0003	0.0004	0.0006	0.0010	0.0011	0.0012	0.0014	0.0018	0.0022
(30,25)	0.0003	0.0004	0.0007	0.0008	0.0011	0.0012	0.0013	0.0014	0.0017	0.0020
(30,30)	0.0003	0.0004	0.0005	0.0007	0.0009	0.0010	0.0011	0.0014	0.0016	0.0020
(35,5)	0.0003	0.0007	0.0009	0.0010	0.0011	0.0012	0.0014	0.0016	0.0019	0.0021
(35,10)	0.0002	0.0004	0.0006	0.0007	0.0011	0.0012	0.0013	0.0015	0.0018	0.0021
(35,15)	0.0002	0.0002	0.0003	0.0006	0.0009	0.0010	0.0011	0.0013	0.0017	0.0021
(35,20)	0.0003	0.0002	0.0003	0.0005	0.0008	0.0010	0.0011	0.0013	0.0016	0.0021
(35,25)	0.0003	0.0004	0.0006	0.0008	0.0008	0.0010	0.0012	0.0014	0.0017	0.0020
(35,30)	0.0003	0.0004	0.0005	0.0006	0.0008	0.0009	0.0011	0.0013	0.0016	0.0020

Figure 5.3.1 and Figures A1-A4 in Appendix show that for a given fixed value of n_r, the estimated standard error decreases if the value of n_v increases. However, as shown in Figure 5.3.2 and Figures A5-A8 in Appendix, an increase of the value of n_r when n_v is fixed does not always result in a decrease in the estimated standard error.

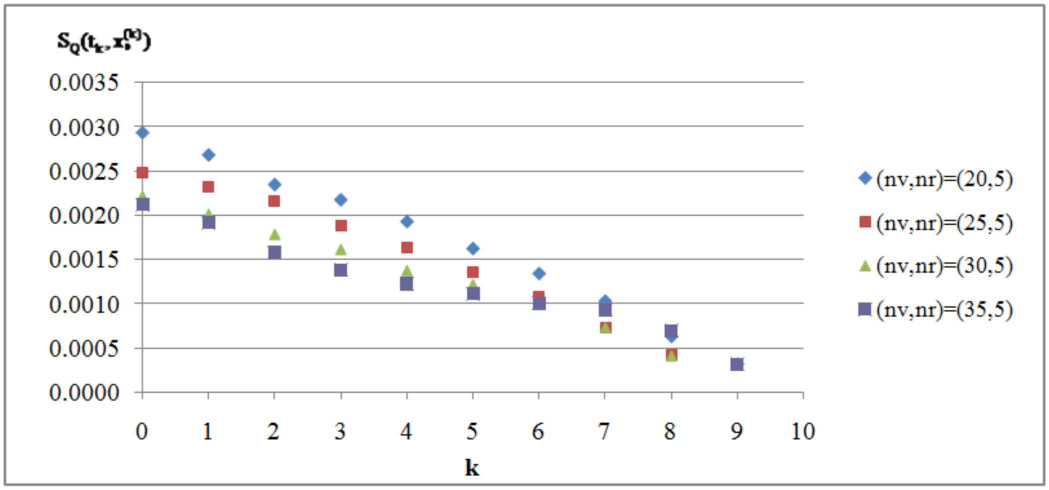

Figure 5.3.1: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_r is fixed but n_v is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3, a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

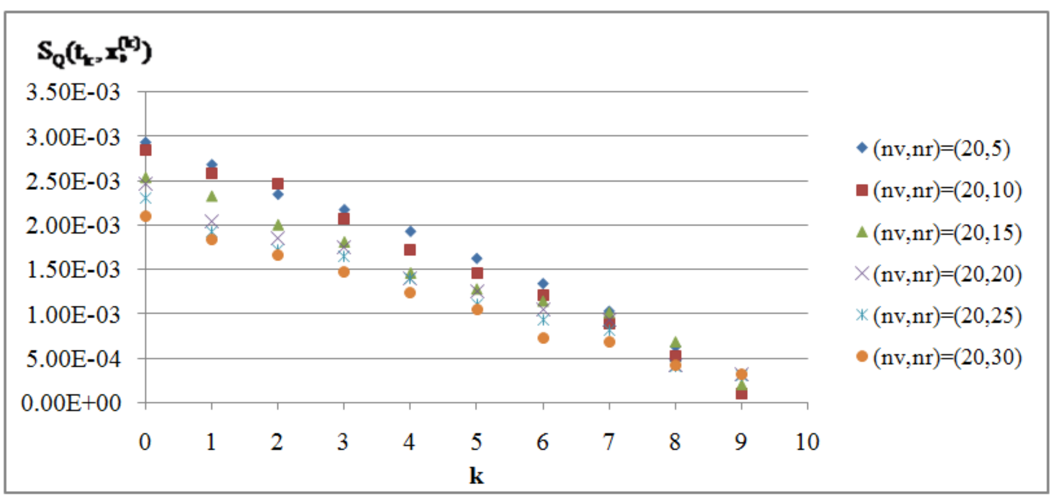

Figure 5.3.2: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_v is fixed but n_r is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

Case 2: Standard error of American option price when $\overline{\mathbf{m}}_3^{(\mathbf{i})} = \mathbf{0.1}$ and $\overline{\mathbf{m}}_4^{(\mathbf{i})} = \mathbf{3.0}$ for i = 1, 2, 3

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 0 for different (n_v, n_r) are shown in Table 5.3.3.

Table 5.3.3: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for different values of (n_v, n_r)

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0.1$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

(n_v,n_r)	k									
	9	8	7	6	5	4	3	2	1	0
(20,5)	0.0003	0.0007	0.0013	0.0015	0.0018	0.0019	0.0020	0.0022	0.0024	0.0028
(20,10)	0.0003	0.0008	0.0013	0.0015	0.0018	0.0019	0.0020	0.0021	0.0023	0.0027
(20,15)	0.0003	0.0007	0.0012	0.0016	0.0017	0.0018	0.0020	0.0023	0.0024	0.0030
(20,20)	0.0004	0.0007	0.0013	0.0016	0.0018	0.0018	0.0022	0.0024	0.0025	0.0033
(20,25)	0.0003	0.0007	0.0012	0.0015	0.0018	0.0017	0.0023	0.0024	0.0026	0.0030
(20,30)	0.0003	0.0008	0.0012	0.0015	0.0017	0.0017	0.0020	0.0023	0.0024	0.0028
(25,5)	0.0003	0.0007	0.0011	0.0014	0.0016	0.0017	0.0019	0.0020	0.0021	0.0026
(25,10)	0.0003	0.0008	0.0010	0.0014	0.0015	0.0016	0.0019	0.0019	0.0022	0.0024
(25,15)	0.0003	0.0006	0.0010	0.0013	0.0015	0.0017	0.0019	0.0022	0.0023	0.0027
(25,20)	0.0003	0.0007	0.0010	0.0013	0.0015	0.0016	0.0018	0.0020	0.0022	0.0025
(25,25)	0.0004	0.0007	0.0010	0.0014	0.0015	0.0016	0.0018	0.0020	0.0023	0.0025
(25,30)	0.0003	0.0008	0.0010	0.0013	0.0015	0.0016	0.0017	0.0020	0.0022	0.0025
(30,5)	0.0003	0.0007	0.0009	0.0012	0.0015	0.0016	0.0016	0.0018	0.0020	0.0023
(30,10)	0.0003	0.0006	0.0010	0.0012	0.0014	0.0015	0.0016	0.0017	0.0019	0.0023
(30,15)	0.0004	0.0006	0.0009	0.0012	0.0013	0.0015	0.0017	0.0018	0.0019	0.0022
(30,20)	0.0003	0.0006	0.0010	0.0012	0.0014	0.0016	0.0017	0.0019	0.0021	0.0025
(30,25)	0.0003	0.0006	0.0009	0.0010	0.0013	0.0015	0.0017	0.0019	0.0021	0.0024
(30,30)	0.0003	0.0005	0.0009	0.0010	0.0012	0.0013	0.0016	0.0018	0.0018	0.0022
(35,5)	0.0003	0.0006	0.0009	0.0010	0.0013	0.0014	0.0016	0.0017	0.0018	0.0022
(35,10)	0.0004	0.0005	0.0008	0.0010	0.0012	0.0013	0.0015	0.0016	0.0017	0.0022
(35,15)	0.0003	0.0005	0.0008	0.0010	0.0012	0.0013	0.0015	0.0016	0.0018	0.0022
(35,20)	0.0004	0.0005	0.0008	0.0010	0.0011	0.0013	0.0015	0.0015	0.0018	0.0021
(35,25)	0.0003	0.0005	0.0008	0.0010	0.0011	0.0013	0.0014	0.0016	0.0017	0.0021
(35,30)	0.0003	0.0005	0.0007	0.0009	0.0010	0.0012	0.0013	0.0015	0.0017	0.0022

The plots of the $S_Q(t_k, \mathbf{x}_0^{(k)})$ against n_v (or n_r) when n_r (or n_v) is fixed are shown in Figures 5.3.3-5.3.4 and Appendix (refer to Figures A9-A16). Figure 5.3.3 and Figures A9-A13 in Appendix show that for a given fixed value of n_r, the estimated standard error decreases if the value of n_v increases. However, as shown in Figure 5.3.4 and Figures A14-A16, an increase of the value of n_r when n_v is fixed does not always result in a decrease in the estimated standard error.

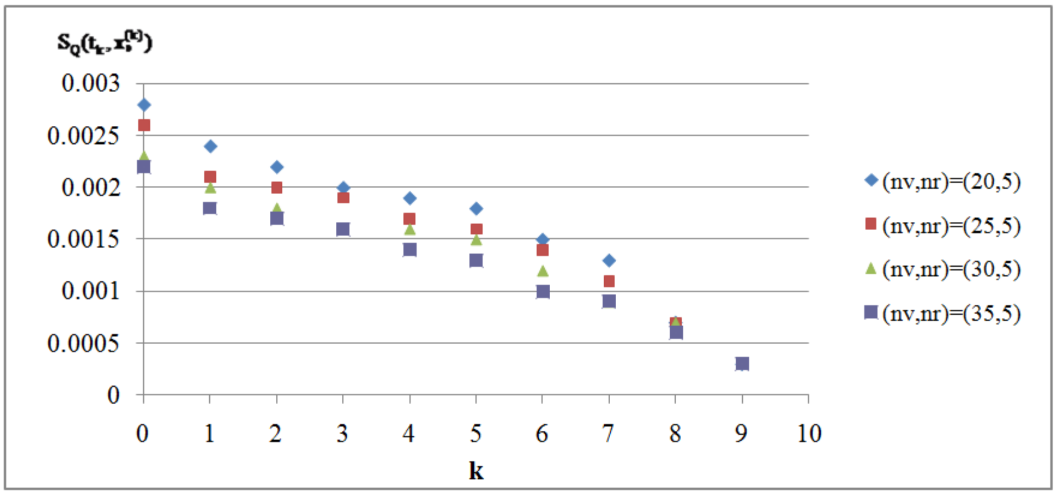

Figure 5.3.3: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_r is fixed but n_v is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0.1$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

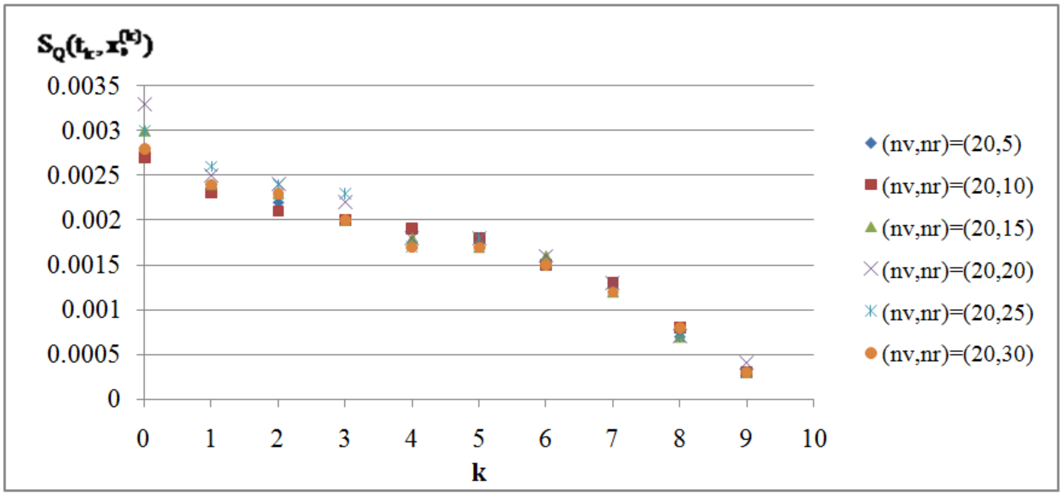

Figure 5.3.4: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_v is fixed but n_r is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0.1$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

Case 3: Standard error of American option price when $\overline{\mathbf{m}}_3^{(\mathbf{i})} = \mathbf{0}$ and $\overline{\mathbf{m}}_4^{(\mathbf{i})} = \mathbf{8.0}$ for i = 1, 2, 3

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 0 for different (n_v, n_r) are shown in Table 5.3.4.

Table 5.3.4: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for different values of (n_v, n_r)

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 8.0$**,** for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

(n_v,n_r)	k									
	9	8	7	6	5	4	3	2	1	0
(20,5)	0.0002	0.0006	0.0009	0.0014	0.0016	0.0018	0.0020	0.0022	0.0024	0.0352
(20,10)	0.0001	0.0007	0.0012	0.0015	0.0018	0.0019	0.0021	0.0023	0.0025	0.0334
(20,15)	0.0002	0.0005	0.0012	0.0015	0.0017	0.0019	0.0021	0.0023	0.0025	0.0291
(20,20)	0.0003	0.0005	0.0011	0.0014	0.0016	0.0018	0.0020	0.0021	0.0023	0.0322
(20,25)	0.0002	0.0004	0.0011	0.0013	0.0016	0.0018	0.0020	0.0021	0.0022	0.0345
(20,30)	0.0003	0.0004	0.0010	0.0013	0.0016	0.0017	0.0019	0.0020	0.0021	0.0336
(25,5)	0.0002	0.0006	0.0008	0.0013	0.0016	0.0017	0.0019	0.0020	0.0025	0.0332
(25,10)	0.0003	0.0006	0.0010	0.0015	0.0017	0.0019	0.0022	0.0025	0.0027	0.0317
(25,15)	0.0002	0.0005	0.0010	0.0016	0.0018	0.0020	0.0021	0.0022	0.0024	0.0257
(25,20)	0.0003	0.0005	0.0009	0.0013	0.0017	0.0019	0.0020	0.0022	0.0023	0.0335
(25,25)	0.0003	0.0004	0.0009	0.0014	0.0017	0.0019	0.0020	0.0021	0.0022	0.0331
(25,30)	0.0002	0.0003	0.0008	0.0013	0.0016	0.0017	0.0018	0.0020	0.0022	0.0327
(30,5)	0.0002	0.0007	0.0009	0.0014	0.0015	0.0016	0.0018	0.0020	0.0023	0.0332
(30,10)	0.0002	0.0005	0.0009	0.0014	0.0016	0.0018	0.0019	0.0020	0.0022	0.0321
(30,15)	0.0002	0.0004	0.0009	0.0013	0.0016	0.0017	0.0018	0.0021	0.0021	0.0336
(30,20)	0.0001	0.0004	0.0008	0.0013	0.0015	0.0017	0.0018	0.0020	0.0021	0.0355
(30,25)	0.0002	0.0004	0.0007	0.0012	0.0015	0.0016	0.0017	0.0019	0.0020	0.0371
(30,30)	0.0002	0.0004	0.0007	0.0011	0.0014	0.0016	0.0018	0.0019	0.0020	0.0329
(35,5)	0.0002	0.0006	0.0009	0.0012	0.0014	0.0015	0.0017	0.0019	0.0023	0.0316
(35,10)	0.0003	0.0005	0.0008	0.0013	0.0015	0.0017	0.0018	0.0019	0.0022	0.0302
(35,15)	0.0002	0.0005	0.0008	0.0011	0.0013	0.0015	0.0017	0.0018	0.0020	0.0334
(35,20)	0.0003	0.0005	0.0007	0.0012	0.0013	0.0016	0.0016	0.0017	0.0020	0.0309
(35,25)	0.0002	0.0004	0.0007	0.0011	0.0014	0.0015	0.0016	0.0018	0.0019	0.0318
(35,30)	0.0002	0.0004	0.0006	0.0011	0.0013	0.0015	0.0016	0.0017	0.0020	0.0307

The plots of the $S_Q(t_k, \mathbf{x}_0^{(k)})$ against n_v (or n_r) when n_r (or n_v) is fixed are shown in Figures 5.3.5-5.3.6 and Appendix (refer to Figures A17-A24). It is found that the increment of $S_Q(t_k, \mathbf{x}_0^{(k)})$ from $S_Q(t_{k+1}, \mathbf{x}_0^{(k+1)})$ when k=8,7,...,1 are relatively small when compared with the increment of $S_Q(0, \mathbf{S}(0))$ from $S_Q(t_1, \mathbf{x}_0^{(1)})$. The "sudden jump" in the last increment usually happens when the distributions of the $v_i^{(k)}$ deviate a lot from normality.

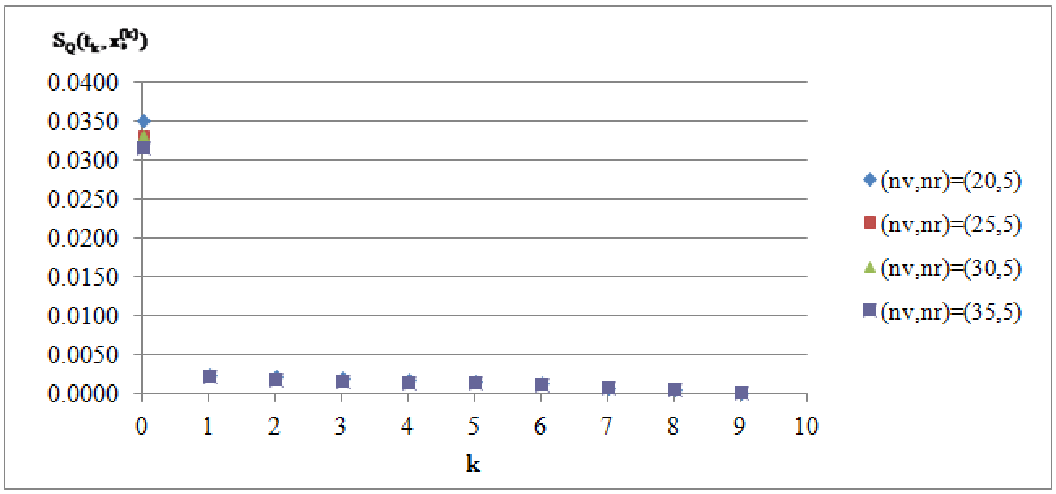

Figure 5.3.5: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_r is fixed but n_v is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 8.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

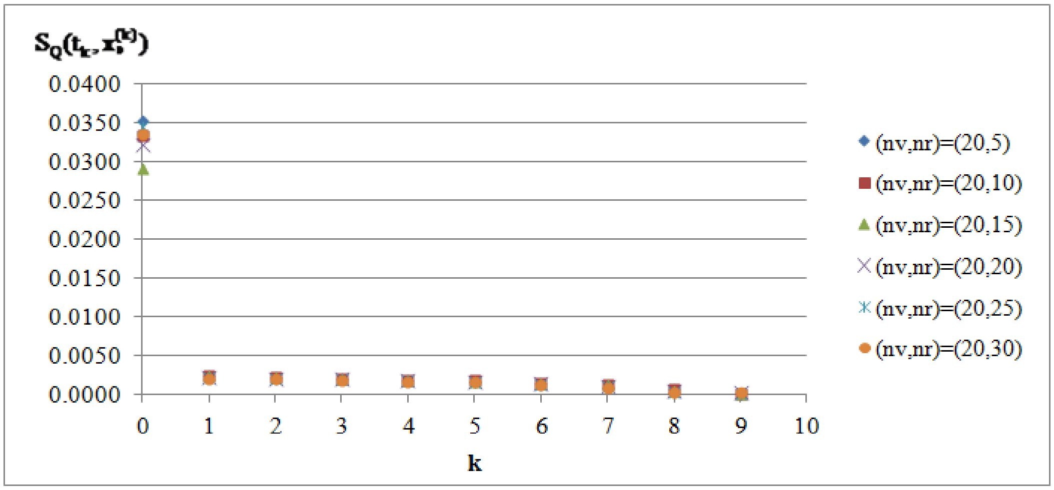

Figure 5.3.6: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_v is fixed but n_r is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 8.0$ for i=1, 2, 3, other parameters are as given in the beginning part of Section 5.3]

Case 4: Standard error of American option price when $\overline{\mathbf{m}}_3^{(1)} = 0.1$, $\overline{\mathbf{m}}_4^{(1)} = 5.0$, $\overline{\mathbf{m}}_3^{(2)} = 0.2$, $\overline{\mathbf{m}}_4^{(2)} = 4.0$, $\overline{\mathbf{m}}_3^{(3)} = 0.2$ and $\overline{\mathbf{m}}_4^{(3)} = 3.8$

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 0 for different (n_v, n_r) are shown in Table 5.3.5.

Table 5.3.5: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for different values of (n_v, n_r)

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(1)} = 0.1$, $\overline{m}_4^{(1)} = 5.0$, $\overline{m}_3^{(2)} = 0.2$, $\overline{m}_4^{(2)} = 4.0$, $\overline{m}_3^{(3)} = 0.2$ and $\overline{m}_4^{(3)} = 3.8$, other parameters are as given in the beginning part of Section 5.3]

(n_v,n_r)	k									
	9	8	7	6	5	4	3	2	1	0
(20,5)	0.0005	0.0011	0.0016	0.0020	0.0024	0.0029	0.0034	0.0037	0.0039	0.0410
(20,10)	0.0005	0.0009	0.0015	0.0018	0.0019	0.0022	0.0025	0.0028	0.0031	0.0371
(20,15)	0.0005	0.0009	0.0015	0.0018	0.0018	0.0020	0.0023	0.0026	0.0030	0.0329
(20,20)	0.0004	0.0008	0.0014	0.0017	0.0018	0.0021	0.0024	0.0029	0.0031	0.0383
(20,25)	0.0005	0.0008	0.0013	0.0016	0.0018	0.0022	0.0025	0.0030	0.0032	0.0385
(20,30)	0.0004	0.0008	0.0012	0.0015	0.0017	0.0022	0.0024	0.0028	0.0030	0.0337
(25,5)	0.0005	0.0010	0.0012	0.0016	0.0020	0.0021	0.0027	0.0033	0.0034	0.0403
(25,10)	0.0005	0.0008	0.0011	0.0014	0.0018	0.0019	0.0024	0.0026	0.0029	0.0348
(25,15)	0.0004	0.0008	0.0010	0.0014	0.0017	0.0019	0.0023	0.0027	0.0029	0.0319
(25,20)	0.0004	0.0007	0.0010	0.0013	0.0017	0.0018	0.0021	0.0024	0.0027	0.0327
(25,25)	0.0003	0.0007	0.0009	0.0013	0.0015	0.0017	0.0022	0.0024	0.0026	0.0356
(25,30)	0.0003	0.0007	0.0011	0.0012	0.0017	0.0020	0.0022	0.0025	0.0028	0.0326
(30,5)	0.0004	0.0008	0.0010	0.0014	0.0017	0.0019	0.0026	0.0029	0.0034	0.0391
(30,10)	0.0003	0.0006	0.0009	0.0012	0.0014	0.0016	0.0025	0.0027	0.0029	0.0317
(30,15)	0.0004	0.0007	0.0010	0.0013	0.0014	0.0016	0.0023	0.0024	0.0026	0.0384
(30,20)	0.0003	0.0007	0.0010	0.0011	0.0013	0.0015	0.0023	0.0024	0.0026	0.0376
(30,25)	0.0003	0.0006	0.0009	0.0012	0.0015	0.0016	0.0021	0.0023	0.0025	0.0355
(30,30)	0.0003	0.0006	0.0009	0.0011	0.0013	0.0015	0.0021	0.0023	0.0024	0.0315
(35,5)	0.0004	0.0005	0.0009	0.0010	0.0014	0.0016	0.0023	0.0024	0.0029	0.0339
(35,10)	0.0003	0.0006	0.0008	0.0010	0.0011	0.0014	0.0017	0.0022	0.0027	0.0349
(35,15)	0.0003	0.0004	0.0008	0.0010	0.0012	0.0013	0.0017	0.0020	0.0026	0.0332
(35,20)	0.0003	0.0005	0.0007	0.0009	0.0011	0.0012	0.0016	0.0021	0.0025	0.0314
(35,25)	0.0003	0.0005	0.0007	0.0008	0.0010	0.0011	0.0014	0.0020	0.0025	0.0325
(35,30)	0.0003	0.0004	0.0007	0.0009	0.0011	0.0012	0.0014	0.0019	0.0024	0.0311

The comparison of the $S_Q(t_k, \mathbf{x}_0^{(k)})$ when (n_v, n_r) varies are shown in Figures 5.3.7-5.3.8 and Appendix (Figures A25-A32). It is found that the increment of $S_Q(t_k, \mathbf{x}_0^{(k)})$ from $S_Q(t_{k+1}, \mathbf{x}_0^{(k+1)})$ when k=8, 7,..., 1 are relatively small when compared with $S_Q(0, \mathbf{S}(0))$ from $S_Q(t_1, \mathbf{x}_0^{(1)})$. The "sudden jump" in the last increment usually happens when the distributions of the $v_i^{(k)}$ deviate a lot from normality.

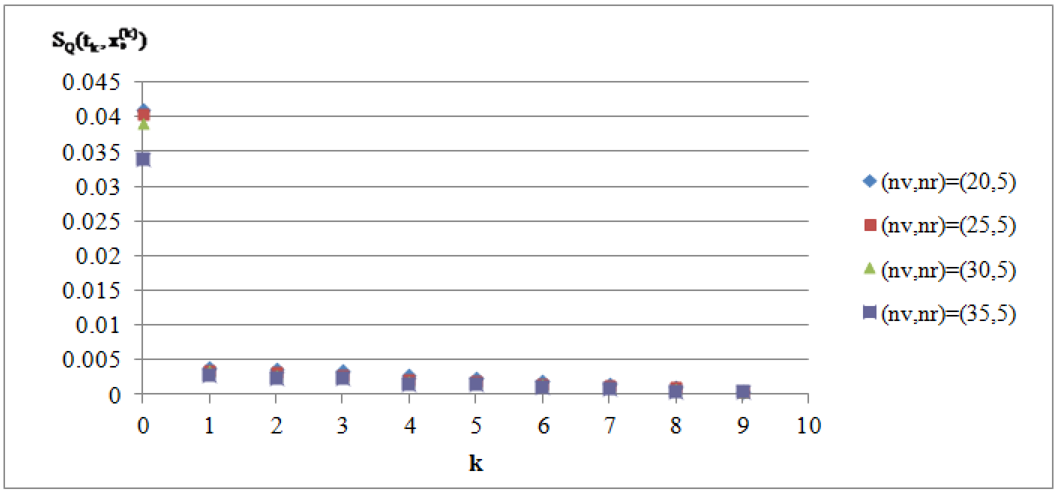

Figure 5.3.7: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_r is fixed but n_v is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(1)} = 0.1$, $\overline{m}_4^{(1)} = 5.0$, $\overline{m}_3^{(2)} = 0.2$, $\overline{m}_4^{(2)} = 4.0$, $\overline{m}_3^{(3)} = 0.2$ and $\overline{m}_4^{(3)} = 3.8$, other parameters are as given in the beginning part of Section 5.3]

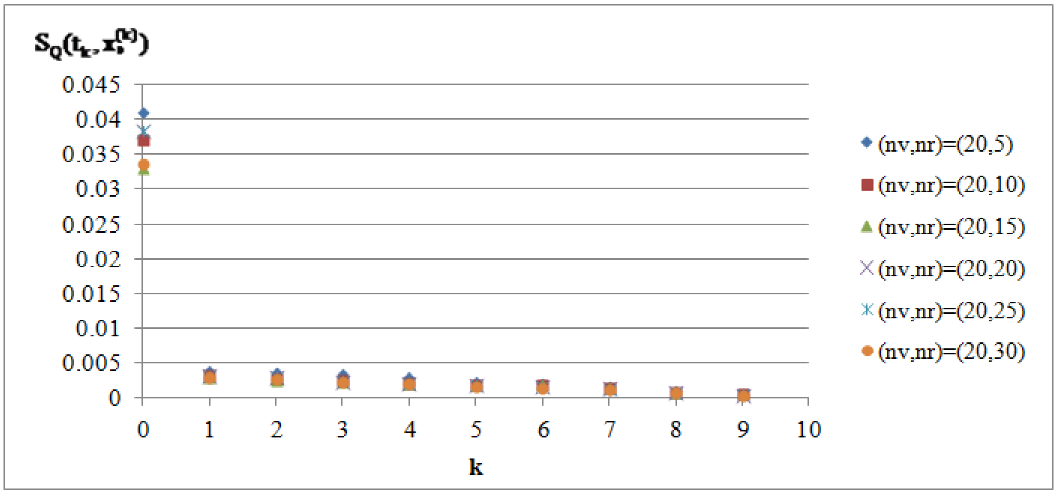

Figure 5.3.8: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ when n_v is fixed but n_r is varied

[Number of underlying assets is 3, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46, $a_1 = 0.3$, $a_2 = 0.3$, $a_3 = 0.4$, $\overline{m}_3^{(1)} = 0.1$, $\overline{m}_4^{(1)} = 5.0$, $\overline{m}_3^{(2)} = 0.2$, $\overline{m}_4^{(2)} = 4.0$, $\overline{m}_3^{(3)} = 0.2$ and $\overline{m}_4^{(3)} = 3.8$, other parameters are as given in the beginning part of Section 5.3]

5.4 Estimation of the standard error of the price of an American option when N=6

Consider the case when T=10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$ and $a_6 = 0.2$.

Suppose $\mathbf{P} = \{corr(w_i^{(k)}, w_j^{(k)})\} = \begin{bmatrix} 1 & 0.01 & 0.045 & 0.08 & 0.05 & 0.1 \\ 0.01 & 1 & 0.05 & 0.03 & 0.1 & 0.07 \\ 0.045 & 0.05 & 1 & 0.1 & 0.075 & 0.09 \\ 0.08 & 0.03 & 0.1 & 1 & 0.07 & 0.05 \\ 0.05 & 0.1 & 0.075 & 0.07 & 1 & 0.04 \\ 0.1 & 0.07 & 0.09 & 0.05 & 0.04 & 1 \end{bmatrix}$, and the values of $\mu_i, \sigma_i, \mathbf{S}^{(0)}, \overline{m}_3^{(i)}$ and $\overline{m}_4^{(i)}$ are given by Table 5.4.1 for i,j=1,2,…,6.

Table 5.4.1: Values of μ_i, σ_i, $\mathbf{S}^{(0)}$, $\overline{m}_3^{(i)}$ and $\overline{m}_4^{(i)}$

i	μ_i	σ_i	$\mathbf{S}^{(0)}$	$\overline{m}_3^{(i)}$	$\overline{m}_4^{(i)}$
1	0.05	0.15	50	0	3.0
2	0.05	0.10	60	0	3.0
3	0.05	0.20	35	0	3.0
4	0.05	0.20	40	0	3.0
5	0.05	0.20	45	0	3.0
6	0.05	0.20	52	0	3.0

In this section, we will show numerical results for the standard error from cases which corresponding respectively to the four values of (n_v, n_r) given by (100, 30), (200, 30), (300, 30) and (400, 30). We will obtain the estimated standard error of $Q(t_k, x_0^{(k)})$ for k=9, 8, 7 and 6, and then use linear extrapolation to find approximately the estimated standard error of $Q(t_k, x_0^{(k)})$ for k=5, 4, 3, ..., 1, 0.

Case 1: **Standard error of American option price when (n_v, n_r) = (100, 30)**

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 6 for (n_v, n_r) = (100, 30) are shown in Table 5.4.2. The plot of $S_Q(t_k, \mathbf{x}_0^{(k)})$ against k together with the fitted straight line is shown in Figure 5.4.1. The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for k=5, 4,…, 0 based on the fitted straight line are shown in Table 5.4.3.

Table 5.4.2: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for (n_v, n_r) = (100, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

k	$S_Q(t_k, \mathbf{x}_0^{(k)})$
9	0.00009
8	0.00089
7	0.00199
6	0.00296

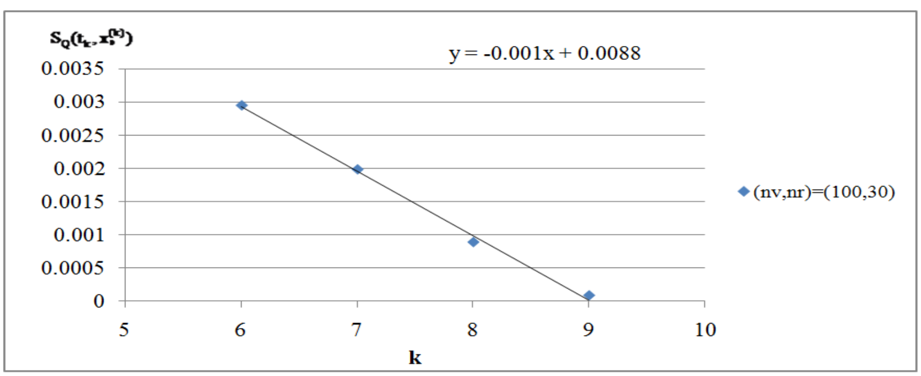

Figure 5.4.1: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ and the fitted equation for (n_v, n_r) = (100, 30)
[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

Table 5.4.3: The estimated values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ obtained by using linear extrapolation for (n_v, n_r) = (100, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, the fitted function is $S_Q(t_k, \mathbf{x}_0^{(k)})$ =-0.001(k)+0.0088, other parameters are as given in the beginning part of Section 5.4]

k	Estimated $S_Q(t_k, \mathbf{x}_0^{(k)})$
5	0.0038
4	0.0048
3	0.0058
2	0.0068
1	0.0078
0	0.0088

Case 2: Standard error of American option price when (n_v, n_r) = (200, 30)

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 6 for (n_v, n_r) = (200, 30) are shown in Table 5.4.4. The plot of $S_Q(t_k, \mathbf{x}_0^{(k)})$ against k together with the fitted straight line is shown in Figure 5.4.2. The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for k=5, 4,…, 0 based on the fitted straight line are shown in Table 5.4.5.

Table 5.4.4: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for (n_v, n_r) = (200, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

k	$S_Q(t_k, \mathbf{x}_0^{(k)})$
9	0.00005
8	0.00066
7	0.00165
6	0.00246

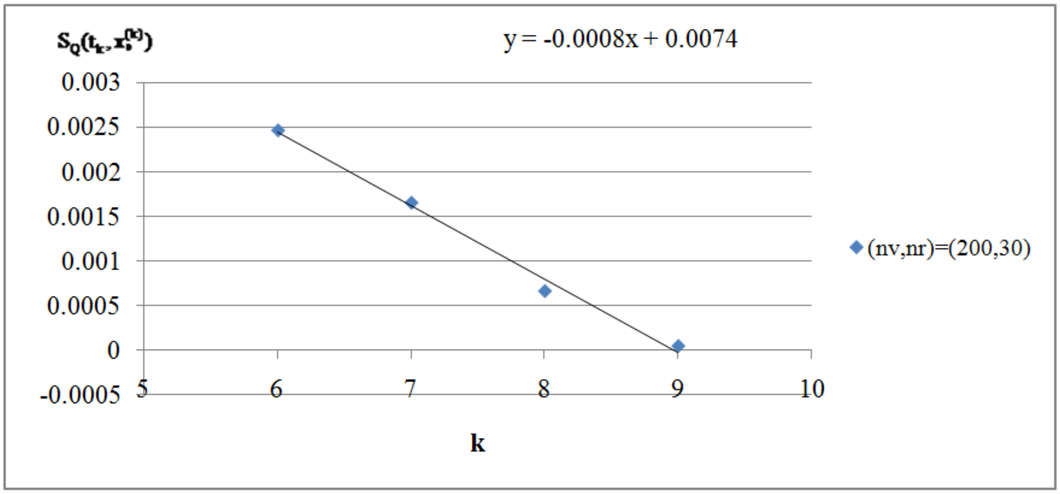

Figure 5.4.2: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ and the fitted equation for $(n_v, n_r) = (200, 30)$

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

Table 5.4.5: The estimated values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ obtained by using linear extrapolation for $(n_v, n_r) = (200, 30)$

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, the fitted function is $S_Q(t_k, \mathbf{x}_0^{(k)})$ =-0.0008(k)+0.0074, other parameters are as given in the beginning part of Section 5.4]

k	Estimated $S_Q(t_k, \mathbf{x}_0^{(k)})$
5	0.0034
4	0.0042
3	0.0050
2	0.0058
1	0.0066
0	0.0074

Case 3: **Standard error of American option price (n_v, n_r) = (300, 30)**

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2, ..., 6 for (n_v, n_r) = (300, 30) are shown in Table 5.4.6. The plot of $S_Q(t_k, \mathbf{x}_0^{(k)})$ against k together with the fitted straight line is shown in Figure 5.4.3. The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for k=5, 4,…, 0 based on the fitted straight line are shown in Table 5.4.7.

Table 5.4.6: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for (n_v, n_r) = (300, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}^{(i)} = 3.0$, for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

k	$S_Q(t_k, \mathbf{x}_0^{(k)})$
9	0.00005
8	0.00054
7	0.00151
6	0.00229

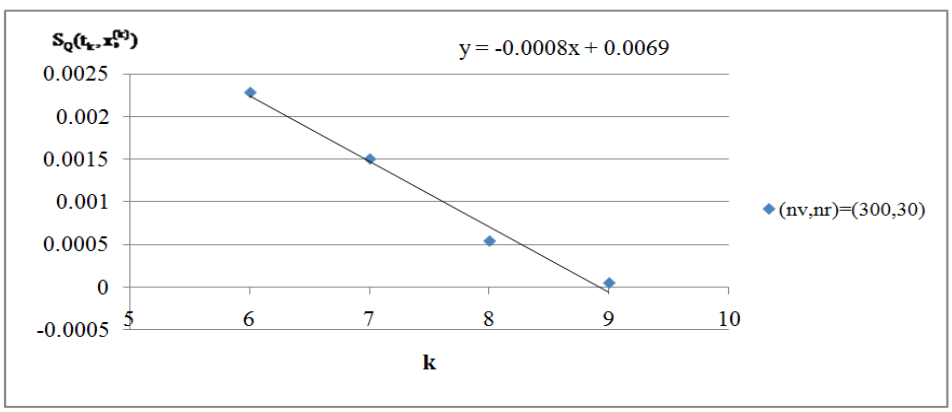

Figure 5.4.3: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ and the fitted equation, for (n_v, n_r) = (300, 30) [Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$ for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

Table 5.4.7: The estimated values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ obtained by using linear extrapolation for (n_v, n_r) = (300, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, the fitted function is $S_Q(t_k, \mathbf{x}_0^{(k)})$ =-0.0008(k)+0.0069, other parameters are as given in the beginning part of Section 5.4]

k	Estimated $S_Q(t_k, \mathbf{x}_0^{(k)})$
5	0.0029
4	0.0037
3	0.0045
2	0.0053
1	0.0061
0	0.0069

Case 4: Standard error of American option price when (n_v, n_r) = (400, 30)

The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$, for k=k*-1, k*-2,..., 6 for (n_v, n_r) = (400, 30) are shown in Table 5.4.8. The plot of $S_Q(t_k, \mathbf{x}_0^{(k)})$ against k together with the fitted straight line is shown in Figure 5.4.4. The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for k=5, 4,…, 0 based on the fitted straight line are shown in Table 5.4.9.

Table 5.4.8: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ for (n_v, n_r) = (400, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

k	$S_Q(t_k, \mathbf{x}_0^{(k)})$
9	0.00005
8	0.00041
7	0.00145
6	0.00215

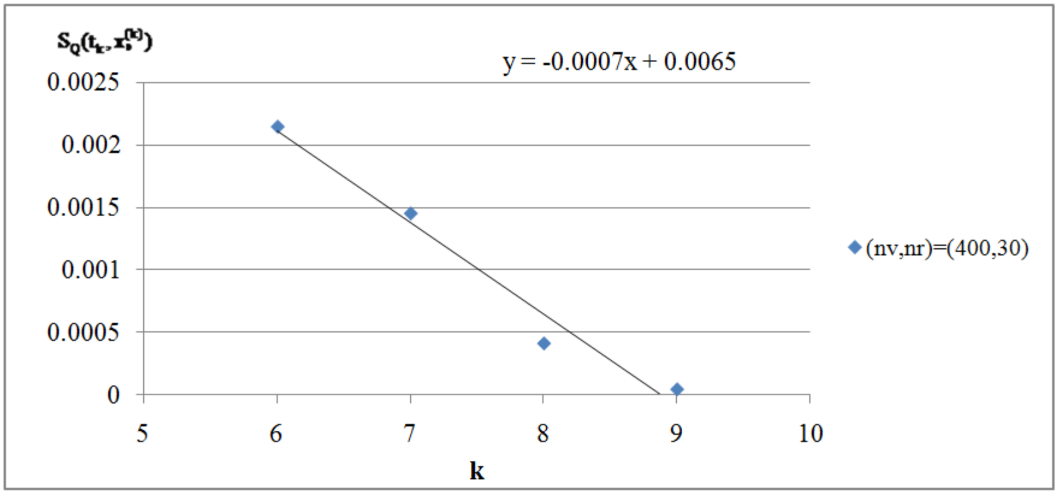

Figure 5.4.4: The values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ and the fitted equation for (n_v, n_r) = (400, 30) [Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, other parameters are as given in the beginning part of Section 5.4]

Table 5.4.9: The estimated values of $S_Q(t_k, \mathbf{x}_0^{(k)})$ obtained by using linear extrapolation for (n_v, n_r) = (400, 30)

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2,…, 6, the fitted function is $S_Q(t_k, \mathbf{x}_0^{(k)})$ =-0.0007(k)+0.0065, other parameters are as given in the beginning part of Section 5.4]

k	Estimated $S_Q(t_k, \mathbf{x}_0^{(k)})$
5	0.0030
4	0.0037
3	0.0044
2	0.0051
1	0.0058
0	0.0065

The estimated standard error of the price at time t=0 based on linear extrapolation in the above four cases are summarized in Table 5.4.10.

Table 5.4.10: The estimated standard error of the price at time t=0 based on linear extrapolation when n_r is fixed but n_v is varying

[Number of underlying assets is 6, k*=10, exercise dates are 1/365, 2/365,…, 10/365, r=0.05, K=46.5, $a_1 = 0.2$, $a_2 = 0.2$, $a_3 = 0.2$, $a_4 = 0.1$, $a_5 = 0.1$, $a_6 = 0.2$, $\overline{m}_3^{(i)} = 0$ and $\overline{m}_4^{(i)} = 3.0$, for i=1, 2, …, 6, other parameters are as given in the beginning part of Section 5.4]

(n_v, n_r)	Estimated standard error of the price at time t=0
(100, 30)	0.0088
(200, 30)	0.0074
(300, 30)	0.0069
(400, 30)	0.0065

Table 5.4.10 shows that the standard error of the American option price decreases as we increase the value of n_v while keeping n_r fixed at 30.