CHAPTER 6

CONCLUSIONS

The main contributions of the thesis include

(A) The introduction of a method based on multivariate quadratic-normal distribution for computing the joint distribution of the vector of time-t asset prices.

(B) The introduction of a method based on regression and numerical integration for pricing multidimensional American options.

(C) The introduction of a method for assessing the accuracy of the computed prices of the American options.

The limitation of the method in (B) is that the method may not be able to price fairly accurately the American options when the number of underlying assets is larger than eight.

Further remarks are as follows.

At a given time t, some of the values of the vector of asset prices are "important" because of their non-negligible probabilities of occurrence. In estimating American option prices, it is important to find out approximately the set which contains "important" values of the vector of asset prices at a given time t. Some of the methods in the literature determine the "important" values from the set of generated paths of vector of asset prices. But in the method proposed in the thesis, we instead make use of the distribution of the vector of asset prices at time t in order to determine the "important" values.

To construct the option value as a function of the vector of asset prices over the "important" region at a given time t, we first transform the space formed by the N asset prices to a space formed by N uncorrelated random variables having respectively the standard normal distributions. The resulting space is next transformed to the N-dimensional polar coordinate system. We approximate the option values for the points along the radial direction by a low degree polynomial. By using a regression procedure, each of the coefficients of the polynomial in terms of the radial distance is next expressed as a low degree polynomial of the polar angles. In this way, we obtain a representation of the option value as a function of the vector of asset prices at time t.

As the variation of the coefficients of the polynomial in terms of the radial distance is not large when the polar angles vary in each quadrant, we do not need to use extremely large number of chosen polar angles in order to express the coefficients of the polynomial in terms of the radial distance as a low degree polynomial function of the polar angles. The relatively small variation of the coefficients of the polynomial in terms of the radial distance has helped us to alleviate the problem of long computing time in the higher dimensional situation.

As the constructed function for the option value at time t_k is based on the selected points along the radial direction, and also on the selected polar angles, the function would change when the sets of selected values vary. In order to measure the extent of the variation of the constructed function for the option value at time t_k, we compute the time-t_{k-1} option values at the vector of mean values of asset prices at time t_{k-1} using the functions based on the different sets of selected values, and find the standard error of the time-t_{k-1} option values. The standard error at time t_0 will then be the standard error of the computed American option price. The standard error helps us to gauge whether the number of selected points along the radial direction and the number of selected polar angles are large enough to achieve the required level of accuracy for the computed American option price.

Although the thesis concentrates on the basket call options, the methods in the thesis may also be adapted to deal with other type of American options.