CHAPTER 1

Introduction

1.1 Introduction

Since early 1860s, there are considerable amount of research work related to panel data analysis that are available in the literature. There are a few terms used to describe panel data analysis. Woolridge (2003) and Hsiao (2005) defined panel data as at least two-dimensional data with a combination of time series and cross section analysis. Gujarati (2003) however categorised the following as panel data: (1) pooled data (pooling time series and cross sectional observations), (2) combination of time series and cross section data, (3) micro panel data, (4) longitudinal data (a study of a variable or group of subjects over time), (5) event history analysis and (6) cohort analysis. On the other hand, Arellano and Honoré (2001) defined a panel data set as a group of observations in which such groups consist of individuals belonging to the same family; a situation in which there is a 'homogeneous' grouping of the data. Thus, in general, panel data analysis refers to the study of observations on many individual units over a specific time interval (Franses, 2002; Nerlove, 2002; Hsiao, 2003 and Yafee, 2003). The individual units can be countries, states, firms, commodities and groups of people or even individuals.

Since the availability of data sources have been greatly raised nowadays, the interest to study a cross-sections of individuals observed over time have been increased among researchers and econometricians (Hsiao, 2003). Panel data can be used to describe the cross sectional behavior by pooling all information together rather than a single time series or cross-section. Panel data can enhance the quality and quantity of data: more informative data, more variability, less collinearity among variables, more degree of freedom and more efficiency rather than using either cross section or time series dimensions (Gujarati, 2003). In other words, if one is willing to consider both these two dimensions, panel analysis can provide a wealthy and powerful study of set of people. Panel data techniques consider fewer time series observations (Breitung and Candelon, 2005). This condition has challenged the traditional time series approach which requires more time spans to study the economics activities over a period of time. This is simply because, the short time span enable researchers to observe the units which are having similar characteristics before it becomes unstable if too long time spans are taken. Therefore, with continual observations of cross-section, panel analysis allows the researcher to study the dynamics of change of cross sectional units with short time series (Gujarati, 2003). The use of panel data in economics enables us to analyze the complex economic phenomena (Kapetonis et. al, 2006). That is why there are vast literatures on this panel analysis in economics. The attractions to obtain the parameter of interest which is assumed to have common values across panels units caused the huge development in this area.

To set the idea, the following panel data is considered: y_{it}, for $i = 1,2,\ldots,N$; $t = 1,2,\ldots,T$, be the i^{th} unit observed at a particular point in time, t. Thus, a typical data set will consist of $N \times T$ data. As an illustration, the United States (US) oil consumption trend data for the periods 1949 - 2004 is utilized (see Figure 1.1). In Figure 1.1, the use of the US oil for several sectors for over more than 50 years is studied. The sectors (panels) considered are transportation, industrial, residential and commercial lots. Here, y_{it} refers to the US petroleum consumption (measured in million barrels) for the sector i at year t, and thus we have $N = 4$ sectors and $T = 55$ years which gives a total of $4 \times 55 = 220$ observations in this study. Since these sectors are of interest, the characteristics of the petroleum used in each sector over a specified time period can be observed by plotting them together.

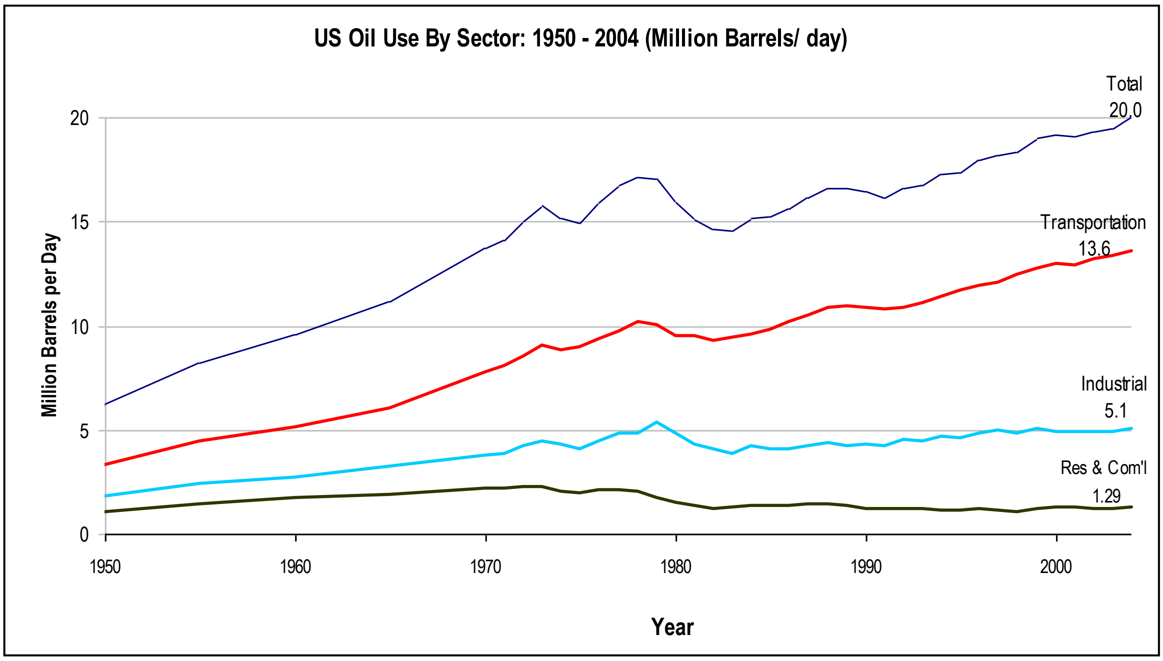

Figure 1.1: Panel Time Series Plots of the US Oil Use

There are two common models in the panel: first is the purely static model and second is the dynamic model. In the pure static case, several independent variables may be used to explain the model. For the simple pure static case, the following linear heterogeneous panel model is considered:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}; \quad i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{1.1}$$

where y_{it} is the response variable, x_{it} is the independent (predictor) variable, α_i, β_i are unknown parameters which varies across i and e_{it} are the random errors. In the dynamic panel model, there is the lagged dependent variable in the model which commonly takes the form as follows:

$$y_{it} = \alpha_i + \beta_i^T y_{it-1} + e_{it}; \quad i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{1.2}$$

where $\alpha_i, \beta_i, e_{it}$ are as in (1.1) while y_{it-1} is the first lagged value of y_{it}. To estimate the parameters of interest, the standard Ordinary Least Squares (OLS) estimation is commonly used in the static case, while the standard Generalized Methods of

Moments[1] (GMM) is used in the dynamic model (Ahn et al., 2001; Wansbeek, 2001; Bond, 2002; Hsiao and Tahmiscioglu, 2008).

In the pure static model, two models that are usually considered: (1) fixed effects (FE) and (2) random effects (RE). In the FE model, all regressors and individual effects are allowed for endogeneity ($y_{it} = \alpha_i + \beta^T x_{it} + e_{it};\ e_{it} \sim iid(0,\sigma^2)$) while RE model assumes exogeneity of all regressors and the random individual effects ($y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}; \beta_i = \beta + v_i$, where $e_{it} \sim iid(0,\sigma^2)$ and $v_i \sim (0,\Omega)$) (Baltagi et al., 2003). The choice between these models can be investigated using the Hausman and Taylor (1981) test.

For the dynamic case, the investigation of the stationary data in the panel data has received a great attention in this area. This is because one of the main characteristics of panel model is that they show nonstationary patterns, although there may be occasions on which linear combinations of the series are stationary. This can be achieved via the unit root test. Due to these interests, this study will focus on the estimation for the pure static case, where we limit our study to the FE model[2] and testing for the unit root in the dynamic model.

1.2 Related Issues

1.2.1 Cross Sectional Dependence (CD)

Panel data analysis has attracted a lot of attention among researchers for some reasonable motivations; one issue that is often addressed in many of the literature is the appropriateness of pooling the units in the sample which results in a pooled model (Davidson and MacKinnon, 1993; Baltagi, 2001; and Green, 2003). In the pooled model, the parameters are assumed to be constant, that are $\alpha_i = \alpha$, $\beta_i = \beta$, and the residuals are identically and independently (iid) distributed with mean zero and a

[1] The GMM estimator is based on method of moments to estimate the model.
[2] The RE model is beyond the scope of the study.

constant variance. Here, the OLS is appropriate, yielding the estimators that are consistent, efficient and unbiased (Coekley et al., 2006).

The pooled model ignores parameter heterogeneity (Coekley et al. 2006). Ignoring such heterogeneity could lead to inconsistent or meaningless estimates of interesting parameter estimates. Therefore, in order to tackle the problem with heterogeneity in parameters (refer model (1.1), two common approaches are usually employed: (1) FE (2) RE. Such models (pooled, FE and RE) however ignore the structure of the data since each observation is treated as independently among the others, $E(e_{it}e_{jt}) = 0$. In general such restriction is invalid since most of the economic data are correlated between cross sectional units; this may arise from a common influence which affects all cross section units.

The cross sectional dependence (CD) is defined when the residuals are correlated across units (test for $E(e_{it}e_{jt}) \neq 0$ for $i \neq j$). The CD among cross sectional units may occur due to the presence of unobserved factors and global shocks that are common to all members. This shock can be correlated with other regressors, whose effects are captured by the disturbances and thus give rise to non-zero off-diagonal elements of the variance covariance matrix. In economic applications, these unobserved factors may be explained by social norms and neighbourhood effects which imply strong interdependencies between cross sectional units.

There are a number of studies that have used an unobserved common factor structure to explain cross dependency (see Pesaran, 2004, 2006; Bai and Ng, 2002; Moon and Perron, 2004; and Philips and Sul, 2003). The common factor structures have several advantages: firstly, the procedure of statistical estimation and inferences are in general well-understood, and secondly, using common factors to explain cross dependency leads to no dimensionality problem which has been found to work well in many empirical studies (Gengenbach et al., 2010).

When the assumption of the independence among residuals is violated, the pooled and other estimators which assume cross sectional independence are inadequate, and could lead to significant size distortions in the presence of neglected CD. To illustrate this, consider model in (1.1). For the pooled model which restricted to independent assumption among the residuals, we have the estimates (1) $\hat{\mathbf{b}} = \left(\hat{\alpha}, \hat{\boldsymbol{\beta}}\right)^T = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$; with $E(\hat{\mathbf{b}}) = \mathbf{b}$ and $Var(\hat{\mathbf{b}}) = (\mathbf{X}^T\mathbf{X})^{-1}\sigma^2$. In the presence of CD, the estimates of $\mathbf{b}$ takes the form of (2) $\hat{\mathbf{b}} = \left(\hat{\alpha}, \hat{\boldsymbol{\beta}}\right)^T = (\mathbf{X}^T\boldsymbol{\Omega}\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\Omega}\mathbf{y}$; with $E(\hat{\mathbf{b}}) = \mathbf{b}$ and $Var(\hat{\mathbf{b}}) = (\mathbf{X}^T\boldsymbol{\Omega}\mathbf{X})^{-1}\sigma^2$, with non-zero off diagonal of $\boldsymbol{\Omega}$. It is observed that, the parameter estimates in both cases, (1) and (2) are unbiased but $Var(\hat{\mathbf{b}})$ in (2) is inefficient due to the effect of the CD on the standard error. This could leads to incorrect inferences to the model and subsequently results in the wrong decision making.

Moreover, with CD, the model becomes more complicated and the limitations of software language for modeling the panel in the presence of cross dependency hinder reliable modeling. By ignoring the cross dependency problem and assuming uncorrelated errors among the cross sectional units (Philips and Sul, 2003), a misspecified model is obtained.

1.2.2 Outliers

Outlier is an observation that is very peculiar and differs from the entire observed data. In many cases, some data points will be deflected away from their expected values than what are deemed reasonable. This can be due to a systematic error, faults in the theory that generated the expected values, or it can simply arise from the cases where some observations are a long way from the centre of the data. Outlying points can therefore indicate faulty data, erroneous procedures, or areas where a certain theory is perhaps not valid. In practice, a small number of outliers are expected in normal distributions.

There are several types of outliers that are commonly handled in statistics applications; namely (1) Additive outliers (AO), (2) Innovation outliers (IO), (3) Level Shift (LS), (4) Temporary Change (TC) and (5) Leverage Point (LP). The AO only affect the level but leave the variance unaffected. The presence of the AO will not be reflected in the values of the adjacent observations but its manifestation can be dramatic and obvious. The IO arises from an inherent form of contamination and can be reflected through the correlation structure of the process in neighbouring observations. The potential IO conspires to conceal itself and the detection of it may become more problematic. On the other hand, the LS will produce an abrupt and permanent step in the series while TC dies out gradually in time. The LP refers to outlying values that are present in the independent (predictor) variables. A large LP value influences the parameter, thus pulling the fitted model towards it and resulting in a bad fit.

The existence of multiple outliers (MO) in the dataset adds to the complexity of identifying and detecting outliers. The time series analysis with MO is hindered by two problems: masking and swamping. While swamping occurs when the observation is not an outlier but is misjudged as an outlier, masking occurs when an outlier is being masked by other observations. These two problems are typically caused by the other adjacent outliers. To illustrate this phenomenon, a linear regression model is considered.

Let $\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} = \mathbf{H}\mathbf{y}$. Specifically, for the i^{th} observation, $\hat{y}_i = \sum_{k=1}^{n} x_i \left(\mathbf{X}^T\mathbf{X}\right)^{-1} x_k^T y_k = \sum_{k=1}^{n} h_{ik} y_k$. In particular, consider an outlier at point $i = j$. $y'_i = y_i + \Delta y_i$, is replaced, where Δy_i takes the form of $\Delta y_i = \begin{cases} 1 & , i = j \\ 0 & \text{otherwise} \end{cases}$. It can be seen that at point $i = j$, $\hat{y}_j = \sum_{k \neq j}^{n} h_{jk} y_k + h_{jj} y'_j = \sum_{k=1}^{n} h_{jk} y_k + h_{jj} \Delta y_j$; thus $\Delta \hat{y}_j = h_{jj} \Delta y_j$.

This is the impact on $\hat{y}_j$ of a change in y_j equals that changes multiplied by h_{jj}. Thus, the regression line will rotate or shift, causing some of the original observations to appear as outliers (swamping). If the additional outliers (say at point $i = m$) are added to the original outlier closely, these outliers may not be picked up as outliers because of the more pronounced rotation or shift of the regression line towards these outliers (masking).

To overcome masking, several proposals are made and this includes those of Atkinson and Marco (2000). These methods however, typically involve removing the 'masking' outliers from the data set before the 'masked' outliers can be identified. The misidentification of outliers may result in biasness to parameter estimates and thus provide an inappropriate model in the panel analysis. Many other useful references for the detection of outliers in the time series model are discussed in the following section.

Chang et al. (1989) introduced the iterative procedures by using the intervention models discussed in Box and Tiao (1975) to determine the outliers' effects for the detections of the AO and IO. Chen and Liu (1993) extended the study by Chang et al. (1989) to other two types of outliers: (i) LS and (ii) TC. Sachez et al. (2000) improved the performance of the previous method of Chen and Liu (1993) by using two tools to distinguish the IO from the LS with the presence of outliers. The first is a better initial parameter estimate that is obtained by cleaning the series of patches of jointly influential observations that are treated as LS. The second is a better significance level that prevents the confusion of LS with IO. Franses and Ghijsels (1999), who extended the study of Chen and Liu (1993) in forecasting stock market volatility in generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, focused on forecasting performance of GARCH models for AO-corrected returns[3]. As a result, they found that the models for the AO-corrected data yielded substantial improvement over GARCH and GARCH-t (when for η_t is conditionally *t*-distributed) for the original returns. Pena (1990) suggested a statistics based on the Mahalanobis distance to measure the influence of outliers on the model parameters and the techniques were shown to be very useful for indicating the robustness of the fitted model. Bidarkota (2003) used different approaches whereby he suggested multi-process mixture models as an alternative model in capturing LS in the presence of outliers.

In the economic and financial data, for example, some observed values may be inconsistent from other observations in a sample of heterogeneous economic units. These isolated or extreme observed values are termed outliers and often have a large impact on the results of the statistical analyses on which the conclusion based on a sample with and without these units may differ drastically (Verardi and Wagner, 2010).

1.3 Problem Statement and Objectives of the Study

Over the years, the interest in the panel data models focused on the issue of cross sectional dependence. However, limited literature can be found on the presence of outliers in the panel. Bramati and Croux (2007) studied the robust regression technique with the fixed effects in contaminated panel. They investigated the robustness of the procedure by means of breakdown point computations with simulated data.

In empirical studies, CD among cross sectional units may occur due to the presence of unobserved factors and global shocks that are common to all panel members. In addition, each cross sectional may have local shocks (outliers) which affect

[3] AO-corrected returns, r_t^*, that is formulated from; "Let's consider the GARCH (1,1) model; $r_t = \eta_t h_t^{1/2}$, where; $h_t = \alpha_0 + \alpha_t r_{t-1}^2 + \beta_t h_{t-1}$, $\eta_t \sim N(0,1)$. Thus, the AO-corrected returns is constructed as $r_t^* = r_t$ for $t \neq \tau$, with the formula $r_t^* = sign(r_t).(r_t^{*2})^{1/2}$ for $t = \tau$."

only these respective individual units. With the presence of outliers, the effect of cross dependency can be more severe. In view of these, it is necessary to assess whether:

- *In the presence of outliers, does cross sectional dependence really exists?*

  The standard cross sectional dependence tests rely on the estimated OLS residuals and it is subjected to the influence of outliers. The presence of multiple outliers may worsen the situation due to masking, and this will affect the presence or absence of CD. Thus, robust versions of CD tests are proposed to overcome such problem.

- *Are the standard approach of estimation and inference employed in the literature remains appropriate in dealing with cross correlated error?*

  Even though there are a number of available approaches (Coakley et al., 2002, 2006; Philips and Sul, 2003; Kapetonis and Pesaran, 2004; Pesaran, 2006) that take into consideration the presence of CD in modeling the panel, the methods may provide inconsistent parameter estimates and inferences when outliers occur in the panel. As such, one needs to filter the effect of this cross section dependence and outliers to the model. Based on this interest, some robust approaches are proposed to deal with the cross sectional problem in the presence of contaminations in the panel data. The properties, test of hypothesis and construction of the confidence intervals for the parameter are also considered.

- *Does the presence of outliers affect the finite sample behavior of the standard estimator?*

  The finite sample behaviors of estimator are commonly evaluated via simulation experiment. The statistical measures obtained from the experiment, for example; bias, MSE; may affected in the presence of outliers. The bias of the parameter estimates may become huge and subsequently result in large MSE. This will lead to inconsistent in parameter estimates as well as its standard error. This problem can be avoided using an appropriate estimation procedure and this can be achieved using robust estimator. Due to that, the performance and robustness of the proposed estimator in parameter estimation is discussed and comparisons are made to some existing approaches in the literatures.

- *Do the standard unit root tests provide reliable result in detecting the presence of a unit root in the presence of outliers and cross dependency?*

  The presence of cross dependency and outliers may affect the stationary of the model in the dynamic model. The CD and outliers' effect may biases the OLS estimator subsequently result in stationary of the model where as it is nonstationary. Based on this issue, a modification of existing unit root test is discussed with the aim to correct for CD, reduce the outliers' effect and yield a reliable result for a unit root test.

1.4 The Contribution of the Thesis

The main contributions of the thesis are listed as follows. Firstly, robust cross dependency tests which are less sensitive to the influence of outliers are proposed. The presence of cross sectional tests reviewed includes the Breusch and Pagan (1980) Lagrange Multiplier Test (LM) and the Pesaran (2004) Cross Dependency Test (PCD). The LM and PCD tests as in Pesaran (2004) are sensitive to outliers in both pure static and dynamic models especially for the case of mild CD effect. These tests produce both type I[4] and type II[5] error in the presence of outliers. Thus, as an alternative to these tests, robust tests which filter the effects of outliers on cross section correction are proposed. Our tests incorporate the robust methods with some modifications to the existing approaches in both types of models. The properties of the tests are derived in terms of asymptotic distribution of the respective proposed tests, while the finite sample behaviour of the proposed tests is examined by means of Monte Carlo experiments.

[4] The test rejects a true null hypothesis (no CD).
[5] The test fails to reject a false null hypothesis (no CD).

Secondly, a robust estimation procedure is proposed to estimate the parameter of interest in pure static model. The objective function of the proposed procedure aims at minimizing overall error yet is robust to influence outliers as well as cross correlated errors. The proxy (the average of dependent variable and the observed regressor) of Pesaran (2006) approach is modified with the aim of computing a robust variance covariance matrix of residuals which yield consistent and unbiased estimators. Since Pesaran's (2006) approach provide inconsistency in parameter estimates in the presence of outliers, an alternative approach that allows for cross sectional dependence in the presence of outliers is introduced.

Thirdly, the properties of the proposed model in terms of the limiting distribution of the proposed estimator are derived. This will provide a new test statistics for hypothesis testing, confidence interval of parameter of interest and overall goodness of fit of the model. To understand the finite sample behavior of the proposed estimator, a Monte Carlo simulation study is conducted. The performance of the proposed method will be measured in terms of bias and consistency.

Fourthly, in coping the presence of outliers and CD in dynamic panels, a modification of Pesaran (2007) unit root test (namely, CIPS) is proposed. The CIPS unit root test seems to provide a good size and power of study when no outlier is present. Because the poor performance of CIPS in the presence of outliers, we modify the CIPS approach and Monte Carlo evidence support a better result of a proposed test in terms of size and power.

For illustration purposes, all proposed procedures are applied to the real data set. The gasoline data of 18 organizations for Economic Co-operation and Development (OECD) countries will be used to estimate the parameter of the model in purely static model while; The Purchasing Power Parity (PPP) data will be used as an empirical application for testing a unit root in dynamic framework. Here, the aim is to illustrate how those approaches solve the real situations of economic dataset and provide a good explanation and discussion based on the output obtained.

1.5 Organization of the Study

The remainder of this study is organized as follows: Chapter 2 will discuss cross dependency tests of Breusch and Pagan (1980) (LM) and Pesaran (2004) (PCD). The details about the proposed cross dependency tests based on the robust tools are also presented in this chapter. The properties of the proposed tests are studied and the performances of these tests are illustrated via the Monte Carlo simulation study. The results of the simulation study of these CD tests are also discussed in this chapter.

Chapter 3 discusses the techniques of the estimation procedures used to estimate the parameter in panel. The pooled model which is restricted to independence assumption of residuals is reviewed. The Common Correlated Effects Mean Group (CMG) of Pesaran (2006) which relaxes such assumption is also discussed. The robust version of the CMG is proposed where the details of this procedure are discussed. The method to evaluate the fit of the model is also discussed further. In inferential statistics, the asymptotic distribution of the RCMG is revised: the test statistics, hypothesis tests as well as the confidence interval of parameter estimates based on the asymptotic distribution are also developed in this chapter.

Chapter 4 will focus on the simulation studies with the aim of illustrating the respective estimation procedures. Several simulation experiments are conducted to study the behaviour of these procedures in terms of the parameter estimates itself. In the presence of outliers, the estimation procedure is illustrated by introducing a small percentage of the contaminated data to the original series. Here, the size and the power of the respective procedures as discussed in the previous chapter are measured. For CI estimation, the quartiles of the parameter estimates using Monte Carlo simulation study are estimated.

Chapter 5 discusses the unit root tests in the panel data in the presence of the cross-sectional dependence. The beginning of the chapter discusses the standard Augmented Dickey-Fuller (ADF) unit root test with the assumption of cross sectional independence within the residuals. With CD, the Common Correlated ADF (CADF) proposed by Pesaran (2007) which uses the simple average of the individual CADF-tests is reviewed and this is subjected to the influence of outliers. A robust unit root test as an alternative to the ADF is introduced while Pesaran's unit root tests and the properties of these tests are discussed. The small sample performance is examined via the Monte Carlo simulation study and the results are given at the end of this chapter.

Chapter 6 presents the analyses using real data with the aim of determining the appropriate procedure and reliable model. Finally, Chapter 7 provides some concluding remarks of this study.

CHAPTER 2

Cross Sectional Dependence Tests

2.1 Introduction

Several statistical procedures have been developed to diagnose the CD in panels. Moran (1948) was the first to provide a test of spatial independence in the context of a pure cross section model. This test is however based on spatial matrix which is inappropriate for finance and economics modeling. Thus, the residuals correlated across units (test for $E(e_{it}e_{jt}) \neq 0$ for $i \neq j$) is analysed based on the average of pair-wise correlation of the residuals $\hat{\rho}_{ij}$. The Lagrange Multiplier (hereafter LM) test of Breusch and Pagan (1980) is based on the squared sample pair-wise correlations $\hat{\rho}_{ij}^2$ of residuals. It has been shown that the test has the correct size only if the number of cross sectional units (N) is less than or equals to the length of the time period (T), which is $N \leq T$. This is due to the fact that $E\left(\hat{\rho}_{ij}^2\right) \leq 1$ by the central limit theorem and is non-centered at zero for large N (see Pesaran, 2004). As such, the size of the test will be distorted as $N \to \infty$. Pesaran (2004) examined the normal approximation version of the LM test resulting in a modified CD test (hereafter PCD) which uses $\hat{\rho}_{ij}$ and is applicable for any values of N and T. The PCD test is very similar to Friedmen's (1937) test which is based on Spearman's rank correlation coefficient R_{ave}[6].

Frees (1995) however modified the version of Friedmen to R_{ave}^2 in order to obtain a better power and size of the CD test in his paper. Meanwhile, Pesaran et al. (2008) proposed a bias-adjusted normal approximation version of the LM test for the

[6] The formula of Friedmen's test is given as $R_{ave} = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{r}_{ij}$, where $\hat{r}_{ij}$ is the sample estimate of the rank correlation coefficient of the residuals.

panel data models with strictly exogenous regressors and normal errors. This method uses the exact mean and variance of the LM test and provides consistency in size and power when the cross section mean of factor loadings[7] is near zero. However, the bias-adjusted LM test is not as robust as the PCD test for the non-normal residuals and/or in the presence of weakly exogenous regressors.

In a recent study, Godfrey and Yamagata (2010) proposed the bootstrap methods for testing H_0 of the cross section independence in the panel. They relaxed the assumption of normality and homoskedasticity of the residuals as described in Pesaran et al. (2008) and the Monte Carlo results indicate that the proposed test give comparable results under classical assumptions and is well-behaved under heteroskedasticity. Moscone and Tosetti (2009) reviewed and compared the performance of several possible CD tests based on the sample pair-wise correlation coefficient and spacing. The Monte Carlo results show that the tests based on spacing are powerful (reject the null hypothesis correctly) under various forms of strong CD. However, these tests have low power (inability to detect the presence of CD) in capturing weak dependency among the residuals[8].

A small fraction of spurious observations can sometimes seriously distort the results of CD due to the estimated residuals obtained from OLS. However, it is more severe when there are bad leverage points (that is, outliers in X-direction). This situation arises in the dynamic panel model. For example, consider model in (2.3): $y_{it} = \alpha_i + \beta_i^T y_{it-1} + e_{it}$. If outlier occurs at $t = s$, it creates two outliers in the regression. Say that (y_{is-1}, y_{is}) is a vertical outlier referring only to the outlyingness in y_{is}, while (y_{is}, y_{is+1}) is the leverage point because their y_{is} value is outlying. If (y_{is}, y_{is+1}) is far from the plane corresponding to the majority of the data, it is said to be a bad leverage

[7] Refer to equation (2.4) to see what factor loading is.
[8] Also known as spatial correlation (see Moscone and Tosetti, 2009).

point. If this point is large enough, the OLS estimator will be biased towards zero (Rousseeuw and Van Zomeren, 1990).

The cross sectional dependence tests of the LM and PCD rely on the OLS residuals, and it is well known that these estimates are sensitive to the presence of outliers. The presence of multiple outliers may worsen the situation due to masking, and this will subsequently affect the OLS fit and the corresponding residuals. The estimated residuals are affected by the outliers' effects which subsequently result in an incorrect value of pair-wise correlation of the residuals $\hat{\rho}_{ij}$ and provide incorrect results for the test statistics. For example, when the panel is free from CD, the presence of outliers will tend to favour the alternative hypothesis that CD is present. Likewise, if CD is observed in the panel, the presence of outliers may lead to the acceptance of the null hypothesis of cross section independence. This phenomenon is shown in the simulation results of the LM and PCD tests when outliers are present in the panel.

In this chapter, robust methods for testing the presence of CD in uncontaminated and contaminated panels are developed. Here, the potential outliers are estimated using robust tools which subsequently reduce the effects of outliers. The properties of the proposed tests are studied and the finite sample behaviour of the tests will be investigated by means of Monte Carlo experiments.

2.2 Model

Consider the panel model given by

$$y_{it} = f(x_{it}, y_{it}; \alpha_i, \beta_i) + e_{it}; \qquad i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{2.1}$$

where $f(x_{it}, y_{it}; \alpha_i, \beta_i)$ is a function of x_{it} and y_{it} with x_{it} denotes the observed regressor (independent variable), y_{it} is the dependent variable, α_i, β_i are parameters which are allowed to vary across each panel member i and e_{it} is the random errors component. The function $f(\mathbf{x},\mathbf{y})$ can be expressed in the form of linear regression model with x_{it} as the explanatory variable:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}; \tag{2.2}$$

or, in the form of a dynamic model:

$$y_{it} = \alpha_i + \beta_i^T y_{it-1} + e_{it}; \tag{2.3}$$

where y_{it-1} is the first lagged value of y_{it} and e_{it} are random errors.

In the presence of CD, e_{it} takes the form

$$e_{it} = \gamma_i^T f_t + \varepsilon_{it}; \qquad i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{2.4}$$

where f_t is the latent factors, γ_i are factor loadings that are common across cross-sectional units i, and ε_{it} is the random errors of e_{it}.

The objective of the tests is to 'measure' CD, that is to decide whether in reality we have a cross section independence (null hypothesis H_0) or the process is indeed cross sectional dependence (alternative hypothesis H_1). Specifically, H_0 and H_1 are defined as follows:

H_0: $E(e_{it}e_{jt}) = 0$ for all $i, j = 1,2,\ldots,N$, $i \neq j$; $t = 1,2,\ldots,T$;

H_1: $E(e_{it}e_{jt}) \neq 0$ for at least a pair of (i, j), $i, j = 1,2,\ldots,N$, $i \neq j$; $t = 1,2,\ldots,T$.

Under H_0, there is cross section independence between the $(i, j)^{\text{th}}$ residuals at time t while the H_1 states that at least a pair of cross sectional units is dependent. To test for H_0, the following usual assumptions are required:

A2.1: The disturbances e_{it} are serially uncorrelated random variables (rv), that is $E(e_{it}e_{is}) = 0$ for all $t, s = 1,2,\ldots,T$, $t \neq s$; $i = 1,2,\ldots,N$; each with mean 0 and the finite variance, $0 < \sigma_i^2 < \infty$.

A2.2: Under the null hypothesis of cross section independence, $e_{it} = \sigma_i \varepsilon_{it}$ for all i and t and with $\varepsilon_{it} \sim iid(0,1)$.

2.3 Existing Approaches of CD tests

2.3.1 The LM test of Breusch and Pagan (1980)

The most common method used to detect CD in the panel data model is the Lagrange Multiplier of Breusch and Pagan test and this test is given as

$$\mathrm{LM} = T\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}^{2} \tag{2.5}$$

where $\hat{\rho}_{ij}$ is the sample pair-wise correlation of the residuals with

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T}\hat{e}_{it}\hat{e}_{jt}}{\left(\sum_{t=1}^{T}\hat{e}_{it}^{2}\right)^{1/2}\left(\sum_{t=1}^{T}\hat{e}_{jt}^{2}\right)^{1/2}} \tag{2.6}$$

and $\hat{e}_{it}$ is the fitted residual obtained from the OLS estimator of model (2.1). Specifically, $\hat{e}_{it} = y_{it} - \hat{y}_{it}$, where y_{it} and $\hat{y}_{it}$ are the observed and fitted values respectively of the dependent variable. Under the null hypothesis, LM tends to a chi-square distribution with $\frac{N(N-1)}{2}$ degrees of freedom when N is fixed and T tends to infinity[9].

2.3.2 The PCD test of Pesaran (2004)

As an alternative to the LM test, Pesaran (2004) proposed a new cross dependency test (PCD) to test for cross sectional dependence in the panel model. This test is applicable to both the pure static and dynamic models and is also robust to structural breaks[10]. The test is then given by

$$\mathrm{PCD} = \sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij} \tag{2.7}$$

[9] See Breusch and Pagan (1980) for details of the test properties.
[10] Pesaran has shown both theoretically and empirically that his test is robust to multiple structural breaks and can also be applied to skewed data.

with $\hat{\rho}_{ij}$ as in (2.6) and that the PCD test is distributed as the $N(0,1)$ under the null hypothesis. A major drawback of the PCD test as argued by Hoyos and Sarafidis (2006) is that it is likely that the sum of negative and positive correlations will cancel out even though $|\hat{\rho}_{ij}|$ is close to 1 which is indicative of the presence of CD. The PCD test lacks power in situations where the average population pair-wise correlations are zero, although the underlying individual population pair-wise correlations are non-zero. This could arise under the alternative hypothesis cross sectional dependence where cross sectional dependence can be characterised as a factor model with a mean of zero factor loadings (see Pesaran, 2004). In addition, it is only applicable for residuals distributed from any symmetric distributions.

2.4 Robust Regression

An OLS estimator is not applicable in models where the fundamental assumptions are violated. Although some analysts adopt 'data transformation' to ensure that the assumptions of the model hold, this approach may not totally eliminate the effect of outliers (Yafee, 2003). While the Box-Cox transformation is successful for the weak CD in many cases, it will fail in the strong CD effect. For this, a small simulation is conducted to support this finding[11].

Inaccurate value of the parameter estimates may distort the results or information wanted. Thus, an alternative is to replace the OLS by robust regression (RREG) (Maronna et al., 2006).

The main purpose of the RREG is to find a reliable fit for a model by limiting the influence of outliers. Chen (2002) addresses the following problems which may occur in regression models:

[11] The result of this analysis is given in Appendix A.

- Outliers occur in Y- direction (dependent variable)
- Outliers occur in X-direction (called leverage observation)
- Outliers occur in both X and Y-directions.

As a first step towards robustness, the OLS estimator is replaced with two techniques of the RREG in order to reduce the influence of outliers in the estimation process: (1) An M-estimator for the pure static model, and (2) a Least Trimmed Squares (LTS) in the dynamic case. The spin-off from the robust fit allows the identification of outliers. Once outliers are detected, these observations can be corrected or 're-estimated'. In general, this procedure is used for exploratory purposes to (1) identify potential outliers, and (2) compute robust $\hat{\rho}_{ij}$.

Case 1: Pure Static Model

In the pure static panel, a robust M-estimation is applied to obtain $\hat{\mathbf{b}}_i = \left(\hat{\alpha}_i, \hat{\beta}_i\right)^T$ in model (2.2) which, typically 'handles' outliers in the dependent variable and assuming that there is no outlier in vertical direction (independent variables). The general M-estimator is defined as the value b_i which minimizes the following criterion:

$$\min \sum_{t=1}^{T} \rho_i \left(\frac{y_{it} - \hat{\mathbf{b}}_i^T \mathbf{x}_{it}}{\hat{\sigma}_i} \right) = \min \sum_{t=1}^{T} \rho_i \left(\frac{\hat{e}_{it}}{\hat{\sigma}_i} \right) \quad \text{for } i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{2.8}$$

with a bounded ρ - function and a high breakdown point preliminary scale $\hat{\sigma}_i$ given in (2.17), for each individual units, i .

For the M-estimator, the ρ_i in (2.8) is a filter function constructed subject to the following properties:

For each $i = 1,2,\ldots,N$,

A2.3: $\rho_i \left(\frac{\hat{e}_{it}}{\hat{\sigma}_i} \right) \geq 0$

A2.4: $\rho_i(0) = 0$

A2.5: $\rho_i\left(\frac{\hat{e}_{it}}{\hat{\sigma}_i}\right)=\rho_i\left(-\frac{\hat{e}_{it}}{\hat{\sigma}_i}\right)$

A2.6: $\rho_i(\infty)=1$ if ρ_i is bounded. (2.9)

with a ψ-function defined as the derivative of a ρ-function such that ψ is an odd function with $\psi\left(\frac{\hat{e}_{it}}{\hat{\sigma}_i}\right)\geq 0$ for $\frac{\hat{e}_{it}}{\hat{\sigma}_i}\geq 0$ (Maronna et al., 2006).

Examples of the ρ - function include the family of M-estimation (such as Huber, Tukeys biweight, and Hampel), Generalized M-estimation (such as Schweppes) and high-breakdown estimation (such as Least Median Squares (LMS) and LTS). Next, how the $\hat{\mathbf{b}}_i$ in (2.8) is obtained, is shown.

Let $\hat{u}_{it}=\frac{\hat{e}_{it}}{\hat{\sigma}_i}$, minimizing (2.8) can be achieved by differentiating (2.8) w.r.t. $\mathbf{b}_i$ thus solving

$$\sum_{t=1}^{T}\psi_i(\hat{u}_{it})\mathbf{x}_{it}=0, \tag{2.10}$$

where $\psi_i(\hat{u}_{it})=\rho_i'(\hat{u}_{it})$ and $\hat{\sigma}_i$ is computed from initial L1[12] scale estimate of $\hat{\mathbf{b}}_i$ based on Median Absolute Deviation formula given in (2.17)[13] .

Using $\sum_{t=1}^{T}\psi_i(\hat{u}_{it})=\hat{u}_{it}w_i(\hat{u}_{it})$, (2.10) can be written as follows:

$$\sum_{t=1}^{T}\hat{u}_{it}w_i(\hat{u}_{it})\mathbf{x}_{it}=0 \qquad \text{for } i=1,2,.....,N \text{ and } t=1,2,.....,T. \tag{2.11}$$

Rewriting $\hat{u}_{it}=\frac{\hat{e}_{it}}{\hat{\sigma}_i}$, the following is obtained:

$$\sum_{t=1}^{T}\frac{\hat{e}_{it}}{\hat{\sigma}_i}w_i(\hat{u}_{it})\mathbf{x}_{it}=0$$

---

[12] The L1 estimator minimize objective function of $\sum_{t=1}^{T}\left|e_{it}\left(\hat{\mathbf{b}}_{\mathbf{i}}\right)\right|=\min$.

[13] Here, $\mathbf{x}_{it}$ is the combination of vector of independent variable and vector of ones.

$$\sum_{t=1}^{T} \frac{\left(y_{it} - \hat{\mathbf{b}}_i^T \mathbf{x}_{it}\right)}{\hat{\sigma}_i} w_i(\hat{u}_{it})\mathbf{x}_{it} = 0 \tag{2.12}$$

$$\hat{\mathbf{b}}_i = \sum_{t=1}^{T} \frac{y_{it} w_i(\hat{u}_{it})\mathbf{x}_{it}}{w_i(\hat{u}_{it})\mathbf{x}_{it}^2} \tag{2.13}$$

In the matrix form, $\hat{\mathbf{b}}_i$ is computed as follows:

$$\hat{\mathbf{b}}_i = \left(\hat{\alpha}_i, \hat{\beta}_i\right)^T = \left(\mathbf{X}_i^T \mathbf{W}_i(\hat{u}_{it})\mathbf{X}_i\right)^{-1} \mathbf{X}_i^T \mathbf{W}_i(\hat{u}_{it})\mathbf{y}_i \tag{2.14}$$

where $\underset{T\times(k+1)}{\mathbf{X}_i} = \begin{bmatrix} 1 & x_{1i1} & x_{2i1} & \dots & x_{ki1} \\ 1 & x_{1i2} & x_{2i2} & \dots & x_{ki2} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{1iT} & x_{2iT} & \dots & x_{kiT} \end{bmatrix}$, $\underset{T\times 1}{\mathbf{y}_i} = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix}$

and $\mathbf{W}_i(\hat{u}_{it})$ is $T \times T$ matrix of weight matrix.

This estimating equation is solved using iteratively reweighted least squares with the initial estimates $\hat{\mathbf{b}}_i^{(0)}$ from OLS. The weights $\mathbf{W}_i\left(\hat{u}_{it}^{(h-1)}\right)$ are computed from the previous estimates using $\hat{e}_{it}^{(h-1)}$ at iteration h. Thus, a new weighted least squares estimation is given as follows:

$$\hat{\mathbf{b}}_i^{(h)} = \left(\mathbf{X}_i^T \mathbf{W}_i\left(\hat{u}_{it}^{(h-1)}\right)_i \mathbf{X}_i\right)^{-1} \mathbf{X}_i^T \mathbf{W}_i\left(\hat{u}_{it}^{(h-1)}\right)\mathbf{y}_i \tag{2.15}$$

and this procedure is repeated until the convergence is observed. For a homogeneous estimate for all i ($\mathbf{b}_1 = \mathbf{b}_2 = ... = \mathbf{b}_N = \mathbf{b}$), $\mathbf{b}_i = \left(\alpha_i, \beta_i\right)^T$ where the estimate of $\mathbf{b}$ is computed as $\hat{\mathbf{b}} = \frac{1}{N}\sum_{i=1}^{N} \hat{\mathbf{b}}_i$.

<u>Case 2: Dynamic Model</u>

In the dynamic model however, the Least Trimmed Squares (LTS) estimator is employed with a high breakdown point. The motivation for choosing the LTS instead of other robust estimators is to detect outliers and bad leverage points. According to Rousseeuw (1984), the LTS estimator is more efficient than other procedures in RREG and he advocates trimming the distribution to eliminate the outliers' effects. Besides, the LTS method has better statistical efficiency and local stability than the LMS (Wang and Suter, 2002).

The LTS estimator is defined through the following criterions:

$$\min \sum_{t=1}^{m} \left| y_{it} - \hat{\mathbf{b}}_i^T x_{it} \right|^2_{(t)} \tag{2.16}$$

that is, minimizing the sum of the absolute squared residuals to obtain a coefficient of interest. Here, m takes the value of $\lfloor T/2 \rfloor + \lfloor (k+2)/2 \rfloor$ with $\lfloor . \rfloor$ denoting the integer value and $|Z|_{(t)}$ representing the $|Z|$ order statistics for $t = 1,2,\ldots,T$.

Once outliers are detected from the robust fit, two different approaches are applied to reduce the outliers' effect and consequently results in the correct value of $\hat{\rho}_{ij}$. The details of the procedures are given in the next section.

2.4.1 Robust Standard Deviation (Scale)

The standard deviation of the residual process in (2.1) is commonly computed as $\hat{\sigma}_i = \left(\dfrac{\sum_{t=1}^{T} \hat{e}_{it}^2}{T-k-1} \right)^{1/2}$, where $\hat{e}_{it}$ is the estimated residual obtained from OLS procedure. In the presence of outliers, the OLS residuals are affected result in a larger value of $\hat{\sigma}_i$. As $\hat{\sigma}_i$ may be overestimated in the presence of outliers, we may need an alternative estimate of the standard deviation. A very robust choice of scale estimate is the Median Absolute Deviation (MAD) with the tuning constant $c = 1.4825$, chosen so that $\hat{\sigma}_i$ is consistent for σ_i at the normal distribution, which yields

$$\hat{\sigma}_i = c \operatorname*{median}_t \left| \hat{e}_{it} - \operatorname*{median}_t \left(\hat{e}_{it} \right) \right| \text{ for } i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T . \tag{2.17}$$

Therefore, the MAD is employed in (2.8) as a robust scale estimates since it is more robust than other scale estimates.

2.4.2 Robust CD tests

The following simple alternative is proposed by replacing $\hat{\rho}_{ij}$ in (2.6) with $\hat{\rho}_{\psi(ij)}$ where:

$$\hat{\rho}_{\psi(ij)} = \hat{\rho}_{\psi(ji)} = \frac{\sum_{t=1}^{T} \psi_{[.]it}\psi_{[.]jt}}{\left(\sum_{t=1}^{T} \psi_{[.]it}^{2}\right)^{1/2} \left(\sum_{t=1}^{T} \psi_{[.]jt}^{2}\right)^{1/2}}, i \neq j \tag{2.18}$$

and $\psi_{[.]it}$ represents either $\psi_{[1]it}$ or $\psi_{[2]it}$, where $\psi_{[1]it}$ and $\psi_{[2]it}$ are the robust standardized residuals using a robust and diagnostic tools, respectively, and this is described in Sections 2.4.2.1 and 2.4.2.2.

2.4.2.1 Robust CD test using a robust tool

Some commonly used methods for 'reducing' the outliers' effects can be found in Huber (1981), Rousseeuw and Leroy (1987), Chen (2002), and Maronna et al. (2006). The following method is based on Huber ρ -function:

$$\rho(u_{it}) = \begin{cases} \dfrac{u_{it}^{2}}{2} & ; |u_{it}| \leq \varpi \\ \varpi\left(|u_{it}| - \dfrac{\varpi}{2}\right) & ; \text{otherwise} \end{cases} \tag{2.19}$$

where u_{it} is standardized by

$$u_{it} = \frac{\hat{e}'_{it}}{\text{MAD}} \tag{2.20}$$

Here, the standardized residual u_{it} is computed using $\hat{e}'_{it}$ as the fitted residual obtained from a RREG divided by an estimate of their robust scale, MAD given in Equation (2.17).

The $\psi_{[1]it}$ is the derivative of (2.19) w.r.t. u_{it}

$$\psi_{[1]it} = \begin{cases} u_{it} & ;|u_{it}| \le \varpi \\ \varpi \operatorname{sign}(u_{it}) & \text{otherwise} \end{cases} \tag{2.21}$$

By setting $\varpi = 1.345$ to achieve a 95% efficiency at normal distribution, any observation with $\left|\frac{\hat{e}'_{it}}{\hat{\sigma}_i}\right| > 1.345$ will be flagged as outliers. While most standardized residuals remain unchanged, those flagged outliers will be replaced by a constant $\varpi \operatorname{sign}(u_{it})$. Notice that as $\varpi \to \infty$, $\psi_{[1]it} \to u_{it}$. In general, this procedure is used for exploratory purposes to identify the potential outliers prior to computing the robust correlation coefficient $\hat{\rho}_{\psi_{[.]}(ij)}$.

By substituting (2.21) into (2.18), two robust versions of the CD test are obtained based on the Huber function, namely:

i) Robust LM Test 1, denoted by RLM1:

$$\text{RLM1} = T\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}^2_{\psi_1(ij)} \tag{2.22}$$

ii) Robust PCD Test 1, denoted by RPCD1:

$$\text{RPCD1} = \sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_1(ij)} \tag{2.23}$$

The behaviour of these tests is discussed in Section 2.4.3.

2.4.2.2 Robust CD test with diagnostic tool

In the second approach, the outliers' effects are detected using the benchmark of standardized residuals u_{it}. Here, $\psi_{[2]it}$ is used in (2.18) and it is given as follows:

$$\psi_{[2]it} = \begin{cases} \hat{\sigma}_i u_{it} & ;|u_{it}| \le d \\ 0 & ;\text{otherwise} \end{cases} \tag{2.24}$$

where $\hat{\sigma}_i$ is a robust scale computed as in (2.17) , u_{it} is computed as in (2.20) and d is chosen from the simulation study reported in Tables 2.1 to 2.2 for the pure static and dynamic models, respectively. The value of d represents the critical value of this procedure and the effect of outliers is removed when the absolute standardized residuals $|u_{it}| > d$. Notice that as $d \to \infty$, $\psi_{[2]it} \to u_{it}$.

Substituting (2.24) in (2.18), yields other sets of robust CD tests as follows:

i) Robust LM Test 2, denoted by RLM2:

$$\mathrm{RLM2} = T\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}^2_{\psi_2(ij)} \tag{2.25}$$

ii) Robust PCD Test 2, denoted by RPCD2:

$$\mathrm{RPCD2} = \sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_2(ij)} \tag{2.26}$$

For the case of the unbalanced panel, $\hat{\rho}_{\psi(ij)}$ in (2.18) is computed as (see Pesaran, 2004)

$$\hat{\rho}_{\psi(ij)} = \hat{\rho}_{\psi(ji)} = \frac{\sum_{t\in T_i\cap T_j}\left(\psi_{[.]it} - \overline{\psi}_{[.]i}\right)\left(\psi_{[.]jt} - \overline{\psi}_{[.]j}\right)}{\left(\sum_{t\in T_i\cap T_j}\left(\psi_{[.]it} - \overline{\psi}_{[.]i}\right)^2\right)^{1/2}\left(\sum_{t\in T_i\cap T_j}\left(\psi_{[.]jt} - \overline{\psi}_{[.]j}\right)^2\right)^{1/2}}$$

$$\text{where } \overline{\psi}_{[.]i} = \frac{\sum_{t\in T_i\cap T_j}\psi_{[.]it}}{T_i \cap T_j} \quad \text{and } \overline{\psi}_{[.]j} = \frac{\sum_{t\in T_i\cap T_j}\psi_{[.]jt}}{T_i \cap T_j} \tag{2.27}$$

2.4.3 The Properties of the Proposed CD tests

The following theorem is needed in Theorem 2.1 and Theorem 2.2.

Central Limit Theorem (CLT)

This theorem states that the sum of a large number of random variable (rv) will have an approximately normal distribution if the sequences of those rv are identically and independently distributed (iid), with some mean and variances. For example, in the panel data model described in (2.1), if $(\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Nt})$ for $i = 1,2,\dots,N$. $t = 1,2,\dots,T$ are iid with mean μ and variances σ^2, the central limit theorem states that:

$$\sqrt{NT}\left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\varepsilon_{it} - \mu\right) \xrightarrow{d} N\left(0, \sigma^2\right)$$

as $N \to \infty$ and $T \to \infty$.

Theorem 2.1: Under assumptions **A2.1- A2.6**, and assuming that $\psi_{[1]it}$ is monotone nondecreasing function[14],

$$E\left(\hat{\rho}_{\psi_1(ij)}\right) = 0$$

and

$$E(\text{RPCD1}) = 0$$

where $\hat{\rho}_{\psi_1(ij)} = \dfrac{\sum_{t=1}^{T}\psi_{[1]it}\psi_{[1]jt}}{\left(\sum_{t=1}^{T}\psi_{[1]it}^2\right)^{1/2}\left(\sum_{t=1}^{T}\psi_{[1]jt}^2\right)^{1/2}}$ and the RPCD1 are defined in (2.23) .

Proof of **Theorem 2.1**[15]*:*

Assuming that assumptions **A2.1-A2.2** is true for robust residuals[16], $\hat{e}'_{it}$ and under assumption **A2.3- A2.5** for the proposed procedure, we have

$$E\left(\psi_1\left(\frac{\hat{e}'_{it}}{\hat{\sigma}_i}\right)\right) = E\left(\psi_1\left(u_{it}\right)\right) = 0 \quad . \tag{2.28}$$

Equation (2.28) holds if $\hat{e}'_{it}$ is symmetric, where $\hat{e}'_{it}$ is the estimated residuals obtained from M-estimator (e_{it} in model (2.1)) and $\hat{\sigma}_i$ is the robust scale estimate, MAD and computed as in (2.17).

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[14] ψ_1 is monotone nondecreasing with $\psi_1(-\infty) < 0 < \psi_1(\infty)$.

[15] The references of this proof are based on Maronna et al. (2006) and Pesaran (2004).

[16] Robust M-residuals coincides with the OLS residuals when $\psi_{[1]it}$ in (2.21) is identity function (McKean et al. (1993)). Thus, the properties of this M-residual will follow OLS properties that result in the assumption of **A2.1-A2.2**. See details in McKean et al. (1993).

Let ξ_{it} and ξ_{jt} be the scaled standardized residuals defined by

$$\xi_{it} = \frac{\psi_1(u_{it})}{\left[\psi_1(u_{it})^T \psi_1(u_{it})\right]^{1/2}} \quad \text{and} \quad \xi_{jt} = \frac{\psi_1(u_{jt})}{\left[\psi_1(u_{jt})^T \psi_1(u_{jt})\right]^{1/2}} \tag{2.29}$$

and rewrite $\hat{\rho}_{\psi_1(ij)} = \dfrac{\sum_{t=1}^{T} \psi_{[1]it}\psi_{[1]jt}}{\left(\sum_{t=1}^{T} \psi_{[1]it}^2\right)^{1/2}\left(\sum_{t=1}^{T} \psi_{[1]jt}^2\right)^{1/2}}$ as $\hat{\rho}_{\psi_1(ij)} = \sum_{t=1}^{T} \xi_{it}\xi_{jt}$.

Using the properties of (2.28) in (2.29), we have $E(\xi_{it}) = 0$ for all i and t.

Assuming that assumptions of **A2.1-A2.2** are true for $\hat{e}'_{it}$, and then $\hat{\rho}_{\psi_1(ij)}$ and $\hat{\rho}_{\psi_1(is)}$ are cross sectional independences for i, j, s such that $i \neq j \neq s$. Specifically,

$$E\left(\hat{\rho}_{\psi_1(ij)}\hat{\rho}_{\psi_1(is)}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T} E\left(\xi_{it}\xi_{jt}\xi_{it'}\xi_{st'}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T} E\left(\xi_{it}\xi_{jt}\right)E\left(\xi_{it'}\right)E\left(\xi_{st'}\right) = 0,$$

Thus, $E\left(\hat{\rho}_{\psi_1(ij)}\right) = 0$, which in turn leads to $E(\text{RPCD1}) = 0$ for any N and all $T > k+1$.

End of proof of **Theorem 2.1**.

<u>**Proposition 2.1:**</u> Under **Theorem 2.1**, and assumptions **A2.1-A2.6**, the asymptotic distribution of RPCD1 is given as follows

$$\sqrt{\frac{2T}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{\psi_1(ij)} \xrightarrow{d} N(0, Z_F^2)$$

as $T \to \infty$ and $N \to \infty$, where Z_F^2 is given in (2.30).

Proof of **Proposition 2.1**[17]*:*

Under H_0, let

$$F_N = \sqrt{\frac{2}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{\psi_1(ij)}.$$

[17] The references of this proof are based on Maronna et al. (2006) and Pesaran (2004).

Using the result in **Theorem 2.1** where $\hat{\rho}_{\psi_1(is)}$ are cross sectional independences for i, j, s such that $i \neq j \neq s$. Specifically,

$$E\left(\hat{\rho}_{\psi_1(ij)}\hat{\rho}_{\psi_1(is)}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T} E\left(\xi_{it}\xi_{jt}\xi_{it'}\xi_{st'}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T} E\left(\xi_{it}\xi_{jt}\right)E\left(\xi_{it'}\right)E\left(\xi_{st'}\right) = 0,$$

and so, $Var\left(\hat{\rho}_{\psi_1(ij)}\right) = E\left(\hat{\rho}^2_{\psi_1(ij)}\right) \leq 1$.

Using the CLT, for all $T > k+1$ where k is the number of independent variables, the following is obtained.

$$F_N \xrightarrow{d} N(0, Z_F^2) \qquad \text{as } N \to \infty$$

with $$Z_F^2 = \lim_{N\to\infty}\left[\frac{2}{N(N-1)}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} E\left[\hat{\rho}^2_{\psi_1(ij)}\right]\right] \leq 1. \tag{2.30}$$

Hence, $$\frac{F_N}{\sqrt{Z_F^2}} = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_1(ij)}}{\sqrt{E\left[\hat{\rho}^2_{\psi_1(ij)}\right]}} \sim N(0,1) \quad \text{as } N \to \infty.$$

Thus,

$$\sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_1(ij)} \xrightarrow{d} N(0, Z_F^2) \text{ as } N \to \infty \ .$$

where Z_F^2 is given in (2.30) and depends on ψ_1 used.

Note that (2.18) can be rewritten as follows:

$$\sqrt{T}\hat{\rho}_{\psi_1(ij)} = \frac{\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_1(u_{it})\psi_1(u_{jt})}{\left(\frac{\sum_{t=1}^{T}\psi_1(u_{it})^2}{T}\right)^{1/2}\left(\frac{\sum_{t=1}^{T}\psi_1(u_{jt})^2}{T}\right)^{1/2}}$$

and obtain $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\psi_1(u_{it})\psi_1(u_{jt}) = o_p(1)$ and $\left(\frac{\sum_{t=1}^{T}\psi_1(u_{it})^2}{T}\right) = 1 + o_p(1)$.

under H_0 and as $T \to \infty$, $\sqrt{T}\hat{\rho}_{\psi_1(ij)} \xrightarrow{d} N(0, Z_F^2)$. Using the results in

$$\sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_1(ij)}$$

will subsequently yield $\sqrt{\frac{2T}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{\psi_1(ij)} \xrightarrow{d} N(0, Z_F^2)$ as $T \to \infty$ and $N \to \infty$.

End of proof of **Proposition 2.1**[18]**.**

Corollary 2.1[19]: Using result in **Theorem 2.1** and **Proposition 2.1:**

$$T\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}^2_{\psi_1(ij)} \xrightarrow{d} \chi^2_{N(N-1)/2}$$

as $N \to \infty$ and $T \to \infty$.

Corollary 2.2: When ϖ tends to infinity, RLM1 coincides with LM and RPCD1 coincides with PCD.

Corollary 2.3: When d tends to infinity, RLM2 coincides with LM and RPCD2 coincides with PCD.

2.5 Finite Sample Behaviours of the Tests of Cross Sectional Dependence

In this section, the Monte Carlo simulation study is used to investigate the finite sample behaviour of the CD tests in the presence of contaminations. In the first part of this experiment, the mean and standard deviation, size and power of the test based on 500 replications for $N = 20, T = 100$ are computed to investigate the performance of the CD tests under the various degrees of CD and various types of outliers. In testing the

[18] Figure 2.1 supports the result that RPCD1 approximate the normal distribution.
[19] Figure 2.2 supports the result that RLM1 approximate the normal distribution.

null hypothesis of the cross sectional independence, the size of the test is defined as the probability of rejecting the null hypothesis of cross sectional independence when no contemporaneous correlated errors exist in the model, that is,

$$\text{The size of the test} = P\left(\text{reject } \mathrm{H}_0 \mid \mathrm{H}_0 \text{ is true}\right)$$

We say that the size of the test is reasonable if it is fairly close to 0.05 (at 5% level of significant). The power measures the probability of correctly rejecting the null when the alternative hypothesis is true (there is cross correlated errors in the model), that is,

$$\text{The power of the test} = P\left(\text{reject } \mathrm{H}_0 \mid \mathrm{H}_0 \text{ is false}\right)$$

and if the power more than 90% (at 5% level of significant), than the test is considered has high power .

Next, 500 runs are performed for each pair of cross sectional units and times with $N = (10,20,30,50,100)$ and $T = (10,20,30,50,100)$ to investigate the size and power of the tests in uncontaminated and contaminated panels with 5% contamination[20]. For the LM, RLM1 and RLM2 tests, only the performance of the tests for $N < T$ is studied because these tests experience size distortion when $N > T$.

2.5.1 The Pure Static Panel Model

Following Pesaran (2004), data generating process (DGP) is considered as specified by:

$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}$; and $e_{it} = \gamma_i^T f_t + \sigma_{it}\varepsilon_{it}$; for $i = 1,2,\ldots,N,\ \ t = -49,\ldots,0,1,2,\ldots,T$

with $\alpha_i \sim iidU(-0.5,0.5)$; $\sigma_{it} = 1$; $x_{it} \sim iidN(0,1)$; $\varepsilon_{it} \sim iidN(0,1)$; $f_t \sim iidN(0,1)$.

β_i is set as follows:

1. Homogeneous slope: $\beta_i = \beta = 1$;

2. Heterogeneous slope: $\beta_i \sim U[0,1]$. (2.31)

[20] All of the computations are conducted using S-PLUS

Under various degrees of CD, the value of γ_i takes the following forms[21]:

$$\left.\begin{array}{l} \text{(i) } \gamma_i = 0 \text{ for cross-sectional independence,} \\ \text{(ii) } \gamma_i \sim iidU(0.1,0.3) \text{ for mild cross dependency and ,} \\ \text{(iii) } \gamma_i \sim iidU(0.5,1.5) \text{ for strong effect of cross dependency.} \end{array}\right\} \tag{2.32}$$

In the presence of contaminations (m_{it}) at time $t = \tau_i$, for each cross-sectional unit i, the residual has the following form:

$$e_{it} = \begin{cases} e_{it} & \text{for } t \neq \tau_i \\ e_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N; \tag{2.33}$$

and in the presence of structural breaks (changes in slope):

$$\beta_{it} = \begin{cases} 0.2 & \text{for } t = -49,\ldots,T/2 \\ 1.0 & \text{for } t = T/2+1,\ldots,T \end{cases} \quad \text{for } i = 1,2,\ldots,N;$$

and for changes in variances:

$$\sigma_{it} = \begin{cases} \sqrt{1.5} & \text{for } t = -49,\ldots,T/2 \\ 1.0 & \text{for } t = T/2+1,\ldots,T \end{cases} \quad \text{for } i = 1,2,\ldots,N; \tag{2.34}$$

2.5.1.1 Results and Discussion

Tables 2.1 and 2.2 provide the cut-off point, d in (2.24) based on three different nominal levels, 10%, 5% and 1% for the pure static and dynamic models, respectively. As expected, when T large, RLM2 and RPCD2 follow a normal distribution and hence $d \approx 1.64, 1.96, 2.58$ at 10%, 5% and 1% nominal level, respectively. These values however, are subjected to the small sample size as shown in the table. This cut-off point d is used in computing the test statistics in investigating the size and power of the tests.

Tables 2.3 to 2.4 summarize the results of the CD test in the regression panel for the case of the homogeneous slope. For uncontaminated data and cross sectional independence (column 1), the average values of test statistics for LM and PCD are

[21] The forms of CD are set to be close to Pesaran (2007) and Coakley et al. (2006).

relatively small, suggesting that the null hypothesis of $\gamma_i = 0$ should not be rejected. The size of these tests is around 5%. The proposed tests (i.e. RLM1, RLM2, RPCD1 and RPCD2) provide the same conclusion, and are comparable with those results obtained from the standard LM and PCD tests. For the case of mild CD effect $\gamma_i \sim iidU(0.1, 0.3)$, it is observed that only the PCD, RPCD1 and RPCD2 tests reject the null of the cross section independence while the LM, RLM1 and RLM2 do not. When the effect of the CD is stronger $\gamma_i \sim iidU(0.5, 1.5)$, all tests reject the null of cross section independence with a high probability.

In the presence of 5% contamination (see columns 2 to 5 of Table 2.3), it is observed that the LM test is over-sized when no CD is present in the panel. The RLM1, RLM2 and RPCD2 however provide the correct size for the study. Similar results are obtained for the RPCD1 test, except in the presence of contamination from $\chi^2_{(30)}$. Notice that both the LM and PCD tests fail to detect the present of mild CD in the contaminated panel. By contrast, the RPCD1 and RPCD2 tests perform well with reasonably good power. For the stronger CD effect, the power of the proposed methods (RLM1, RLM2, RPCD1 and RPCD2) continue to perform well compared to the LM and PCD tests in both the uncontaminated and contaminated panels. Similar results are obtained as the percentage of contaminations increases to 10% (Table 2.4). For the heterogeneous panel, the similar findings are observed and these are reported in Tables 2.5 to 2.6.

In the presence of the structural breaks, all tests attain the correct size for the case of the pure static model (Table 2.11). The PCD, RPCD1 and RPCD2 tests outperform the LM, RLM1 and RLM2 tests in term of power for the case of mild CD effect.

2.5.2 Dynamic Panel Model

The DGP for the first order dynamic panel model is considered as follows: $y_{it} = \alpha_i + \beta_i^T y_{it-1} + e_{it}$; with $\alpha_i = \mu_i(1-\beta_i)$ and $e_{it} = \gamma_i^T f_t + \varepsilon_{it}$ for $i = 1,2,\ldots,N$. $t = -49,\ldots,0,1,2,\ldots,T$. $\mu_i \sim iidU[0,0.02]$ is set with $\varepsilon_{it}, f_t \sim N(0,1)$ and β_i are given as in (2.32). In investigating the power and size of the tests, the degrees of the cross sectional dependence in the DGP for this model will be similar with (2.32). In the presence of contaminations in the dynamic case, only the additive outliers (AO)[22] type that affects only y_{it} is considered. Thus, the notation of this will take the form of:

$$y_{it} = \begin{cases} y_{it} & \text{for } t \neq \tau_i \\ y_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N. \tag{2.35}$$

Based on 500 simulation runs[23], the mean and standard error of the test statistics are computed for a pair of sample size $N = 20, T = 100$ in the first experiment, together with the respective power and size of study under five conditions namely, in which the residuals e_{it} are distributed as (i) $N(0,1)$; (ii) $(1-\delta)N(0,1) + \delta\chi^2_{30}$; (iii) $(1-\delta)N(0,1) + \delta N(4,4)$; (iv) $(1-\delta)N(0,1) + \delta LN(1,2)$; (v) $(1-\delta)N(0,1) + \delta$ Cauchy (0,16) , where δ is the percentage of contamination chosen as 0.05 and 0.10, respectively. In the second experiment, in order to provide insight on the effect of N cross sectional units and time T on the size and power of the test, residuals e_{it} are simulated from $N(0,1)$ for the uncontaminated and $(1-\delta)N(0,1) + \delta$ Cauchy (0,16) for the contaminated panels, respectively with $\delta = 0.05$. The pair of sample size chosen is $N = (10,20,30,50,100)$, and $T = (10,20,30,50,100)$. The performance is investigated when $\beta_i = \beta$ (that is when the slope is homogeneous across cross sectional units) and

[22] AO affect the level but leave the variance unaffected.
[23] Only 500 runs are used due to time consuming especially when sample size are large, for example when $(N,T) = (100,100)$ which equivalent to 10000 samples. Besides that, with 500 runs, the results slightly consistent if we take larger replications, and due to the time constraint, we fix to this number.

when β_i differs across i (heterogeneous case) under the various levels of cross dependency in the panel: no CD, mild CD effects and strong CD effects.

2.5.2.1 Results and Discussion

The results of the dynamic panel are summarized in Tables 2.7 to 2.8, and 2.9 to 2.10 for the homogeneous and heterogeneous slopes, respectively. In the case of the homogeneous slope and no contaminations, all tests have the correct size when $\gamma_i = 0$ (see column 1 Tables 2.7 and 2.8). The power of the tests for the case of mild CD is reasonably well except for the RLM, RLM1 and RLM2 tests. These tests however provide a good power for the case of a strong CD effect, and are comparable with those obtained for the PCD, RPCD1 and RPCD tests in uncontaminated data.

For the homogeneous case and in the presence of outliers, the LM, RLM1 and RLM2 tests suffer from size distortion (for both 5% and 10% contaminations from $\chi^2_{(30)}$ and Cauchy distribution (see columns 2 and 5 of Tables 2.7 and 2.8). Similar results are obtained for the RPCD1 and RPCD2 tests. These tests fail for a larger size and heavy tailed contamination. The PCD test however provides a reasonable size of the test in the presence of outliers (columns 2-5 of Tables 2.7 to 2.8). The proposed method, the RPCD1 and RPCD2 tests have reasonably good powers when CD is observed in panel (that is $\gamma_i \sim iidU(0.1,0.3)$ and $\gamma_i \sim iidU(0.5,1.5)$) and which outperform the PCD, LM, RLM1 and RLM2 tests for the case of mild CD in contaminated panel. Similar results are obtained for the size and power of the tests for the case of the heterogeneous slope in the uncontaminated data (see column 1 of Tables 2.9 to 2.10). In the presence of outliers, the size of the LM, RLM1 and RLM2 tests suffer from size distortion as in the homogeneous slope case. The PCD, RPCD1, RPCD2 tests however have the correct size for the test with values approximately 0.05 (as shown in columns 2 to 5 of Tables 2.9 to 2.10). In the case of mild CD effect, the RPCD1 and RPCD2 tests provide good results in the presence of 5% contamination while the LM, RLM1, RLM2 and PCD tests fail to do so (see Table 2.9). The power remains high with 10% contamination except for the RPCD1 test when AOs are from χ^2_{30} distribution (see Table 2.10). All tests attain high power for the case of the strong CD in the presence of 5% contamination (Table 2.9) but not for LM and PCD when the percentage of contamination increases to 10% (Table 2.10).

In the presence of the structural breaks (Table 2.12), similar results are obtained as in the pure static case. The LM, RLM1 and RLM2 tests suffer from size distortion in the presence of structural breaks while the RPCD1 and RPCD2 outperform PCD in term of size of the test in the presence of structural breaks in the dynamic model. All tests however provide the correct detection of CD when a strong CD is observed in the panel.

For overall performance of size and power of CD tests, the results are reported in Tables 2.13 to 2.14 (pure static model) and Tables 2.15 to 2.16 (dynamic), respectively. The results of the Monte Carlo study are summarized as follows: First, in the contaminated data (see Tables 2.14 and 2.16), the RLM1, RLM2, RPCD1 and RPCD2 tests retain similar power as in the uncontaminated version (see Tables 2.13 and 2.15). Secondly, the RLM1 and RLM2 tests however generally have lower power in the presence of a mild CD except when $T \to \infty$ for both the uncontaminated (see Tables 2.13 and 2.15) and contaminated panels (see Tables 2.14 and 2.16). Thirdly, while the power of the test for the PCD and LM tests decreases as $T \to \infty$ in the presence of mild CD and outliers, as shown in Tables 2.14 and 2.16, the RPCD1 and RPCD2 tests continue to attain good power. Finally, it is concluded that the proposed method (RPCD1 and RPCD2) yields comparable size and power of study to those obtained in the PCD approach for both cases when N large relative to is T and when T is large relative to N.

Table 2.1: Critical Region for *d* in the Pure Static Panel Model (Heterogeneous slope)

T / *N*	10	20	30	50	100
			10%		
10	1.818	1.796	1.786	1.788	1.783
20	1.707	1.707	1.707	1.697	1.700
30	1.690	1.682	1.681	1.681	1.678
50	1.668	1.667	1.665	1.663	1.664
100	1.657	1.655	1.654	1.652	1.656
			5%		
10	2.264	2.218	2.208	2.199	2.184
20	2.077	2.069	2.066	2.054	2.053
30	2.048	2.025	2.026	2.020	2.017
50	2.003	1.997	1.996	1.994	1.993
100	1.983	1.977	1.977	1.974	1.979
			1%		
10	3.269	3.143	3.110	3.088	3.056
20	2.878	2.814	2.814	2.788	2.785
30	2.772	2.727	2.714	2.708	2.703
50	2.681	2.663	2.662	2.656	2.654
100	2.629	2.614	2.614	2.612	2.617

Note: The results are the value of d in (2.24) for three nominal levels, 1%, 5% and 10% computed based on 5000 replications.
The model for the purely static panel is $y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}$; and $e_{it} = \gamma_i f_t + \sigma_{it}\varepsilon_{it}$; with $\alpha_i \sim iidU(-0.5,0.5)$; $\sigma_{it} = 1$; $x_{it} \sim iidN(0,1)$ $\varepsilon_{it} \sim iidN(0,1)$; $f_t \sim iidN(0,1)$; $\beta_i \sim U[0,1]$.

Table 2.2: Critical Region for *d* in the Dynamic Panel Model (Heterogeneous slope)

T / *N*	10	20	30	50	100
			10%		
10	1.968	1.946	1.940	1.937	1.940
20	1.780	1.775	1.783	1.770	1.774
30	1.739	1.739	1.736	1.730	1.733
50	1.701	1.697	1.702	1.701	1.701
100	1.676	1.674	1.673	1.673	1.674
			5%		
10	2.552	2.522	2.513	2.502	2.503
20	2.246	2.244	2.245	2.232	2.238
30	2.169	2.162	2.166	2.157	2.155
50	2.092	2.083	2.089	2.089	2.088
100	2.032	2.032	2.029	2.027	2.029
			1%		
10	3.676	3.571	3.565	3.546	3.533
20	3.078	3.048	3.056	3.035	3.034
30	2.915	2.890	2.889	2.877	2.883
50	2.770	2.755	2.763	2.761	2.765
100	2.677	2.669	2.668	2.671	2.669

Note: The results are the value of d in (2.24) for three nominal levels, 1%, 5% and 10% computed based on 5000 replications.
The model for the dynamic panel is $y_{it} = \alpha_i + \beta_i^T y_{it-1} + e_{it}$; with $\alpha_i = \mu_i(1-\beta_i)$ and $e_{it} = \gamma_i f_t + \varepsilon_{it}$; $\mu_i = \varepsilon_{i0} + \eta_i$ with $\varepsilon_{it}, \eta_i, f_t \sim N(0,1)$; $\beta_i \sim U[0,1]$.

Table 2.3: CD Test in the Pure Static Model (Homogeneous slope) (5% contamination)

Test	CD case/ e_{it}	$N(0,1)$	0.95 $N(0,1)$+ $0.05\chi^2_{30}$	0.95 $N(0,1)$+ 0.05N(4,4)	0.95 $N(0,1)$ + 0.05LN(1,2)	0.95 N(0,1)+ 0.05 Cauchy (0,16)
LM	No CD $\gamma_i = 0$	191.696 (18.679) **0.048**	193.203 (39.209) **0.212**	192.487 (48.081) **0.260**	193.137 (65.059) **0.296**	192.161 (82.285) **0.302**
RLM1		191.778 (19.215) **0.056**	189.139 (19.551) **0.048**	191.755 (20.182) **0.064**	191.208 (19.793) **0.056**	192.117 (18.528) **0.048**
RLM2		191.645 (19.312) **0.054**	191.064 (19.220) **0.050**	191.631 (19.361) **0.058**	191.603 (19.150) **0.054**	191.551 (19.101) **0.044**
PCD		-0.010 (0.999) **0.050**	0.043 (1.039) **0.056**	-0.030 (0.979) **0.046**	-0.020 (0.974) **0.042**	0.072 (1.003) **0.044**
RPCD1		-0.002 (1.010) **0.050**	0.763 (1.020) **0.134**	0.010 (0.963) **0.048**	0.126 (0.986) **0.050**	-0.030 (1.003) **0.042**
RPCD2		0.017 (1.034) **0.056**	0.061 (1.025) **0.052**	0.002 (1.007) **0.060**	0.022 (1.011) **0.062**	0.062 (0.949) **0.038**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	221.866 (26.609) **0.450**	192.555 (38.005) **0.200**	192.796 (48.830) **0.240**	194.370 (67.681) **0.278**	189.078 (74.700) **0.296**
RLM1		218.010 (25.779) **0.400**	216.869 (26.749) **0.354**	213.530 (23.328) **0.312**	215.813 (24.677) **0.372**	212.000 (24.036) **0.298**
RLM2		205.551 (22.146) **0.202**	205.393 (23.018) **0.202**	207.392 (22.370) **0.234**	206.680 (22.420) **0.220**	206.910 (22.306) **0.220**
PCD		5.174* (1.757) **0.984**	0.157 (1.056) **0.062**	0.521 (1.123) **0.114**	0.920 (1.217) **0.196**	0.166 (1.060) **0.076**
RPCD1		4.910* (1.741) **0.964**	4.907* (1.614) **0.966**	4.218* (1.558) **0.932**	4.578* (1.595) **0.962**	4.283* (1.649) **0.924**
RPCD2		3.739* (1.584) **0.878**	3.480* (1.523) **0.858**	3.738* (1.418) **0.902**	3.602* (1.457) **0.878**	3.895* (1.570) **0.896**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4373.464* (885.003) **1.000**	210.470 (43.692) **0.340**	404.418* (122.165) **0.972**	728.332* (283.102) **0.992**	234.439* (84.262) **0.524**
RLM1		4023.240* (859.100) **1.000**	2993.436* (597.366) **1.000**	3037.903* (605.708) **1.000**	3388.093* (665.294) **1.000**	3076.213* (645.163) **1.000**
RLM2		2504.035* (666.212) **1.000**	2160.785* (529.390) **1.000**	2403.561* (563.367) **1.000**	2305.680* (551.954) **1.000**	2518.730* (605.188) **1.000**
PCD		65.558* (7.111) **1.000**	3.845* (1.671) **0.824**	12.128* (3.378) **1.000**	15.923* (5.241) **1.000**	3.476* (2.119) **0.764**
RPCD1		60.857* (7.156) **1.000**	52.023* (5.850) **1.000**	52.328* (5.888) **1.000**	55.535* (6.108) **1.000**	52..679* (6.190) **1.000**
RPCD2		47.639* (6.965) **1.000**	44.115* (6.004) **1.000**	46.678* (6.119) **1.000**	45.629* (6.136) **1.000**	47.859* (6.407) **1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.4: CD Test in the Pure Static Model (Homogeneous slope) (10% contamination)

Test	CD case/ e_{it}	N (0,1)	0.90 N (0,1)+ 0.10χ^2_{30}	0.90 N (0,1)+ 0.10N(4,4)	0.90 N (0,1) + 0.10LN(1,2)	0.90 N(0,1)+ 0.10Cauchy (0,16)
LM	No CD $\gamma_i = 0$	191.696	192.392	190.536	186.813	190.862
		(18.679)	(23.090)	(33.318)	(71.063)	(76.327)
		0.048	**0.104**	**0.154**	**0.274**	**0.288**
RLM1		191.778	191.904	191.190	189.909	192.059
		(19.215)	(20.223)	(19.899)	(19.862)	(19.453)
		0.056	**0.062**	**0.072**	**0.052**	**0.044**
RLM2		191.645	192.448	192.394	191.976	191.820
		(19.312)	(18.938)	(19.857)	(19.827)	(19.969)
		0.054	**0.056**	**0.068**	**0.048**	**0.060**
PCD		-0.010	0.050	-0.073	-0.079	-0.004
		(0.999)	(1.033)	(1.052)	(0.974)	(0.968)
		0.050	**0.054**	**0.050**	**0.038**	**0.036**
RPCD1		-0.002	2.763*	0.106	0.532	-0.030
		(1.010)	(1.019)	(0.990)	(1.038)	(0.955)
		0.050	**0.774**	**0.044**	**0.100**	**0.042**
RPCD2		0.017	-0.017	0.014	0.067	-0.028
		(1.034)	(0.983)	(0.998)	(1.050)	(0.956)
		0.056	**0.046**	**0.040**	**0.072**	**0048**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	221.866	192.392	191.439	195.228	189.858
		(26.609)	(22.162)	(34.942)	(79.600)	(76.777)
		0.450	**0.104**	**0.176**	**0.302**	**0276**
RLM1		218.010	226.475*	207.771	213.367	206.137
		(25.779)	(27.622)	(22.967)	(23.913)	(22.214)
		0.400	**0.540**	**0.244**	**0.330**	**0.200**
RLM2		205.551	202.969	206.529	205.159	207.428
		(22.146)	(21.530)	(21.969)	(21.937)	(23.127)
		0.202	**0.160**	**0.218**	**0.208**	**0..228**
PCD		5.174*	0.135	0.171	0.280	0.027
		(1.757)	(0.985)	(1.027)	(1.058)	(0.937)
		0.984	**0.052**	**0.058**	**0.074**	**0.046**
RPCD1		4.910*	6.269*	3.624*	4.478*	3.541*
		(1.741)	(1.481)	(1.504)	(1.535)	(1.557)
		0.964	**0.996**	**0.874**	**0.962**	**0.844**
RPCD2		3.739*	3.154*	3.698*	3.464*	3.851*
		(1.584)	(1.478)	(1.445)	(1.430)	(1.567)
		0.878	**0.788**	**0.890**	**0.876**	**0.904**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4373.464*	195.876	233.448*	291.731*	192.135
		(885.003)	(24.065)	(45.549)	(110.697)	(75.803)
		1.000	**0.148**	**0.540**	**0.714**	**0.286**
RLM1		4023.240*	2379.144*	2246.318*	2844.122*	2236.337*
		(859.100)	(469.926)	(464.425)	(573.148)	(479.913)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RLM2		2504.035*	1834.594*	2245.567*	2093.207*	2414.202*
		(666.212)	(458.405)	(504.123)	(503.889)	(550.847)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
PCD		63.558*	1.669	5.560*	5.735*	0.738
		(7.111)	(1.199)	(2.038)	(2.837)	(1.104)
		1.000	**0.386**	**0.982**	**0.934**	**0.180**
RPCD1		60.857*	46.006*	44.263*	50.487*	44.160*
		(7.156)	(5.139)	(5.268)	(5.743)	(5.431)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RPCD2		47.639*	40.339*	44.943*	43.238*	46.749*
		(6.965)	(5.692)	(5.710)	(5.895)	(5.978)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\text{PCD}|, |\text{RPCD1}|, |\text{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.5: CD Test in the Pure Static Model (Heterogeneous slope) (5% contamination)

Test	CD case/ e_{it}	$N(0,1)$	0.95 $N(0,1)$+ $0.05\chi^2_{30}$	0.95 $N(0,1)$+ $0.05N(4,4)$	0.95 $N(0,1)$ + $0.05LN(1,2)$	0.95 $N(0,1)$+ 0.05 Cauchy (0,16)
LM	No CD $\gamma_i = 0$	192.030	192.581	191.581	188.421	185.691
		(19.234)	(38.275)	(45.886)	(63.636)	(76.961)
		0.056	**0.180**	**0.242**	**0.252**	**0.272**
RLM1		191.808	189.398	191.385	190.953	191.025
		(19.913)	(19.494)	(17.848)	(18.735)	(19.462)
		0.046	**0.048**	**0.042**	**0.042**	**0.066**
RLM2		190.944	192.186	190.513	191.287	192.215
		(20.511)	(19.054)	(18.298)	(18.994)	(18.619)
		0.050	**0.054**	**0.040**	**0.048**	**0.050**
PCD		-0.001	-0.022	0.019	0.030	-0.038
		(1.006)	(1.031)	(1.008)	(1.008)	(1.003)
		0.042	**0.054**	**0.066**	**0.052**	**0.052**
RPCD1		0.014	0.733	0.058	0.202	-0.003
		(1.030)	(1.009)	(1.001)	(1.035)	(1.022)
		0.066	**0.134**	**0.052**	**0.070**	**0.056**
RPCD2		-0.001	-0.013	0.026	0.018	0.011
		(1.047)	(0.979)	(1.010)	(1.008)	(1.046)
		0.062	**0.042**	**0.058**	**0.050**	**0.056**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	221.247	188.415	190.858	189.204	194.555
		(27.763)	(36.037)	(43.828)	(59.166)	(84.603)
		0.428	**0.162**	**0.226**	**0.244**	**0.328**
RLM1		218.330	215.971	214.518	216.534	213.401
		(26.219)	(27.004)	(24.075)	(24.408)	(25.076)
		0.380	**0.354**	**0.326**	**0.346**	**0.306**
RLM2		205.936	203.596	207.463	206.771	208.202
		(22.291)	(22.344)	(22.736)	(22.798)	(23.160)
		0.212	**0.176**	**0.216**	**0.212**	**0.250**
PCD		5.192*	0.109	0.592	0.898	0.211
		(1.717)	(1.042)	(0.994)	(1.097)	(1.016)
		0.986	**0.066**	**0.096**	**0.166**	**0.046**
RPCD1		4.938*	4.789*	4.287*	4.634*	3.723*
		(1.665)	(1.645)	(1.564)	(1.621)	(1.523)
		0.972	**0.970**	**0.944**	**0.968**	**0.926**
RPCD2		3.761*	3.369*	3.827*	3.723*	3.744*
		(1.526)	(1.510)	(1.541)	(1.523)	(1.581)
		0.884	**0.822**	**0.898**	**0.878**	**0.872**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4393.100*	205.248	403.501*	712.852*	239.534*
		(792.487)	(41.902)	(122.032)	(274.245)	(95.069)
		1.000	**0.270**	**0.980**	**0.994**	**0.548**
RLM1		4040.739*	2925.314*	3044.824*	3398.364 *	3008.858*
		(763.288)	(598.863)	(632.519)	(706.001)	(630.438)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RLM2		2511.355*	2088.451*	2414.057*	2321.304*	2452.455*
		(608.222)	(523.827)	(599.387)	(595.814)	(611.719)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
PCD		63.789*	3.368*	12.095*	15.738*	3.479*
		(6.346)	(1.587)	(3.313)	(5.112)	(2.222)
		1.000	**0.798**	**1.000**	**1.000**	**0.752**
RPCD1		61.087*	51.295*	52.355*	55.584*	52.026*
		(6.340)	(5.858)	(6.087)	(6.427)	(6.097)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RPCD2		47.806*	43.234*	46.776*	45.778*	47.111*
		(6.301)	(6.034)	(6.403)	(6.468)	(6.482)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.6: CD Test in the Pure Static Model (Heterogeneous slope) (10% contaminations)

Test	CD case/ e_{it}	$N(0,1)$	$0.90\,N(0,1)+0.10\chi^2_{30}$	$0.90\,N(0,1)+0.10N(4,4)$	$0.90\,N(0,1)+0.10LN(1,2)$	$0.90\,N(0,1)+0.10\text{Cauchy}(0,16)$
LM	No CD $\gamma_i = 0$	192.030 (19.234) **0.056**	192.392 (23.090) **0.104**	188.826 (34.273) **0.154**	190.376 (73.393) **0.282**	190.769 (80.260) **0.290**
RLM1		191.808 (19.913) **0.046**	191.904 (20.223) **0.062**	191.014 (17.830) **0.038**	190.036 (18.404) **0.036**	191.465 (19.966) **0.062**
RLM2		190.944 (20.511) **0.050**	192.448 (18.938) **0.056**	190.925 (18.589) **0.046**	190.759 (18.393) **0.048**	192.289 (18.577) **0.054**
PCD		-0.001 (1.006) **0.042**	0.050 (1.033) **0.054**	0.045 (0.954) **0.040**	0.034 (0.984) **0.044**	0.036 (0.942) **0.032**
RPCD1		0.014 (1.030) **0.066**	2.763* (1.019) **0.774**	0.127 (0.981) **0.060**	0.599 (0.995) **0.086**	0.000 (1.022) **0.052**
RPCD2		-0.001 (1.047) **0.062**	-0.017 (0.983) **0.046**	0.031 (1.020) **0.064**	0.086 (1.000) **0.054**	0.048 (1.029) **0.058**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	221.247 (27.763) **0.428**	190.709 (21.801) **0.074**	191.558 (31.546) **0.144**	185.986 (71.183) **0.256**	191.700 (78.097) **0.300**
RLM1		218.330 (26.219) **0.380**	225.516* (27.645) **0.490**	208.878 (22.800) **0.236**	213.742 (25.387) **0.332**	207.921 (22.289) **0.228**
RLM2		205.936 (22.291) **0.212**	210.387 (20.563) **0.138**	207.556 (23.079) **0.236**	204.671 (22.301) **0.182**	208.190 (23.219) **0.242**
PCD		5.192* (1.717) **0.986**	0.010 (0.978) **0.046**	0.202 (0.984) **0.062**	0.228 (0.981) **0.056**	0.087 (1.025) **0.050**
RPCD1		4.938* (1.665) **0.972**	6.163* (1.544) **1.000**	3.657* (1.445) **0.886**	4.524* (1.570) **0.962**	3.569* (1.491) **0.864**
RPCD2		3.761* (1.526) **0.884**	3.077* (1.423) **0.760**	3.730* (1.536) **0.872**	3.553* (1.505) **0.850**	3.734* (1.529) **0.852**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4393.100* (792.487) **1.000**	193.285 (23.379) **0.068**	223.171* (41.805) **0.564**	281.515 * (108.271) **0.686**	194.066 (77.414) **0.322**
RLM1		4040.739* (763.288) **1.000**	2315.331* (464.728) **1.000**	2247.844* (474.317) **1.000**	2845.936* (597.721) **1.000**	2192.461* (464.100) **1.000**
RLM2		2511.355* (608.222) **1.000**	1762.913 * (454.738) **1.000**	2251.347* (535.675) **1.000**	2112.504* (529.529) **1.000**	2345.785* (559.377) **1.000**
PCD		63.789* (6.346) **1.000**	1.520 (1.179) **0.346**	5.562* (1.977) **0.970**	5.556* (2.773) **0.924**	0.804 (1.183) **0.170**
RPCD1		61.087* (6.340) **1.000**	45.287* (5.121) **1.000**	44.305* (5.321) **1.000**	50.497* (5.977) **1.000**	43.680* (5.299) **1.000**
RPCD2		47.806* (6.301) **1.000**	39.385* (5.770) **1.000**	45.008* (5.988) **1.000**	43.469* (6.092) **1.000**	45.950* (6.098) **1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\text{PCD}|, |\text{RPCD1}|, |\text{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.7: CD Test in the Dynamic Model (Homogeneous slope) (5% contamination)

Test	CD case/ e_{it}	N (0,1)	0.95 N (0,1)+ $0.05\chi^2_{30}$	0.95 N (0,1)+ 0.05N(4,4)	0.95 N (0,1) + 0.05LN(1,2)	0.95 N(0,1)+ 0.05 Cauchy (0,16)
LM	No CD $\gamma_i = 0$	193.223	272.167*	226.788*	234.271*	222.825
		(19.290)	(41.936)	(32.166)	(62.862)	(69.395)
		0.070	**0.898**	**0.518**	**0.522**	**0.434**
RLM1		192.070	332.084*	193.613	192.320	201.967
		(19.305)	(100.223)	(19.460)	(19.629)	(23.504)
		0.064	**0.898**	**0.072**	**0.070**	**0.156**
RLM2		191.579	414.084*	191.714	190.842	208.911
		(18.962)	(154.835)	(18.836)	(18.836)	(31.331)
		0.054	**0.946**	**0.074**	**0.044**	**0.264**
PCD		0.044	-0.035	0.137	0.110	0.052
		(1.019)	(1.138)	(1.126)	(1.145)	(1.079)
		0.054	**0.086**	**0.082**	**0.096**	**0.058**
RPCD1		0.033	0.227	0.051	0.039	0.033
		(1.040)	(1.249)	(1.016)	(1.016)	(1.008)
		0.066	**0.122**	**0.058**	**0.060**	**0.060**
RPCD2		-0.020	0.040	-0.004	-0.033	0.035
		(1.050)	(1.395)	(1.021)	(1.029)	(1.041)
		0.052	**0.138**	**0.052**	**0.064**	**0.052**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	219.169	277.042*	229.804*	234.479*	223.640*
		(25.296)	(45.814)	(32.931)	(60.867)	(70.986)
		0.390	**0.894**	**0.556**	**0.542**	**0.448**
RLM1		215.135	335.996*	207.623	208.211	213.505
		(25.204)	(101.654)	(21.576)	(22.815)	(24.619)
		0.340	**0.912**	**0.242**	**0.278**	**0.344**
RLM2		204.300	416.239*	202.883	203.396	220.947
		(23.253)	(159.955)	(21.628)	(21.174)	(31.056)
		0.194	**0.962**	**0.170**	**0.160**	**0.434**
PCD		4.790*	0.690	1.700	0.959	0.414
		(1.545)	(1.507)	(1.357)	(1.296)	(1.265)
		0.970	**0.188**	**0.384**	**0.212**	**0.108**
RPCD1		4.492*	2.629*	3.428*	3.656*	3.237*
		(1.535)	(1.676)	(1.428)	(1.438)	(1.423)
		0.966	**0.612**	**0.848**	**0.872**	**0.778**
RPCD2		3.288*	2.614*	3.183*	3.088*	3.240*
		(1.423)	(1.866)	(1.430)	(1.433)	(1.504)
		0.818	**0.614**	**0.802**	**0.820**	**0.784**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4094.888*	356.117*	1121.806*	576.969*	276.786*
		(558.689)	(78.386)	(275.015)	(207.040)	(81.992)
		1.000	**0.986**	**1.000**	**0.998**	**0.746**
RLM1		3711.907*	1546.011*	2328.707*	2594.715*	1978.073*
		(529.526)	(322.698)	(404.005)	(436.187)	(367.426)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RLM2		2350.155*	1796.809*	1980.675*	2160.231*	2148.955*
		(433.641)	(381.177)	(362.187)	(397.134)	(417.062)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
PCD		61.423*	9.356*	28.102*	13.873*	5.999*
		(4.687)	(3.807)	(4.562)	(4.291)	(2.907)
		1.000	**0.996**	**1.000**	**1.000**	**1.000**
RPCD1		58.240*	33.157*	45.157*	47.950*	40.710*
		(4.632)	(4.935)	(4.532)	(4.619)	(4.574)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RPCD2		45.254*	35.681*	41.427*	43.378*	42.767*
		(4.727)	(5.484)	(4.383)	(4.587)	(4.926)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\text{PCD}|, |\text{RPCD1}|, |\text{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels

Table 2.8: CD Test in the Dynamic Model (Homogeneous slope) (10% contamination)

Test	CD case/ e_{it}	$N(0,1)$	0.90 $N(0,1)$+ $0.10\chi^2_{30}$	0.90 $N(0,1)$+ $0.10N(4,4)$	0.90 $N(0,1)$ + $0.10LN(1,2)$	0.90 $N(0,1)$+ 0.10Cauchy (0,16)
LM	No CD $\gamma_i = 0$	193.223 (19.290) **0.070**	245.432* (32.611) **0.752**	248.422* (31.526) **0.778**	235.066* (63.383) **0.548**	206.779 (72.799) **0.352**
RLM1		192.070 (19.305) **0.064**	1151.167* (234.788) **1.000**	202.515 (20.715) **0.158**	198.565 (20.600) **0.128**	526.170* (138.780) **1.000**
RLM2		191.579 (18.962) **0.054**	1760.005* (394.666) **1.000**	200.742 (21.203) **0.158**	198.763 (21.626) **0.146**	772.479* (229.797) **1.000**
PCD		0.044 (1.019) **0.054**	0.020 (1.095) **0.070**	-0.007 (1.163) **0.086**	0.005 (1.084) **0.076**	0.051 (1.040) **0.062**
RPCD1		0.033 (1.040) **0.066**	1.201 (2.369) **0.330**	-0.005 (0.996) **0.040**	-0.010 (1.003) **0.046**	-0.090 (1.593) **0.158**
RPCD2		-0.020 (1.050) **0.052**	0.193 (2.992) **0.458**	-0.016 (1.002) **0.046**	-0.007 (0.984) **0.042**	-0.070 (1.915) **0.230**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	219.169 (25.296) **0.390**	249.092* (36.436) **0.774**	247.841* (32.200) **0.770**	235.769* (67.427) **0.504**	207.658 (69.264) **0.366**
RLM1		215.135 (25.204) **0.340**	1153.719* (244.910) **1.000**	209.053 (21.711) **0.252**	207.695 (21.918) **0.242**	516.953* (136.603) **0.998**
RLM2		204.300 (23.253) **0.194**	1754.184* (415.755) **1.000**	207.109 (20.550) **0.202**	208.551 (23.120) **0.246**	761.311* (220.801) **1.000**
PCD		4.790* (1.545) **0.970**	0.530 (1.360) **0.156**	1.080 (1.348) **0.224**	0.479 (1.250) **0.126**	0.282 (1.117) **0.086**
RPCD1		4.492* (1.535) **0.966**	3.356* (3.543) **0.604**	2.433* (1.300) **0.612**	2.779* (1.335) **0.732**	1.873 (2.277) **0.386**
RPCD2		3.288* (1.423) **0.818**	2.821* (4.328) **0.524**	2.725* (1.288) **0.778**	2.934* (1.318) **0.728**	2.345* (2.841) **0.466**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4094.888* (558.689) **1.000**	325.333* (78.566) **0.942**	628.911* (142.346) **1.000**	312.730* (89.181) **0.866**	232.863* (74.233) **0.526**
RLM1		3711.907* (529.526) **1.000**	1533.176* (393.360) **1.000**	1429.117* (284.475) **1.000**	1751.728* (337.832) **1.000**	1001.766* (200.920) **1.000**
RLM2		2350.155* (433.641) **1.000**	2228.397* (642.014) **1.000**	1458.926* (279.526) **1.000**	1839.931* (343.874) **1.000**	1531.163* (343.821) **1.000**
PCD		61.423* (4.687) **1.000**	8.127* (4.125) **0.972**	18.809* (3.438) **1.000**	7.554* (2.975) **0.988**	3.616* (2.538) **0.718**
RPCD1		58.240* (4.632) **1.000**	26.759* (8.604) **1.000**	33.966* (4.105) **1.000**	38.213* (4.348) **1.000**	24.379* (4.793) **1.000**
RPCD2		45.254* (4.727) **1.000**	31.168* (11.004) **1.000**	34.713* (4.026) **1.000**	39.521* (4.339) **1.000**	30.814* (6.463) **1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.9: CD Test in the Dynamic Model (Heterogeneous slope) (5% contamination)

Test	CD case/ e_{it}	$N(0,1)$	0.95 $N(0,1)$+ 0.05χ^2_{30}	0.95 $N(0,1)$+ 0.05$N(4,4)$	0.95 $N(0,1)$ + 0.05$LN(1,2)$	0.95 $N(0,1)$+ 0.05 Cauchy (0,16)
LM	No CD $\gamma_i = 0$	192.801	193.619	195.131	198.096	187.042
		(19.270)	(36.917)	(24.103)	(65.938)	(79.281)
		0.072	**0.180**	**0.130**	**0.292**	**0.270**
RLM1		191.379	201.462	191.830	191.567	193.348
		(18.833)	(21.175)	(18.855)	(19.432)	(19.810)
		0.054	**0.102**	**0.056**	**0.058**	**0.064**
RLM2		192.449	207.159	193.169	192.652	197.640
		(20.027)	(24.537)	(19.220)	(19.111)	(20.385)
		0.076	**0.106**	**0.072**	**0.048**	**0.100**
PCD		0.047	0.025	0.134	0.104	0.060
		(1.019)	(1.056)	(1.030)	(1.025)	(0.949)
		0.054	**0.062**	**0.054**	**0.052**	**0.036**
RPCD1		0.023	0.433	0.181	0.119	0.041
		(1.016)	(1.027)	(1.044)	(1.021)	(1.061)
		0.050	**0.086**	**0.050**	**0.058**	**0.064**
RPCD2		-0.045	0.020	-0.002	-0.061	0.106
		(1.028)	(1.041)	(1.070)	(1.054)	(1.032)
		0.050	**0.058**	**0.064**	**0.062**	**0.054**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	218.843	193.708	201.372	201.490	187.100
		(25.244)	(37.075)	(26.817)	(67.102)	(76.126)
		0.382	**0.192**	**0.194**	**0.314**	**0.264**
RLM1		214.211	215.105	207.048	207.809	207.468
		(24.651)	(25.892)	(22.115)	(23.211)	(23.943)
		0.322	**0.344**	**0.224**	**0.246**	**0.226**
RLM2		204.995	219.089	203.353	203.315	211.195
		(22.673)	(27.303)	(21.277)	(21.666)	(23.390)
		0.186	**0.410**	**0.180**	**0.184**	**0.286**
PCD		4.806*	0.222	2.168*	0.958	0.242
		(1.563)	(1.067)	(1.297)	(1.209)	(0.995)
		0.966	**0.070**	**0.546**	**0.214**	**0.050**
RPCD1		4.502*	3.636*	3.742*	3.862*	3.433*
		(1.537)	(1.393)	(1.457)	(1.472)	(1.484)
		0.960	**0.882**	**0.898**	**0.906**	**0.840**
RPCD2		3.313*	3.152*	3.130*	3.176*	3.238*
		(1.451)	(1.417)	(1.428)	(1.422)	(1.457)
		0.840	**0.798**	**0.802**	**0.806**	**0.800**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4102.829*	218.240	1511.915*	649.811*	229.897*
		(556.187)	(44.283)	(345.707)	(258.493)	(81.880)
		1.000	**0.986**	**1.000**	**0.998**	**0.546**
RLM1		3720.083*	2073.993*	2612.715*	2785.489*	2263.177*
		(528.175)	(369.601)	(438.711)	(457.294)	(395.828)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RLM2		2355.418*	2031.676*	2008.971*	2166.818*	2124.033*
		(429.196)	(423.122)	(376.656)	(401.229)	(420.813)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
PCD		61.483*	4.169*	34.478*	14.757*	3.666*
		(4.661)	(1.722)	(4.780)	(5.020)	(1.910)
		1.000	**0.996**	**1.000**	**1.000**	**0.808**
RPCD1		58.309*	42.234*	48.233*	49.938*	44.497*
		(4.609)	(4.336)	(4.578)	(4.636)	(4.441)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
RPCD2		45.302*	41.436*	41.702*	43.418*	42.801*
		(4.637)	(5.000)	(4.510)	(4.634)	(4.938)
		1.000	**1.000**	**1.000**	**1.000**	**1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\text{PCD}|, |\text{RPCD1}|, |\text{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.10: CD Test in the Dynamic Model (Heterogeneous slope) (10% contamination)

Test	CD case/ e_{it}	$N(0,1)$	0.90 $N(0,1)$+ 0.10χ^2_{30}	0.90 $N(0,1)$+ 0.10$N(4,4)$	0.90 $N(0,1)$ + 0.10$LN(1,2)$	0.90 $N(0,1)$+ 0.10Cauchy (0,16)
LM	No CD $\gamma_i = 0$	192.801 (19.270) **0.072**	192.053 (23.416) **0.098**	194.701 (23.895) **0.130**	193.210 (70.995) **0.290**	192.081 (73.366) **0.302**
RLM1		191.379 (18.833) **0.054**	235.089* (26.693) **0.284**	193.625 (18.956) **0.058**	192.753 (19.023) **0.064**	220.842 (24.062) **0.454**
RLM2		192.449 (20.027) **0.076**	310.237* (69.156) **0.360**	198.928 (19.679) **0.116**	197.171 (19.880) **0.104**	237.434* (27.113) **0.684**
PCD		0.047 (1.019) **0.054**	0.022 (1.083) **0.066**	0.034 (1.053) **0.060**	0.025 (0.997) **0.048**	0.082 (1.000) **0.048**
RPCD1		0.023 (1.016) **0.050**	-0.013 (1.094) **0.068**	0.692 (1.044) **0.116**	0.441 (1.034) **0.088**	0.009 (1.059) **0.050**
RPCD2		-0.045 (1.028) **0.050**	2.850* (2.246) **0.063**	0.002 (1.054) **0.064**	-0.019 (1.057) **0.056**	0.042 (1.084) **0.066**
LM	Mild CD $\gamma_i \sim U(0.1,0.3)$	218.843 (25.244) **0.382**	192.082 (23.343) **0.086**	197.851 (25.304) **0.168**	193.385 (71.255) **0.286**	193.035 (73.234) **0.306**
RLM1		214.211 (24.651) **0.322**	251.553* (29.301) **0.832**	206.616 (22.534) **0.218**	205.492 (21.432) **0.206**	235.412* (27.499) **0.666**
RLM2		204.995 (22.673) **0.186**	333.800* (71.208) **0.992**	208.550 (22.400) **0.248**	206.717 (22.294) **0.224**	257.031* (30.490) **0.878**
PCD		4.806* (1.563) **0.966**	0.115 (1.083) **0.062**	1.357 (1.243) **0.692**	0.302 (1.059) **0.078**	0.093 (1.035) **0.054**
RPCD1		4.502* (1.537) **0.960**	2.391* (1.455) **0.590**	3.542* (1.407) **0.898**	3.495* (1.390) **0.888**	2.782* (1.517) **0.690**
RPCD2		3.313* (1.451) **0.840**	5.249* (2.215) **0.948**	2.852* (1.427) **0.738**	2.951* (1.428) **0.754**	3.018* (1.675) **0.720**
LM	Strong CD $\gamma_i \sim U(0.5,1.5)$	4102.829* (556.187) **1.000**	198.375 (25.937) **0.164**	867.096* (209.001) **1.000**	273.998* (95.339) **0.672**	195.672 (72.223) **0.318**
RLM1		3720.083* (528.175) **1.000**	1329.286* (283.044) **1.000**	1904.317* (342.683) **1.000**	2102.362* (375.769) **1.000**	1513.449* (272.890) **1.000**
RLM2		2355.418* (429.196) **1.000**	1743.721* (368.083) **1.000**	1642.788* (305.008) **1.000**	1903.784* (355.380) **1.000**	1775.999* (355.474) **1.000**
PCD		61.483* (4.661) **1.000**	2.108* (1.334) **0.544**	24.528* (3.904) **1.000**	5.572* (2.563) **0.932**	0.955 (1.165) **0.176**
RPCD1		58.309* (4.609) **1.000**	31.538* (4.364) **1.000**	40.502* (4.195) **1.000**	42.763* (4.378) **1.000**	35.159* (3.875) **1.000**
RPCD2		45.302* (4.637) **1.000**	37.132* (4.858) **1.000**	37.320* (4.083) **1.000**	40.406* (4.391) **1.000**	38.284* (4.701) **1.000**

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (bold) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\text{PCD}|, |\text{RPCD1}|, |\text{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.11: Results of the CD Tests in the Presence of Structural Break (SB) in the Pure Static Model

γ_i	Conditions	LM	RLM1	RLM2	PCD	RPCD1	RPCD2
$\gamma_i = 0$	No SB	192.030	191.808	190.944	-0.001	0.014	-0.001
		(19.234)	(19.913)	(20.511)	(1.006)	(1.030)	(1.047)
	With SB	197.930	193.878	191.945	-0.022	-0.025	-0.004
		(18.680)	(18.817)	(18.643)	(0.990)	(0.969)	(0.943)
$\gamma_i \sim U(0.1,0.3)$	No SB	221.247	218.330	205.936	5.192*	4.938*	3.761*
		(27.763)	(26.219)	(22.291)	(1.717)	(1.665)	(1.526)
	With SB	212.590	208.384	201.113	3.794*	3.618*	2.759*
		(24.355)	(24.090)	(22.238)	(1.516)	(1.505)	(1.383)
$\gamma_i \sim U(0.51.5)$	No SB	4393.100*	4040.739*	2511.355*	63.789*	61.087*	47.806*
		(792.49)	(763.29)	(608.22)	(6.35)	(6.34)	(6.30)
	With SB	3155.866*	2912.080*	1799.027*	53.170*	50.981*	39.545*
		(687.58)	(656.32)	(482.87)	(6.60)	(6.53)	(6.05)
				Size			
$\gamma_i = 0$	No SB	0.056	0.046	0.050	0.042	0.066	0.062
	With SB	0.102	0.078	0.056	0.030	0.034	0.038
				Power			
$\gamma_i \sim U(0.1,0.3)$	No SB	0.428	0.380	0.212	0.986	0.972	0.884
	With SB	0.322	0.244	0.150	0.894	0.876	0.690
$\gamma_i \sim U(0.51.5)$	No SB	1.000	1.000	1.000	1.000	1.000	1.000
	With SB	1.000	1.000	1.000	1.000	1.000	1.000

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (second row) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.12: Results of the CD Tests in the Presence of Structural Break (SB) in the Dynamic Panel Model

γ_i	Conditions	LM	RLM1	RLM2	PCD	RPCD1	RPCD2
$\gamma_i = 0$	No SB	193.223	192.070	191.579	0.044	0.033	-0.020
		(19.290)	(19.305)	(18.962)	(1.019)	(1.040)	(1.050)
	With SB	277.538*	240.229*	209.656*	0.138	0.145	0.174
		(28.550)	(25.261)	(23.584)	(1.180)	(1.083)	(1.076)
$\gamma_i \sim U(0.1,0.3)$	No SB	219.169	215.135	204.300	4.790*	4.492*	3.288*
		(25.296)	(25.204)	(23.253)	(1.545)	(1.535)	(1.423)
	With SB	292.176*	254.998*	218.818*	3.804*	3.705*	2.753*
		(32.010)	(27.857)	(22.845)	(1.744)	(1.627)	(1.456)
$\gamma_i \sim U(0.51.5)$	No SB	4094.888*	3711.907*	2350.155*	61.423*	58.240*	45.254*
		(558.689)	(529.526)	(433.641)	(4.687)	(4.632)	(4.727)
	With SB	3124.143*	2892.372*	1765.898*	52.437*	50.443*	38.312*
		(594.189)	(549.518)	(379.724)	(5.659)	(5.400)	(4.821)
				Size			
$\gamma_i = 0$	No SB	0.070	0.064	0.054	0.054	0.066	0.052
	With SB	0.984	0.746	0.296	0.102	0.080	0.076
				Power			
$\gamma_i \sim U(0.1,0.3)$	No SB	0.390	0.340	0.194	0.970	0.966	0.818
	With SB	0.994	0.886	0.420	0.858	0.862	0.668
$\gamma_i \sim U(0.51.5)$	No SB	1.000	1.000	1.000	1.000	1.000	1.000
	With SB	1.000	1.000	1.000	1.000	1.000	1.000

Note: The results are the sample mean, standard deviation (parentheses) and rejection rates (second row) of the tests based on 500 replications. The H_0 is rejected (marked with *) if $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 223.160$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

Table 2.13: Performance of the CD Tests in the Pure Static Model (Uncontaminated panel)

	Size: $\gamma_i = 0$(No CD)					**Power :** $\gamma_i \sim iidU(0.1,0.3)$(Mild CD)					$\gamma_i \sim iidU(0.5,1.5)$(Strong CD)				
T / N	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
								LM							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.068	-	-	-	-	0.084	-	-	-	-	0.994	-	-	-	-
30	0.054	0.082	-	-	-	0.080	0.210	-	-	-	1.000	1.000	-	-	-
50	0.070	0.068	0.106	-	-	0.120	0.242	0.382	-	-	1.000	1.000	1.000	-	-
100	0.070	0.056	0.070	0.068	-	0.186	0.428	0.678	0.942	-	1.000	1.000	1.000	1.000	-
								RLM1							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.096	-	-	-	-	0.070	-	-	-	-	0.990	-	-	-	-
30	0.060	0.082	-	-	-	0.090	0.202	-	-	-	1.000	1.000	-	-	-
50	0.076	0.070	0.082	-	-	0.104	0.228	0.316	-	-	1.000	1.000	1.000	-	-
100	0.070	0.046	0.074	0.060	-	0.178	0.380	0.648	0.924	-	1.000	1.000	1.000	1.000	-
								RLM2							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.074	-	-	-	-	0.066	-	-	-	-	0.962	-	-	-	-
30	0.052	0.072	-	-	-	0.084	0.126	-	-	-	0.998	0.992	-	-	-
50	0.066	0.048	0.078	-	-	0.100	0.194	0.268	-	-	1.000	1.000	0.998	-	-
100	0.064	0.050	0.066	0.064	-	0.154	0.212	0.596	0.870	-	1.000	1.000	1.000	1.000	-
								PCD							
10	0.060	0.034	0.046	0.032	0.058	0.160	0.356	0.440	0.702	0.924	0.978	0.998	1.000	1.000	1.000
20	0.042	0.064	0.034	0.062	0.046	0.196	0.536	0.704	0.910	0.998	1.000	1.000	1.000	1.000	1.000
30	0.050	0.044	0.048	0.046	0.060	0.310	0.626	0.894	0.982	1.000	1.000	1.000	1.000	1.000	1.000
50	0.056	0.056	0.044	0.038	0.044	0.440	0.814	0.960	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.044	0.042	0.050	0.064	0.050	0.634	0.986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD1							
10	0.058	0.034	0.060	0.038	0.058	0.148	0.332	0.460	0.674	0.910	0.976	0.998	1.000	1.000	1.000
20	0.046	0.056	0.028	0.062	0.034	0.180	0.496	0.654	0.888	0.998	1.000	1.000	1.000	1.000	1.000
30	0.044	0.050	0.042	0.048	0.054	0.286	0.630	0.822	0.978	1.000	1.000	1.000	1.000	1.000	1.000
50	0.048	0.050	0.044	0.040	0.040	0.414	0.796	0.948	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.046	0.066	0.044	0.054	0.052	0.594	0.972	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD2							
10	0.052	0.048	0.058	0.042	0.052	0.110	0.200	0.262	0.564	0.842	0.860	0.982	1.000	1.000	1.000
20	0.040	0.042	0.040	0.048	0.032	0.102	0.274	0.448	0.788	0.988	0.988	1.000	1.000	1.000	1.000
30	0.034	0.044	0.048	0.038	0.038	0.136	0.378	0.536	0.946	1.000	1.000	1.000	1.000	1.000	1.000
50	0.052	0.040	0.040	0.030	0.034	0.210	0.526	0.702	0.990	1.000	1.000	1.000	1.000	1.000	1.000
100	0.040	0.062	0.046	0.062	0.052	0.314	0.884	0.918	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note: rejection rates of the tests are based on 500 replications.

Table 2.14: Performance of the CD Tests in the Pure Static Model (Contaminated panel)

	Size: $\gamma_i = 0$(No CD)					**Power :** $\gamma_i \sim iidU(0.1,0.3)$(Mild CD)					$\gamma_i \sim iidU(0.5,1.5)$(Strong CD)				
T / N	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
								LM							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.146	-	-	-	-	0.158	-	-	-	-	0.584	-	-	-	-
30	0.184	0.216	-	-	-	0.204	0.212	-	-	-	0.482	0.724	-	-	-
50	0.190	0.212	0.240	-	-	0.178	0.232	0.252	-	-	0.308	0.498	0.664	-	-
100	0.168	0.272	0.198	0.076	-	0.160	0.328	0.190	0.042	-	0.230	0.548	0.446	1.000	-
								RLM1							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.068	-	-	-	-	0.084	-	-	-	-	0.986	-	-	-	-
30	0.054	0.070	-	-	-	0.064	0.130	-	-	-	1.000	1.000	-	-	-
50	0.070	0.062	0.050	-	-	0.096	0.196	0.286	-	-	1.000	1.000	1.000	-	-
100	0.054	0.066	0.066	0.056	-	0.140	0.306	0.570	1.000	-	1.000	1.000	1.000	1.000	-
								RLM2							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.060	-	-	-	-	0.082	-	-	-	-	0.950	-	-	-	-
30	0.048	0.060	-	-	-	0.070	0.102	-	-	-	0.972	0.968	-	-	-
50	0.052	0.054	0.048	-	-	0.088	0.166	0.250	-	-	0.998	0.990	0.994	-	-
100	0.050	0.050	0.046	0.052	-	0.128	0.250	0.556	1.000	-	1.000	1.000	1.000	1.000	-
								PCD							
10	0.060	0.034	0.036	0.064	0.086	0.096	0.160	0.208	0.234	0.468	0.740	0.948	0.988	1.000	1.000
20	0.026	0.054	0.036	0.038	0.038	0.058	0.112	0.150	0.100	0.324	0.626	0.916	0.982	1.000	1.000
30	0.056	0.042	0.042	0.060	0.052	0.080	0.076	0.098	0.114	0.308	0.518	0.862	0.978	1.000	1.000
50	0.056	0.036	0.056	0.088	0.058	0.058	0.084	0.076	0.052	0.212	0.416	0.814	0.950	1.000	1.000
100	0.036	0.052	0.052	0.076	0.066	0.046	0.046	0.040	0.042	0.146	0.392	0.752	0.926	1.000	1.000
								RPCD1							
10	0.040	0.044	0.070	0.058	0.074	0.142	0.270	0.446	0.916	0.926	0.970	0.998	1.000	1.000	1.000
20	0.074	0.064	0.062	0.052	0.056	0.176	0.472	0.690	1.000	1.000	0.998	1.000	1.000	1.000	1.000
30	0.044	0.070	0.068	0.066	0.060	0.284	0.640	0.830	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	0.070	0.062	0.050	0.068	0.054	0.376	0.846	0.978	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.078	0.056	0.066	0.056	0.062	0.584	0.926	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD2							
10	0.038	0.050	0.042	0.056	0.038	0.114	0.186	0.384	0.882	0.908	0.868	0.928	0.886	1.000	1.000
20	0.060	0.064	0.046	0.048	0.066	0.128	0.324	0.490	0.976	0.992	0.990	0.998	1.000	1.000	1.000
30	0.040	0.060	0.062	0.044	0.044	0.178	0.404	0.604	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	0.058	0.062	0.054	0.068	0.048	0.252	0.600	0.828	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.058	0.056	0.046	0.052	0.052	0.352	0.872	0.974	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note: rejection rates of the tests are based on 500 replications

Table 2.15: Performance of the CD Tests in the Dynamic Model (Uncontaminated panel)

	Size: $\gamma_i = 0$(No CD)					**Power :** $\gamma_i \sim iidU(0.1,0.3)$(Mild CD)					$\gamma_i \sim iidU(0.5,1.5)$(Strong CD)				
T / N	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
								LM							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.094	-	-	-	-	0.114	-	-	-	-	1.000	-	-	-	-
30	0.086	0.104	-	-	-	0.094	0.204	-	-	-	1.000	1.000	-	-	-
50	0.062	0.074	0.128	-	-	0.106	0.234	0.420	-	-	1.000	1.000	1.000	-	-
100	0.066	0.072	0.048	0.054	-	0.188	0.382	0.728	0.904	-	1.000	1.000	1.000	1.000	-
								RLM1							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.064	-	-	-	-	0.098	-	-	-	-	0.996	-	-	-	-
30	0.070	0.084	-	-	-	0.084	0.152	-	-	-	1.000	1.000	-	-	-
50	0.048	0.056	0.064	-	-	0.104	0.182	0.326	-	-	1.000	1.000	1.000	-	-
100	0.060	0.054	0.040	0.048	-	0.166	0.322	0.640	0.882	-	1.000	1.000	1.000	1.000	-
								RLM2							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.078	-	-	-	-	0.062	-	-	-	-	0.870	-	-	-	-
30	0.062	0.068	-	-	-	0.074	0.122	-	-	-	0.958	0.994	-	-	-
50	0.066	0.054	0.046	-	-	0.080	0.140	0.290	-	-	0.992	1.000	1.000	-	-
100	0.054	0.076	0.044	0.042	-	0.108	0.186	0.560	0.732	-	1.000	1.000	1.000	1.000	-
								PCD							
10	0.038	0.056	0.058	0.060	0.044	0.156	0.312	0.460	0.640	0.942	0.988	1.000	1.000	1.000	1.000
20	0.054	0.058	0.050	0.040	0.056	0.208	0.508	0.794	0.916	0.996	1.000	1.000	1.000	1.000	1.000
30	0.042	0.036	0.028	0.048	0.058	0.300	0.600	0.892	0.980	1.000	1.000	1.000	1.000	1.000	1.000
50	0.058	0.048	0.040	0.056	0.042	0.368	0.828	0.990	0.996	1.000	1.000	1.000	1.000	1.000	1.000
100	0.046	0.054	0.050	0.046	0.050	0.624	0.966	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD1							
10	0.038	0.056	0.064	0.044	0.056	0.116	0.270	0.422	0.580	0.928	0.982	1.000	1.000	1.000	1.000
20	0.048	0.052	0.046	0.060	0.056	0.170	0.420	0.656	0.880	0.998	1.000	1.000	1.000	1.000	1.000
30	0.036	0.030	0.030	0.042	0.050	0.276	0.566	0.854	0.970	1.000	1.000	1.000	1.000	1.000	1.000
50	0.046	0.042	0.040	0.058	0.048	0.334	0.780	0.984	0.992	1.000	1.000	1.000	1.000	1.000	1.000
100	0.050	0.050	0.056	0.050	0.042	0.602	0.960	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD2							
10	0.046	0.062	0.052	0.044	0.056	0.100	0.164	0.278	0.356	0.734	0.854	0.980	0.994	1.000	1.000
20	0.048	0.044	0.030	0.044	0.048	0.114	0.254	0.546	0.692	0.990	0.988	1.000	1.000	1.000	1.000
30	0.042	0.034	0.036	0.044	0.040	0.178	0.370	0.662	0.862	1.000	1.000	1.000	1.000	1.000	1.000
50	0.054	0.030	0.038	0.046	0.052	0.196	0.542	0.884	0.958	1.000	1.000	1.000	1.000	1.000	1.000
100	0.050	0.050	0.058	0.056	0.052	0.386	0.840	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note: rejection rates of the tests are based on 500 replications.

Table 2.16: Performance of the CD Tests in the Dynamic Model (Contaminated panel)

	Size: $\gamma_i = 0$(No CD)					**Power :** $\gamma_i \sim iidU(0.1,0.3)$(Mild CD)					$\gamma_i \sim iidU(0.5,1.5)$(Strong CD)				
T / N	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
								LM							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.146	-	-	-	-	0.146	-	-	-	-	0.650	-	-	-	-
30	0.166	0.198	-	-	-	0.170	0.202	-	-	-	0.468	0.628	-	-	-
50	0.166	0.228	0.222	-	-	0.158	0.226	0.212	-	-	0.270	0.538	0.636	-	-
100	0.172	0.270	0.410	0.326	-	0.160	0.264	0.214	0.238	-	0.254	0.546	0.556	0.694	-
								RLM1							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.086	-	-	-	-	0.090	-	-	-	-	0.986	-	-	-	-
30	0.078	0.108	-	-	-	0.090	0.236	-	-	-	0.998	1.000	-	-	-
50	0.104	0.100	0.102	-	-	0.134	0.246	0.416	-	-	1.000	1.000	1.000	-	-
100	0.102	0.064	0.108	0.086	-	0.160	0.266	0.690	0.888	-	1.000	1.000	1.000	1.000	-
								RLM2							
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0.080	-	-	-	-	0.096	-	-	-	-	0.952	-	-	-	-
30	0.082	0.088	-	-	-	0.100	0.208	-	-	-	0.962	0.994	-	-	-
50	0.090	0.098	0.088	-	-	0.138	0.238	0.398	-	-	0.996	1.000	1.000	-	-
100	0.098	0.100	0.194	0.090	-	0.154	0.286	0.642	0.864	-	1.000	1.000	1.000	1.000	-
								PCD							
10	0.060	0.046	0.040	0.060	0.074	0.082	0.118	0.218	0.320	0.620	0.694	0.932	0.992	1.000	1.000
20	0.048	0.054	0.060	0.050	0.054	0.056	0.108	0.116	0.184	0.522	0.636	0.914	0.990	1.000	1.000
30	0.042	0.040	0.048	0.050	0.040	0.064	0.078	0.108	0.126	0.324	0.530	0.876	1.000	1.000	1.000
50	0.060	0.058	0.044	0.070	0.034	0.070	0.056	0.060	0.144	0.182	0.454	0.858	1.000	1.000	1.000
100	0.044	0.036	0.050	0.042	0.060	0.050	0.050	0.058	0.108	0.124	0.468	0.808	1.000	1.000	1.000
								RPCD1							
10	0.048	0.058	0.060	0.072	0.078	0.096	0.202	0.340	0.534	0.818	0.912	0.990	1.000	1.000	1.000
20	0.034	0.044	0.046	0.060	0.058	0.134	0.318	0.626	0.692	0.982	0.998	1.000	1.000	1.000	1.000
30	0.044	0.060	0.048	0.082	0.076	0.192	0.430	0.712	0.888	1.000	1.000	1.000	1.000	1.000	1.000
50	0.052	0.078	0.040	0.078	0.068	0.296	0.618	0.920	0.994	1.000	1.000	1.000	1.000	1.000	1.000
100	0.056	0.064	0.052	0.066	0.056	0.412	0.840	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000
								RPCD2							
10	0.046	0.066	0.078	0.070	0.066	0.108	0.126	0.260	0.412	0.634	0.826	0.974	0.980	1.000	1.000
20	0.054	0.054	0.064	0.068	0.088	0.108	0.266	0.500	0.608	0.966	0.990	1.000	0.996	1.000	1.000
30	0.056	0.054	0.050	0.048	0.048	0.156	0.366	0.584	0.814	0.990	0.998	1.000	1.000	1.000	1.000
50	0.062	0.056	0.046	0.030	0.050	0.248	0.500	0.916	0.920	1.000	1.000	1.000	1.000	1.000	1.000
100	0.070	0.054	0.048	0.052	0.054	0.364	0.800	0.984	0.990	1.000	1.000	1.000	1.000	1.000	1.000

Note: rejection rates of the tests are based on 500 replications.

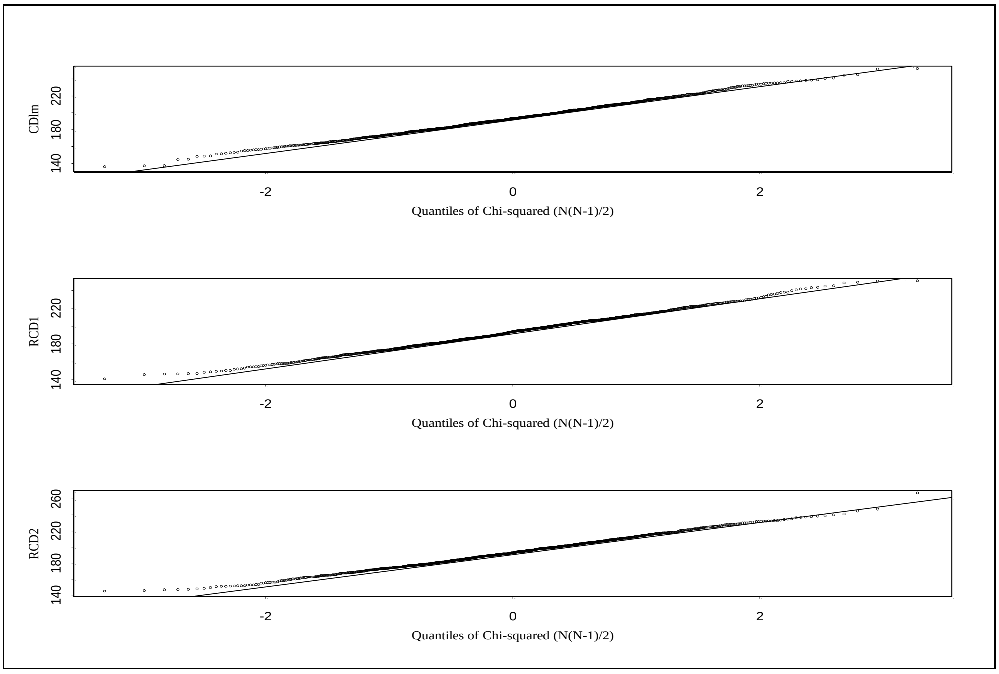

Figure 2.1: Quantile-Quantile Plots of LM, RLM1, RLM2

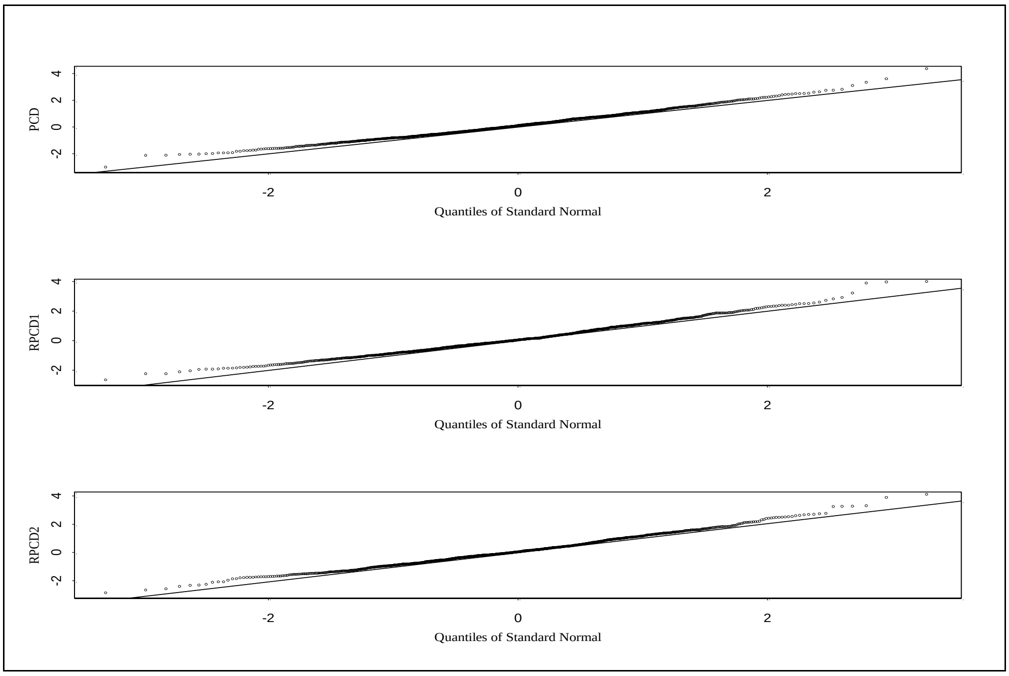

Figure 2.2: Quantile-Quantile Plots of PCD, RPCD1, RPCD2

2.6 Conclusion

This chapter proposes the robust versions of the LM test of Breusch and Pagan and the PCD test of Pesaran for the cross sectional dependence. The proposed tests are robust to the effect of the spurious observation in the data. The asymptotic distribution of the proposed test and the finite sample behaviour are provided resulting from the Monte Carlo analysis. From the results, it can be concluded that the RPCD1 is oversized in the presence of 10% outliers from $\chi^2_{(30)}$ distribution for the pure static model. Since observations drawn from $\chi^2_{(30)}$ are always positive, the RPCD1 tends to reject the null in favor of the alternative. Other proposed tests yield reasonable size and good power in most scenarios in the pure static model. In the dynamic model, it is observed that the proposed tests (RLM1, RLM2, RPCD1 and RPCD2) exceed the reasonable size (that is 0.05) in the presence of larger size and heavy tailed contaminations ($\chi^2_{(30)}$ and Cauchy). The presence of outliers affect the absence of CD subsequently results in the incorrect results for the test statistics. The powers of the tests are not as high as in the pure static model and this illustrates that the proposed tests do not perform well in the dynamic model in the presence of outliers. The powers for the dynamic model are expected to be less than in the pure static model when the independence among the regressor is assumed.

Although the PCD test is quite robust in detecting the CD in the presence of breaks, this test fails to detect the mild CD in the contaminated panels. The proposed RPCD1 test, on the other hand, is capable of detecting the presence of the CD in the presence of outliers even for the case of the mild CD effect.

CHAPTER 3

Parameter Estimation and Inferences in Panel Model

3.1 Introduction

There are a vast number of studies on panel data modeling in the presence of cross sectional dependence (Coakley et al., 2002; Bai and Ng, 2002; Philips and Sul, 2003; Moon and Perron, 2004; Kapetonis and Pesaran, 2004; Coakley et al., 2006; Pesaran, 2006; and Noman, 2008)[24]. This is due to the recent development in panel data analysis in view of the fact that most economic data are cross correlated in panel framework; independent assumption of the residual among cross sectional units is therefore no longer appropriate.

The presence of contemporaneous cross correlation among the disturbances, e_{it} finds support from many empirical applications in macroeconomics, finance and international finance (Moon and Perron, 2004). Many studies have been conducted by characterising CD using factor structure (see Bai and Ng, 2002; Coakley et al., 2002; Philips and Sul, 2003). In order to correct the CD in modeling and estimating the panel, Coakley et al. (2002) and Stock and Watson (2002) proposed the principal component[25] approach to obtain unobserved factor in order to accommodate the presence of CD in the panel. Kapetonis and Pesaran (2004) applied the principal component procedure in modeling the standard Arbitrage Pricing Theory (APT) Model of the company asset returns. Bai and Ng (2002) and Moon and Perron (2004) determined the number of such unobserved factors using the selection criteria such as the Akaike's information

[24] Some literature focuses on (i) estimation of panel model in the presence of cross sectional dependence and (ii) tests on the existence of unit root in the presence of cross dependency among the residuals.
[25] The residuals are obtained using principal component procedure and are considered as unobserved factor.

criterion[26] (AIC) and Bayesian information criterion[27] (BIC). This approach however, provides unreliable parameter estimates since the residuals are obtained from the procedures based on the OLS method to explain the CD.

In this chapter, the techniques used to estimate the panel data model, both with and without the presence of cross sectional dependence are discussed. These techniques include the standard method that uses OLS and the proposed procedure which is based on robust parameter estimation. The properties of the proposed estimator are derived. From this a robust hypothesis tests is proposed and robust confidence interval for the parameter estimates is constructed. In the final section, the goodness of fit of the proposed method is discussed.

3.2 Estimation Procedure

3.2.1 Pooled Model

The standard technique used to model and estimate the parameter of the model is to pool the cross sectional units of the data together. The pooled model is considered since this model is restricted to the assumption of cross sectional independence among the residuals and parameter homogeneity in the model. Coakley et al. (2006) showed in their study that FE and RE provide similar results in terms of sample mean, standard deviation and mean squared error (MSE) under several scenarios of CD and these approaches are listed under Appendix B and not considered in this study[28].

A simple pooled regression model is considered as follows:

$$y_{it} = \alpha + \boldsymbol{\beta}^T \mathbf{x}_{it} + e_{it} \qquad ;\ i = 1,2,\ldots,N \text{ and } t = 1,2,\ldots,T \tag{3.1}$$

---

[26] AIC is computed as $\text{AIC} = 2k - 2\ln(L)$ with k as the number of parameters in the model, and L as the maximized value of the likelihood function (will be discussed in the next page) for the estimated model.

[27] BIC is computed as $\text{BIC} = \ln(\hat{\sigma}^2) + \frac{k}{NT}\ln(NT)$ with k defined as above, $\hat{\sigma}^2$ is computed as the variance of the estimated residuals, N and T are the number of cross sectional units and time period, respectively.

[28] For interested reader, refer Coakley et al. (2006).

where y_{it} is the observation on the i^{th} cross section unit at time t, $\mathbf{x}_{it}$ represents the $k \times 1$ vector independent variable (regressor) on the i^{th} cross section unit at time t and e_{it} is the random error components on the i^{th} cross section unit at time t. In this pooled model, the parameters $\mathbf{b} = (\alpha, \boldsymbol{\beta})^T$ are assumed constant across i and t. The following assumptions are considered in the pooled model (Stock and Watson, 2006):

A3.1: $e_{it} \sim iidN(0,1)$

A3.2: $E(\mathbf{x}_{it} e_{it}) = 0$

A3.3: $E\left(e_{it} \middle| \mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT}\right) = 0$

A3.4: $E(e_{is} e_{it}) = 0$

A3.5: $E(e_{it} e_{jt}) = 0$

for $i, j = 1,2, \ldots, N.$ $s,t = 1,2,\ldots,T$ with $s \neq t$ and $i \neq j$.

Assumptions **A3.1**, **A3.2** and **A3.4** are the usual assumptions for the linear regression with residuals, independent and identically (iid) normal distribution with mean zero and constant variance, and that the regressors are uncorrelated with the residuals. Assumption **A3.3** imposes strictly exogenous where the residuals and regressors are independent; while assumption **A3.5** ensures no cross sectional dependence between residuals.

When the assumptions **A3.1-A3.5** are satisfied, the Maximum Likelihood Estimator (MLE) is used to estimate the parameter of the model. The likelihood function of (3.1) is given as follows:

$$L = \prod_{i=1}^{N} \prod_{t=1}^{T} f\left(\mathbf{x}_{it}, \mathbf{b}\right) \tag{3.2}$$

Under assumption **A3.1**, (3.2) becomes:

$$L = \prod_{i=1}^{N} \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{e_{it}}{\sigma}\right)^2} \tag{3.3}$$

Take the natural log to (3.3), yields:

$$\ln L = \sum_{i=1}^{N}\sum_{t=1}^{T}\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{e_{it}}{\sigma}\right)^2}\right). \tag{3.4}$$

Expanding (3.4) gives the following:

$$\ln L = -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma^2) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{e_{it}}{\sigma}\right)^2. \tag{3.5}$$

With $e_{it} = y_{it} - \mathbf{b}^T\mathbf{x}_{it}$, (3.5) can be written as follows:

$$\ln L = -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma^2) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{y_{it} - \mathbf{b}^T\mathbf{x}_{it}}{\sigma}\right)^2 \tag{3.6}$$

Here, the value of $\mathbf{b} = (\alpha, \boldsymbol{\beta})^T$ that maximizes (3.6) can be obtained by differentiating w.r.t. $\mathbf{b}$,

$$\frac{\partial \ln L}{\partial \mathbf{b}} = -\frac{1}{2}2\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{y_{it} - \mathbf{b}^T\mathbf{x}_{it}}{\sigma}\right) - \mathbf{x}_{it};$$

and thus solving

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{y_{it} - \mathbf{b}^T\mathbf{x}_{it}}{\sigma}\right)\mathbf{x}_{it} = 0$$ [29],

this yields $\hat{\mathbf{b}}$ as follows:

$$\hat{\mathbf{b}} = (\hat{\alpha}, \hat{\boldsymbol{\beta}})^T = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{x}_{it}y_{it}}{\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{x}_{it}^2} \tag{3.7}$$

In matrix notation, (3.7) is equivalent to the following:

$$\hat{\mathbf{b}} = (\hat{\alpha}, \hat{\boldsymbol{\beta}})^T = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} \tag{3.8}$$

[29] Under assumption $e_{it} \sim iidN(0,1)$, we have $\sigma = 1$.

$$\text{where } \underset{NT\times(k+1)}{\mathbf{X}} = \begin{bmatrix} 1 & x_{111} & x_{211} & \cdots & x_{k11} \\ 1 & x_{112} & x_{212} & \cdots & x_{k12} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{11T} & x_{21T} & \cdots & x_{k1T} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{1N1} & x_{2N1} & \cdots & x_{kN1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{1NT} & x_{2NT} & \cdots & x_{kNT} \end{bmatrix}, \quad \underset{NT\times 1}{\mathbf{y}} = \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1T} \\ \vdots \\ y_{N1} \\ \vdots \\ y_{NT} \end{bmatrix}$$

Under assumption **A3.1**, this MLE coincides with OLS. Specifically, the OLS minimizes the sum of the squared residuals of model (3.1). If $\mathbf{X}$ is full rank, the asymptotic variance for $\hat{\mathbf{b}}$ is

$$Var(\hat{\mathbf{b}}) = \hat{\sigma}^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \quad \text{where} \quad \hat{\sigma}^2 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^2}{NT-k-1}. \tag{3.9}$$

where k is the number of parameters in the model. Thus, the asymptotic distribution of the pooled estimates, $\hat{\mathbf{b}}$ is:

$$\left(\hat{\mathbf{b}}-\mathbf{b}\right) \sim N\left(0, \sqrt{Var\left(\hat{\mathbf{b}}\right)}\right)$$

where $Var(\hat{\mathbf{b}})$ is given in (3.9).

OLS is widely used as a statistical tool in econometric analysis. Despite its powerful properties, efficiency and accuracy under the standard assumptions of pooled model (**A3.1-A3.5**), this method lacks robustness. The OLS estimator of pooled model is consistent but is inefficient estimates in the presence of CD. Notice that, in the presence of CD, the Assumption **A3.5** is violated and hence the estimates of $\mathbf{b}$ takes the form, $\hat{\mathbf{b}} = \left(\hat{\alpha}, \hat{\boldsymbol{\beta}}\right) = \left(\mathbf{X}^T\boldsymbol{\Omega}\mathbf{X}\right)^{-1}\mathbf{X}^T\boldsymbol{\Omega}\mathbf{y}$, with non-zero off diagonal of $\boldsymbol{\Omega}$. Since the estimated scale of $\hat{\mathbf{b}}$ is based on the square residuals (refer (3.9)), the presence of outliers influence the estimates resulting in bias estimates and increase in variance. Thus, the pooled model is inappropriate for computing the parameter of interest when contemporaneous correlated errors (CD) and outliers exist in the data.

3.2.2 Common Correlated Effects Mean Group (CMG)

In order to correct for cross dependency, Pesaran (2006) introduced two approaches of the common correlated effects in modeling the panel. The first approach namely, the Common Correlated Effects Mean Group Estimators (CMG) is commonly used in many applications such as by Coakley et al. (2006) and Noman (2008). In the second approach, Pesaran proposed the pooled version of the common correlated effects where the parameters of interest are assumed constant for all cross-sectional units.

Referring to Equations (2.2) and (2.4), model (3.1) will take the following form in the presence of the CD:

$$y_{it} = \alpha_i + \boldsymbol{\beta}_i^T \mathbf{x}_{it} + \gamma_i^T f_t + \varepsilon_{it} \tag{3.10}$$

where y_{it} denotes the observation on the i^{th} cross section unit at time t for $i = 1,2,...,N;\ t = 1,2,...,T,\ \ \mathbf{x}_{it}$ is a $k \times 1$ vector of independent variable on the i^{th} cross section unit at time t . Here, α_i, $\boldsymbol{\beta}_i$ are parameters which differ across i, while f_t is the unobserved factor, γ_i are the factor loadings which are common across i, and ε_{it} represents random error.

In order to eliminate the effect of cross dependency, Pesaran used the cross section averages of the dependent variable ($\bar{y}_t$) and observed regressor ($\bar{\mathbf{x}}_t$), to explain the unobserved factor, f_t. This f_t is derived as follows:

Taking the cross section averages of both the sides of (3.10), the following is obtained:

$$\frac{\sum_{i=1}^{N} y_{it}}{N} = \frac{\sum_{i=1}^{N} \alpha_i}{N} + \frac{\sum_{i=1}^{N} \boldsymbol{\beta}_i^T \mathbf{x}_{it}}{N} + \frac{\sum_{i=1}^{N} \gamma_i^T f_t}{N} + \frac{\sum_{i=1}^{N} \varepsilon_{it}}{N} \tag{3.11}$$

It can be rewritten as:

$$\bar{y}_t = \bar{\alpha} + \boldsymbol{\beta}_i^T \bar{\mathbf{x}}_t + \bar{\gamma}^T f_t + \bar{\varepsilon}_t \tag{3.12}$$

where $\bar{y}_t = \dfrac{\sum_{i=1}^{N} y_{it}}{N}, \bar{\mathbf{x}}_t = \dfrac{\sum_{i=1}^{N} \mathbf{x}_{it}}{N}, \bar{\alpha} = \dfrac{\sum_{i=1}^{N} \alpha_i}{N}, \bar{\gamma} = \dfrac{\sum_{i=1}^{N} \gamma_i}{N}, \bar{\varepsilon}_t = \dfrac{\sum_{i=1}^{N} \varepsilon_{it}}{N}$

Taking $\bar{\gamma}^T f_t$ to the left hand side, yields:

$$\bar{\gamma}^T f_t = \bar{y}_t - \bar{\alpha} - \boldsymbol{\beta}_i^T \bar{\mathbf{x}}_t - \bar{\varepsilon}_t , \tag{3.13}$$

Multiplying both sides with $\left(\bar{\gamma}\bar{\gamma}^T\right)^{-1}\bar{\gamma}$, (3.13) becomes:

$$f_t = \left(\bar{\gamma}\bar{\gamma}^T\right)^{-1}\bar{\gamma}\left(\bar{y}_t - \bar{\alpha} - \boldsymbol{\beta}^T \bar{\mathbf{x}}_t - \bar{\varepsilon}_t\right) \tag{3.14}$$

Using the assumptions in Pesaran (2006), the followings are considered:

B3.1: Unobserved factor, f_t and factor loadings, γ_i is iid for all i and t.

B3.2: ε_{it} is iid for all i and t, serially uncorrelated with mean zero and a finite variance, $\sigma_i^2 < k$ (number of regressor), and a finite fourth-order cumulant.

B3.3: $\|\boldsymbol{\beta}_i\| < k$

The assumption of **B3.2** implies the asymptotic distribution of $\bar{\varepsilon}_t$, that is:

$$\bar{\varepsilon}_t \sim N\left(0, \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2\right), \tag{3.15}$$

and in terms of convergence rate, the variance of $\bar{\varepsilon}_t$ can be rewritten as:

$$Var\left(\bar{\varepsilon}_{wt}\right) = \mathrm{O}\left(\sum_{i=1}^{N} w_i^2\right) = \mathrm{O}\left(\frac{1}{N}\right) \tag{3.16}$$

where w_i is the weight that satisfies conditions (Pesaran, 2006) as in the following:

(i) $w_i = \mathrm{O}\left(\frac{1}{N}\right)$ (ii) $\sum_{i=1}^{N} w_i = |k|$[30]

As shown in Pesaran (2006), $\bar{\varepsilon}_t$ converges as[31]:

$$\bar{\varepsilon}_t \xrightarrow{qm} 0 \text{ as } N \to \infty \text{ , for each } t,$$

and under assumption **B3.1** that is γ is iid for all i and t, yields:

$$\bar{\gamma} \xrightarrow{p} \gamma \text{ as } N \to \infty \quad . \tag{3.17}$$

[30] k is the number of specific regressor which is assumed known, m is the number of unobserved factor which is assumed to be unknown but both values are fixed.
[31] The detail of this proof has been shown in Pesaran (2006).

Suppose the rank of $\gamma = m$, using the result in (3.17), the following[32] is obtained:

$$f_t - \left(\gamma\gamma^T\right)^{-1}\gamma\left(\bar{y}_t - \bar{\alpha} - \boldsymbol{\beta}_i^T\bar{\mathbf{x}}_t\right) \xrightarrow{p} 0 \text{ as } N \rightarrow \infty \tag{3.18}$$

where $\xrightarrow{qm}$, $\xrightarrow{p}$ denotes as convergence in quadratic mean and convergence in probability, respectively.

This suggests that using $\overline{\mathbf{H}} = \left(1, \bar{\mathbf{x}}_t, \bar{y}_t\right)$, f_t and α_i can be eliminated. Averaging across the cross section may reduce the effects of CD among the residuals. Here, $\boldsymbol{\beta}_i$ can be estimated consistently by augmenting the dependent variable; y_{it} on the observed regressor; $\mathbf{x}_{it}$ with vector of ones and cross section averages of respective dependent and observed variable $\bar{y}_t, \bar{\mathbf{x}}_t$; using OLS.

The individual parameter estimate of the common correlated effects in (3.10) is given by:

$$\hat{\boldsymbol{\beta}}_i = (\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i)^{-1}\mathbf{X}_i^T\mathbf{M}\mathbf{y}_i \tag{3.19}$$

where $\mathbf{M} = \mathbf{I}_t - \overline{\mathbf{H}}\left(\overline{\mathbf{H}}^T\overline{\mathbf{H}}\right)^{-1}\overline{\mathbf{H}}^T$ and $\mathbf{I}_t$ is a unit matrix of order $T \times T$. To compute factor loading γ , $\overline{\mathbf{H}} = (\mathbf{1}, \overline{\mathbf{X}}_t, \bar{y}_t)$ is used, where $\overline{\mathbf{H}}$ is a combination of vector of ones, average of independent variables $\left(\bar{\mathbf{x}}_t\right)$ and dependent variables $\left(\bar{y}_t\right)$ and it is given as follows:

$$\overline{\mathbf{H}} = \begin{bmatrix} 1 & \bar{\mathbf{x}}_1 & \bar{y}_1 \\ 1 & \bar{\mathbf{x}}_2 & \bar{y}_2 \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \bar{\mathbf{x}}_T & \bar{y}_T \end{bmatrix}. \tag{3.20}$$

[32] This result is obtained as in Pesaran (2006)

Then, the CMG estimates, $\hat{\boldsymbol{\beta}}_{CMG}$ is the average of parameter estimates $\hat{\boldsymbol{\beta}}_i$ that is

$$\hat{\boldsymbol{\beta}}_{CMG} = \frac{\sum_{i=1}^{N} \hat{\boldsymbol{\beta}}_i}{N}.$$

Following Pesaran (2006), the asymptotic distribution of CMG is

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_{CMG} - \boldsymbol{\beta}\right) \xrightarrow{d} N\left(0, \hat{\boldsymbol{\Sigma}}_{CMG}\right) \text{as } (N,T) \to \infty$$

with $\xrightarrow{d}$ denotes convergence in distribution; and

$$\hat{\boldsymbol{\Sigma}}_{CMG} = \frac{\sum_{i=1}^{N} (\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{\beta}}_{CMG})(\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{\beta}}_{CMG})^T}{N-1}. \tag{3.21}$$

The CMG seems to be robust and stands out for homogeneous as well as heterogeneous slope experiments, and does not seem to depend on whether the rank condition is satisfied (Pesaran, 2006). Coakley et al. (2006) found that the CMG estimator stands out as the most robust in the sense that it is the preferred choice in rather general (non) stationary settings where regressors and errors share common factors and their factor loadings are possibly dependent.

However, this procedure is subjected to the influence of outliers since the OLS is used to model the data. A single influential outlier will automatically pull the fitted line towards it and result in poor parameter estimates. The standard error of CMG estimates is observed as lack robustness of standard deviation due to OLS estimated residuals. Moreover, there is the influence of the outlying observation on the mean and subsequently on $\mathbf{M}$. The aim is limit the influence of outliers and this can be achieved through the use of robust measures. Thus, a robust version of RCMG is proposed in the next subsection.

3.2.3 Robust Estimation Procedure (RCMG)

Following Peter et al. (1982), a general version of the M-estimator criterion for model (3.10) can be written as follows:

$$\min_{\boldsymbol{\beta}_i} \sum_{t=1}^{T} \hat{\sigma}_i u_i(\mathbf{x}_{it}) v_i^2(\mathbf{x}_{it}) \rho_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right) \text{for each } i = 1,2,\ldots,N. \tag{3.22}$$

where $v_i(\mathbf{x}_{it})$ and $u_i(\mathbf{x}_{it})$ are positive functions that are related to the position of the $\mathbf{x}_{it}$ in the $\mathbf{X}$-space. Here, $\rho(\mathrm{t})$ is a differential convex function (with minimum at 0) and is known as the robustifying criterion function while $\hat{\sigma}_i$ is the robust scale defined in (2.17) in Chapter 2. The objective function in (3.22) is introduced with the aim of minimizing a function related to standardized residuals. Therefore, (3.22) is minimized by differentiating ρ_i w.r.t. $\boldsymbol{\beta}_i$;

$$\frac{\partial}{\partial \boldsymbol{\beta}_i} \sum_{t=1}^{T} \hat{\sigma}_i u_i(\mathbf{x}_{it}) v_i^2(\mathbf{x}_{it}) \rho_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right) \quad \text{for each } i = 1,2,\ldots,N;$$

yields the following:

$$\sum_{t=1}^{T} \hat{\sigma}_i u_i(\mathbf{x}_{it}) v_i^2(\mathbf{x}_{it}) \psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_{it}, \text{ where } \psi(t) = \rho'(t). \tag{3.23}$$

Using a weight function given by

$$w(t) = \frac{\psi(t)}{t},$$

and solving (3.23) equals to 0, the M-estimates $\boldsymbol{\beta}_i$ can be computed as follows:

$$\sum_{t=1}^{T} u_i(\mathbf{x}_{it}) w_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\hat{e}_{it}(\boldsymbol{\beta}_i)\mathbf{x}_{it} = 0 \tag{3.24}$$

Several different robust M-estimators can be obtained by substituting the different choices of $u_i(\mathbf{x}_{it}), v_i(\mathbf{x}_{it})$ and $\psi_i(\mathbf{x}_{it})$. The choice will result in how much the data are scarified to achieve certain efficiency at the true underlying model assumption.

In order to provide robustness to the effects of the outlying observation occurring in the $\mathbf{X}$ and $\mathbf{y}$ directions, the high breakdown point estimator is suggested. Let

$$z_{it} = d_i(\mathbf{x}_{it})\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i}, \tag{3.25}$$

and following the work of Hung et. al (2008), a generalization of the M-estimator can be obtained by substituting $u_i(\mathbf{x}_{it}) = 1$ and $v_i(\mathbf{x}_{it}) = \dfrac{1}{d_i(\mathbf{x}_{it})}$ in (3.23), yields the following:

$$\sum_{t=1}^{T} w_i(z_{it})\hat{e}_{it}\mathbf{x}_{it} = 0. \tag{3.26}$$

The $d_i(\mathbf{x}_{it})$ is given as a measure of the outlyingness "$\mathbf{X}$" from its mean value and is defined as follows:

Case I: single regressor variable

$$d(\mathbf{x}_{it}) = \frac{|\mathbf{X}_{it} - \mu_{\mathbf{X}}|}{\sigma_{\mathbf{X}}}$$

where $\mu_{\mathbf{X}}$ is a robust location of $\mathbf{x}_{it}$ (median($\mathbf{x}_{it}$)) and $\sigma_{\mathbf{X}}$ is a robust scale (MAD) and is given by $\sigma_{\mathbf{X}} = 1.4825 \operatorname{median}_t \left|\mathbf{x}_{it} - \operatorname{median}_t(\mathbf{x}_{it})\right|$.

Case II: multiple regressors

$$d(\mathbf{x}_{it}) = \sqrt{(\mathbf{x}_{it} - \mu_{\mathbf{X}})^T \mathbf{V}^{-1}(\mathbf{x}_{it} - \mu_{\mathbf{X}})} \tag{3.27}$$

where $\mu_{\mathbf{X}}$ is a robust location of $\mathbf{x}_{it}$ (median($\mathbf{x}_{it}$)) and $\mathbf{V}$ is a matrix of robust variance covariance matrix of $\mathbf{x}_{it}$ (Minimum Volume Ellipsoid).

Following the procedure to the residuals of Bai and Ng (2002)[33], we have

$$\hat{\mathbf{e}}_i = \mathbf{M}(\mathbf{y}_i - \hat{\boldsymbol{\beta}}_i^T \mathbf{X}_i) \tag{3.28}$$

[33] This can be achieved if m (number of unobserved factors) is fixed.

where $\hat{\mathbf{e}}_i$ is the fitted value of $\mathbf{e}_i$, for $i = 1,2,\ldots,N$ of model (3.10), and $\mathbf{M}$ is computed as:

$$\mathbf{M} = \mathbf{I}_t - \overline{\mathbf{H}}\left(\overline{\mathbf{H}}^T\overline{\mathbf{H}}\right)^{-1}\overline{\mathbf{H}}^T. \qquad (3.29)$$

In order to limit the influence of outliers in $\mathbf{M}$, the robust version given by $\mathbf{M}^* = \mathbf{I}_t - \overline{\mathbf{H}}^*\left(\overline{\mathbf{H}}^{*T}\overline{\mathbf{H}}^*\right)^{-1}\overline{\mathbf{H}}^{*T}$ is introduced; $\mathbf{I}_t$ is an identity T by T matrix with the value of $\overline{\mathbf{H}}^* = \left(1, \psi\left(\overline{\mathbf{X}}_t\right), \psi\left(\overline{\mathbf{y}}_t\right)\right)$. Here $\psi(.)$ are some filter functions for the adjusted value of location for dependent and independent variables. The values of $\psi(.)$ are set as follows:

$$\psi\left(\bar{y}_t\right) = \begin{cases} \bar{y}_t & ;\text{if } \left|\bar{y}_t\right| \le \text{c} \\ sign\left(\bar{y}_t\right) \times \left|\operatorname*{median}_i\left(y_{1t},\ldots\ldots, y_{Nt}\right)\right| & ;\text{elsewhere} \end{cases} \qquad (3.30)$$

where c is the critical value, chosen to achieve specified level of efficiency, and it is computed as $3\hat{\sigma}_{\bar{y}_t}$ with $\hat{\sigma}_{\bar{y}_t}$ is a robust scale given by $\hat{\sigma}_{\bar{y}_t} = 1.4825 \operatorname*{median}_t\left|\bar{y}_t - \operatorname*{median}_t\left(\bar{y}_t\right)\right|$.

Thus, by replacing $\mathbf{M}^*$ in (3.28), and substituting $\hat{\mathbf{e}}_i = \mathbf{M}^*\left(\mathbf{y}_i - \hat{\boldsymbol{\beta}}_i^T\mathbf{X}_i\right)$ into (3.26), yields

$$\mathbf{X}_i^T\mathbf{M}^*\mathbf{W}_i\left(z_{it}\right)\left(\mathbf{y}_i - \mathbf{X}_i\hat{\boldsymbol{\beta}}_i\right) = 0 \qquad (3.31)$$

and final estimates of $\boldsymbol{\beta}_i$ can be obtained via the following:

$$\hat{\boldsymbol{\beta}}_i = (\mathbf{X}_i^T\mathbf{G}_i\mathbf{X}_i)^{-1}\mathbf{X}_i^T\mathbf{G}_i\mathbf{y}_i \qquad (3.32)$$

where $\mathbf{G}_i = \mathbf{M}^*\,\mathbf{W}_i\left(z_{it}\right)$. The final estimates of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}}_{RCMG} = \dfrac{\sum_{i=1}^{N}\hat{\boldsymbol{\beta}}_i}{N}$.

The algorithm of RCMG estimator is given as follows:

(i) Compute the fitted residuals

$$\hat{e}_{it}^{(0)} = y_{it} - \hat{y}_{it}^{(0)} = y_{it} - \mathbf{M}^*\left(\mathrm{y}_i - \mathbf{X}_i\hat{\boldsymbol{\beta}}_i^{(0)}\right); \qquad (3.33)$$

where $\hat{\boldsymbol{\beta}}_i^{(0)}$ is the initial values obtained from LTS[34]-estimates, $\mathbf{M}^* = \mathbf{I}_t - \overline{\mathbf{H}}^* \left(\overline{\mathbf{H}}^{*T} \overline{\mathbf{H}}^* \right)^{-1} \overline{\mathbf{H}}^{*T}$ and $\overline{\mathbf{H}}^* = \left(1, \psi\left(\overline{\mathbf{X}}_t\right), \psi\left(\overline{\mathbf{y}}_t\right)\right)$ with $\psi(.)$ is given in (3.30).

(ii) Compute $z_i^{(0)} = d_i(\mathbf{x}_{it}) \dfrac{\hat{e}_i^{(0)}}{\sigma_e^{(0)}}$ where $\hat{e}_i^{(0)}$ is computed from (3.33) and

$$\sigma_e^{(0)} = 1.4825 \operatorname*{median}_t \left| e_{it}^{(0)} - \operatorname*{median}_t \left(e_{it}^{(0)} \right) \right| \tag{3.34}$$

with $d(\mathbf{x}_{it})$ given in (3.27).

(iii) Compute

$$\mathbf{W}_i^{(0)} = w_i\left(z_i^{(0)}\right) = \frac{\psi_i\left(z_i^{(0)}\right)}{z_i^{(0)}} \tag{3.35}$$

and $\psi(.)$ in (3.35) is given by Huber function

$$\psi_i\left(z_i^{(0)}\right) = \begin{cases} z_i^{(0)} & ; \left| z_i^{(0)} \right| \le \varpi \\ \varpi \operatorname{sign}(z_i^{(0)}) & \text{otherwise} \end{cases} \tag{3.36}$$

ϖ is set to 1.345 at 95% efficiency at normal distribution.

(iv) The value of $\hat{\boldsymbol{\beta}}_i^{(1)}$ is computed as

$$\hat{\boldsymbol{\beta}}_i^{(1)} = \left(\mathbf{X}_i^T \mathbf{G}_i^{(0)} \mathbf{X}_i \right)^{-1} \mathbf{X}_i^T \mathbf{G}_i^{(0)} \mathbf{y}_i \tag{3.37}$$

where $\mathbf{G}_i = \mathbf{M}^* \, \mathbf{W}_i^{(0)}\left(z_{it}\right)$. Thus, for h^{th} iteration in (3.37);

$$\hat{\boldsymbol{\beta}}_i^{(h)} = \left(\mathbf{X}_i^T \mathbf{G}_i^{(h-1)} \mathbf{X}_i \right)^{-1} \mathbf{X}_i^T \mathbf{G}_i^{(h-1)} \mathbf{y}_i$$

(v) Repeat the procedure (i)– (iv) until the value of $\hat{\boldsymbol{\beta}}_i^{(h)}$ converges; that is

$$\left| \frac{\hat{\boldsymbol{\beta}}_i^{(h)} - \hat{\boldsymbol{\beta}}_i^{(h-1)}}{\hat{\boldsymbol{\beta}}_i^{(h-1)}} \right| < Q \text{ and } Q \text{ is some constant value.} \tag{3.38}$$

[34] The objective function of LTS estimates has been defined in Equation (2.16) in Chapter 2.

3.3 Inferences

In this section, the properties of the parameter estimates through its asymptotic distribution are described. The test statistics and confidence interval are constructed based on the asymptotic distribution of the proposed estimation procedure, RCMG. Other studies on inferences of parameter estimates can be found in Kapetonis and Pesaran (2004) and Pesaran (2006).

3.3.1 Asymptotic Properties of RCMG

Preliminaries

(1) Assumptions for ρ_i (filter function); for each $i = 1,2,\ldots,N$

C3.1: $\rho_i\left(\dfrac{\hat{e}_{it}}{\hat{\sigma}_i}\right) \geq 0$

C3.2: $\rho_i(0) = 0$

C3.3: $\rho_i(\infty) = 1$ if ρ_i is bounded.

(2) Law of Large Number

Let $x_1, x_2, \ldots, x_N$ be an independent trial process, with finite expected value $\mu = E(x_j)$ and finite variance $\sigma^2 = V(x_j)$. Let $S_N = X_1 + X_2 + \ldots\ldots + X_N$, then for any $\in > 0$,

$$P\left(\left|\frac{S_N}{N} - \mu\right| \geq \in\right) \to 0 \quad \text{as} \quad N \to \infty$$

Equivalently,

$$P\left(\left|\frac{S_N}{N} - \mu\right| < \in\right) \to 1 \quad \text{as} \quad N \to \infty .$$

(3) Slutsky's Theorem (Maronna et al. (2006))

Let a_N, b_N and x_N be the sequence of random variables, and also let x be a random variable, and a, b are constants. Suppose $a_N \xrightarrow{p} a$, $b_N \xrightarrow{p} b$ converges in probability as $N \to \infty$, and that $x_N \xrightarrow{d} x$ (converges in distribution) as $N \to \infty$, then

$$a_N x_N + b_N \Rightarrow ax + b \text{ as } N \to \infty.$$

The asymptotic distribution of RCMG is given as follows:

Theorem 3.1: Under assumptions **A3.2-A3.4**, **A3.6**, **B3.1, B3.2** and **C3.1-C3.3**,

$$\sqrt{T}\left(\hat{\boldsymbol{\beta}}_i - \boldsymbol{\beta}\right) \xrightarrow{d} N(0, \mathbf{v}_i)$$

as $N \to \infty$ and $T \to \infty$,

where

$$\mathbf{v}_i = \left(\mathbf{X}_i^T \mathbf{M}^* \mathbf{X}_i\right)^{-1} \hat{\sigma}_i^{\,2} \frac{E\left(\psi_i\left(\dfrac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\dfrac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2}$$

for $i = 1, 2, \ldots, N$; where $v_i(\mathbf{x}_{it}) = \dfrac{1}{d_i(\mathbf{x}_{it})}$.

Proof of **Theorem 3.1**.

Suppose the Generalized M-estimates of $\boldsymbol{\beta}_i$ satisfies the following:

$$E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right) = 0. \tag{3.39}$$

Taking the Taylor of expansion of order 1 to (3.39) as a function of $\boldsymbol{\beta}_i$ about $\boldsymbol{\beta}_0$ is

$$\sum_{t=1}^{T} \psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i - \frac{1}{\sigma_i}\sum_{t=1}^{T} \psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T \mathbf{M}^* \mathbf{x}_i\left(\boldsymbol{\beta}_i - \boldsymbol{\beta}_0\right) \tag{3.40}$$

Solving (3.40) equals to 0,

$$0=\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i-\frac{1}{\sigma_i}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i(\boldsymbol{\beta}_i-\boldsymbol{\beta}_0) \tag{3.41}$$

Averaging over t , yields

$$0=\frac{1}{T}\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i-\frac{1}{\hat{\sigma}_i}(\boldsymbol{\beta}_i-\boldsymbol{\beta}_0)\frac{1}{T}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i \tag{3.42}$$

and the following estimate is obtained:

$$(\boldsymbol{\beta}_i-\boldsymbol{\beta}_0)=\frac{\frac{1}{T}\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i}{-\frac{1}{\hat{\sigma}_i}\frac{1}{T}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i} \tag{3.43}$$

multiplying $\sqrt{T}$ to both sides of (3.43), yields

$$\sqrt{T}(\boldsymbol{\beta}_i-\boldsymbol{\beta}_0)=\frac{\sqrt{T}\,\frac{1}{T}\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i}{-\frac{1}{\hat{\sigma}_i}\frac{1}{T}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i} \tag{3.44}$$

The random variable $\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)$ is iid with mean 0 because $E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)=0$.

Therefore, for large T, the CLT (defined in Chapter 2, Section 2.4.3) implies that the distribution of

$$\sqrt{T}\,\frac{1}{T}\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i\xrightarrow{d}N(0,a)\ \text{with}\ a=E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i$$

and the Law of Large Number implies that

$$\left(\frac{1}{T}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i\right)\xrightarrow{p}E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\mathbf{X}_i^T\mathbf{M}^*\mathbf{X}_i\,.$$

Let

$$A_T = \sqrt{T}\,\frac{1}{T}\sum_{t=1}^{T}\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i,$$

$$B_T = \left(-\frac{1}{\hat{\sigma}_i}\left(\frac{1}{T}\sum_{t=1}^{T}\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i^T\mathbf{M}^*\mathbf{x}_i\right)\right),$$

Let $A = \sqrt{T}E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\mathbf{x}_i\right)$, and $B = \frac{1}{-\hat{\sigma}_i}E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\mathbf{X}_i^T\mathbf{M}^*\mathbf{X}_i$

Then by Slutsky's Lemma, $\frac{A_T}{B_T}\xrightarrow{d}\frac{A}{B}$ for large T. Under CLT and Law of large numbers, the following is obtained:

$$\frac{A}{B}\xrightarrow{d}N\left(0,\frac{a}{B^2}\right)$$

as $N\to\infty$ and $T\to\infty$, as stated.

End of proof of **Theorem 3.1.**

Thus, by definition of RCMG estimates, the following is obtained:

$$\sqrt{T}\left(\hat{\boldsymbol{\beta}}_{RCMG}-\boldsymbol{\beta}\right)\xrightarrow{d}N\left(0,\hat{\Sigma}_{RCMG}\right) \tag{3.45}$$

where N is a number of cross sectional units in the panel and $\hat{\Sigma}_{RCMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ with $\mathbf{v}_i$ given in Theorem 3.1.

From this, the hypothesis test and confidence interval for the parameter estimates can be derived as follows:

Hypothesis Test

Here, we are interested to test whether the slope estimates of the proposed model, $\hat{\boldsymbol{\beta}}_{RCMG}$ is close to the true value, $\boldsymbol{\beta}_0$. Thus, the hypothesis is defined as follows:

Null hypothesis $\quad \mathrm{H}_0 : \hat{\boldsymbol{\beta}}_{RCMG} = \boldsymbol{\beta}_0$

Alternative hypothesis $\quad \mathrm{H}_1 : \hat{\boldsymbol{\beta}}_{RCMG} \neq \boldsymbol{\beta}_0 \qquad (3.46)$

Since $\hat{\boldsymbol{\beta}}_{RCMG}$ is asymptotically normally distributed and variance of $\hat{\boldsymbol{\beta}}_{RCMG}$ is known, the test statistics that can be used for the above hypothesis test is the standard z-test which is as follows:

$$z = \frac{\left(\hat{\boldsymbol{\beta}}_{RCMG} - \boldsymbol{\beta}\right)}{se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right)} \qquad (3.47)$$

where $\hat{\boldsymbol{\beta}}_{RCMG}$ is the parameter estimates obtained from the proposed estimation procedure, and $se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right)$ is the standard error of $\hat{\boldsymbol{\beta}}_{RCMG}$ where $se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right) = \frac{1}{\sqrt{T}}\left(\hat{\Sigma}_{RCMG}\right)^{1/2}$, $\hat{\Sigma}_{RCMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ and $\mathbf{v}_i$ is given in **Theorem 3.1**. H_0 is not rejected if $|z| < z_{0.05/2} = 1.96$ and rejected for $|z| > z_{0.05/2} = 1.96$.

<u>Confidence Interval</u>

For a confidence interval, a $100(1-\alpha)\%$ CI for the slopes estimates $\hat{\boldsymbol{\beta}}_{RCMG}$ is given by:

$$\left[\hat{\boldsymbol{\beta}}_{RCMG} - z_{\alpha/2}se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right) < \boldsymbol{\beta} < \hat{\boldsymbol{\beta}}_{RCMG} + z_{\alpha/2}se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right)\right] \qquad (3.48)$$

where $\hat{\boldsymbol{\beta}}_{RCMG}$ is computed as in (3.37), $se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right) = \frac{1}{\sqrt{T}}\left(\hat{\Sigma}_{RCMG}\right)^{1/2}$, $\hat{\Sigma}_{RCMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ at a α level of significant.

3.3.2 A Simple Measure of Robustness

There are several measures of robustness are used in the literature to establish whether the estimates have good properties (Wilcox, 2012). The most common one is the breakdown point and it is defined as the minimum fraction of outlying data that can cause the estimator to take an arbitrary value. A second measure of robustness is the influence function where is defined as the change in an estimate caused by insertion of outlying data as a function of the distance of the data from the (uncorrupted) estimate.

It is really difficult to find the finite sample breakdown point of RCMG estimator since the sophisticated mathematical techniques are required. Thus, Imon (2011) introduced a very simple rule for finding the breakdown point of estimators. The rule assumes that

- There are outliers only in the Y-direction.
- The numbers of observations are deleted/weighted are known before fitting the model.
- The number of parameters (k) to be estimated.

According to Imon (2011), the Generalized M-estimator is a bounded influence estimator and it is popular among the statistician. The weakness of this estimator is that it possess a breakdown point of $\frac{1}{k}$. Thus, when k is large, the breakdown point is small and may not be larger than the lowest breakdown point of $\frac{1}{N}$ [35] (breakdown point for OLS estimator). Therefore, the performance of proposed estimator is investigated by considering the small number of regressor (k) (refer Monte Carlo experiments in Chapter 4), it is believed that proposed Generalized M-estimator will produce very good results[36].

Finally, we can measure the robustness of the proposed estimator in terms of efficiency. The efficiency is defined as the ratio of the minimum variance in an estimate to the actual variance of a (robust) estimate, with the minimum variance being determined by a target distribution such as the normal distribution (Lindgren, 1993). According to Walpole et al. (2012), any estimator is an efficient estimator among all the estimators of β if they produce a minimum variance. Notice that, the MLE is

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[35] N is the number of sample size.
[36] We will discuss the breakdown properties for a larger number of regressos in our forthcoming paper.

always an efficient estimator under the iid assumption of residuals. In the presence of CD, MLE is no longer an efficient estimator. Therefore, the efficiency of the proposed estimator can be investigated dividing the $Var(\hat{\boldsymbol{\beta}}_{RCMG})$ with $Var(\hat{\boldsymbol{\beta}}_{CMG})$. If the value is less than 1, we say that RCMG is efficient estimator than CMG, or vice versa. Consider[37]

$Var(\hat{\boldsymbol{\beta}}_{Pool}) = \hat{\sigma}^2\left(\mathbf{X}^T\mathbf{M}\mathbf{X}\right)^{-1}; Var(\hat{\boldsymbol{\beta}}_{CMG}) = \frac{1}{N^2}\sum_{i=1}^{N}\left(\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}$ and

$$Var(\hat{\boldsymbol{\beta}}_{RCMG}) = \frac{1}{TN^2}\sum_{i=1}^{N}\left(\mathbf{X}_i^T\mathbf{M}^*\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}\,\frac{E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2}.$$

For an illustration, we investigate $\frac{Var(\hat{\boldsymbol{\beta}}_{RCMG})}{Var(\hat{\boldsymbol{\beta}}_{CMG})}$. We have

$$\frac{Var(\hat{\boldsymbol{\beta}}_{RCMG})}{Var(\hat{\boldsymbol{\beta}}_{CMG})} = \frac{\frac{1}{TN^2}\sum_{i=1}^{N}\left(\mathbf{X}_i^T\mathbf{M}^*\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}\,\frac{E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2}}{\frac{1}{N^2}\sum_{i=1}^{N}\left(\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}}.$$

It is observed that $\frac{1}{N^2}\sum_{i=1}^{N}\left(\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}$ can be cancelled out if $\mathbf{M}^* = \mathbf{M}$ (it is true for uncontaminated panels), and yields

$$\frac{Var(\hat{\boldsymbol{\beta}}_{RCMG})}{Var(\hat{\boldsymbol{\beta}}_{CMG})} = \frac{\frac{1}{T}\sum_{i=1}^{N}\frac{E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2}}{1}.$$

---

[37] Details of $Var(\hat{\boldsymbol{\beta}}_{RCMG})$ is $\frac{\hat{\Sigma}_{RCMG}}{T}$ where $\hat{\Sigma}_{RCMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ with $\mathbf{v}_i$ is given in **Theorem 3.1** is $\mathbf{v}_i = \left(\mathbf{X}_i^T\mathbf{M}^*\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}\,\frac{E\left(\psi_i\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\frac{\hat{e}_{it}(\boldsymbol{\beta}_i)}{\hat{\sigma}_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2}$ while $Var(\hat{\boldsymbol{\beta}}_{CMG})$ is derived from $\hat{\Sigma}_{CMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ with $\mathbf{v}_i = \left(\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}$.

Notice that, in the presence of CD, it is believed that $\hat{\boldsymbol{\beta}}_{CMG}$ will produces minimum variance among the estimation procedures (pooled and RCMG) in uncontaminated panel[38]. However, in the presence of outliers, it is believed that RCMG will produces minimum variance since $\mathbf{M}$ in CMG procedure is affected by the influence of outliers.

3.4 Goodness of Fit of the Model

The respective estimation procedure is evaluated using several commonly used measures; that are: (1) coefficient of determination R^2; (2) a robust version of R^2; (3) cross validation criteria CV, and; (4) a robust version of CV. Here, the focus is to (i) assess the goodness-of the fit to the data, and (ii) find the best fitting model. The details of these measures are discussed in the next section.

3.4.1 Coefficient of Determination

The first measure is to use the ratio of the explained variance to the total variance given by:

$$R^2 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(\hat{y}_{it} - \bar{y}\right)^2}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(y_{it} - \bar{y}\right)^2} = 1 - \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(y_{it} - \hat{y}_{it}\right)^2}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(y_{it} - \bar{y}\right)^2} \tag{3.49}$$

Basically, this measure gives the ratio of the "regression sum of squares", $SSR = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\hat{y}_{it} - \bar{y}\right)^2$, to the "total sum of squares", given by $SST = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(y_{it} - \bar{y}\right)^2$. The SST can be decomposed into two components: SSR and the "residuals sum of squares", SSE.

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[38] Refer Table 4.13 in Chapter 4 where when the bias is so small, $\text{RMSE} = \sqrt{\text{var}(\hat{\beta})}$ and this result is proven that CMG is efficient under CD and without outliers.

Here, $SST = SSR + SSE = \sum_{t=1}^{T}\sum_{i=1}^{N}(\hat{y}_{it} - \bar{y})^2 + \sum_{t=1}^{T}\sum_{i=1}^{N}(y_{it} - \hat{y}_{it})^2$. The R^2 is computed as a measure goodness-of-fit, to quantify the percentage of the uncertainty in the data that is explained by the regression model. The R^2 ranges from 0 to 1. If the value of the R^2 is close to 0, this indicates a poor fit. However, if the fitted model is a good fit, the value of the large R^2 is obtained, and a perfect model if $R^2 = 1$.

In the presence of an influential outlier, the SSE components will be larger due to a poor fit model, thus resulting in a small R^2. To limit the influence of such outlier on the resulting model, a robust version of R^2 is employed on which it is based on the absolute values of the fitted residuals (Croux and Dehon, 2003) and yields;

$$RR^2 = 1 - \left(\frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\left|y_{it} - \hat{y}_{it}\right|}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left|y_{it} - \underset{it}{\text{median}}(y_{it})\right|} \right)^2 \tag{3.50}$$

where $SST = \sum_{t=1}^{T}\sum_{i=1}^{N}(y_{it} - \bar{y})^2$ is replaced in (3.50) by $\sum_{t=1}^{T}\sum_{i=1}^{N}\left|y_{it} - \underset{it}{\text{median}}(y_{it})\right|$.

3.4.2 Cross Validation (CV) Criteria

To find the 'best' model, the cross validation techniques of Herwatz and Xu (2006) are employed. The CV techniques are widely used in applied non- and semi-parametric modelling. The details of the techniques are discussed as follows:

For each cross sectional units i,

(1) CV based on averages of absolute value of residuals:

$$\text{CV}_i = \frac{\sum_{t=1}^{T}\left|y_{it} - \hat{y}_{(-i)t}\right|}{T}; \tag{3.51}$$

(2) CV based on squared residuals:

$$\mathrm{C}V_i^2 = \frac{\sum_{t=1}^{T}\left(y_{it} - \hat{y}_{(-i)t}\right)^2}{T}. \tag{3.52}$$

Here $\hat{y}_{(-i)t} = \hat{\mathbf{b}}_{(-i)}^T \mathbf{x}_{(-i)t}$ is computed, where $\hat{\mathbf{b}}_{(-i)}$ the estimated parameter that is obtained is for a particular model after removing the i th pair dependent and independent variables. Morell et al. (2010) stated that although the absolute distances are less affected by outliers than the squared ones, outliers still have an impact on the estimation of $\hat{y}_{(-i)t}$ in (3.51). Thus, Zheng and Yang (1998) introduced the median of the CV (here denoted as RCV) as follows:

$$RCV_i = \mathrm{median}\left\{\left|y_{i1} - \hat{y}_{(-i)1}\right|, \left|y_{i2} - \hat{y}_{i2}\right|, \ldots, \left|y_{iT} - \hat{y}_{(-i)T}\right|\right\} \tag{3.53}$$

A possible disadvantage of RCV is that substantial information may be lost since only the median of the residual is used. An alternative to RCV is to trim a proportion λ of the residuals square which is particularly large in the sample, thus yielding the following:

$$RCV_i^2 = \frac{1}{[\lambda T]} \sum_{t=1}^{[\lambda T]} \left(y_{(-i)t} - \hat{y}_{(-i)t}\right)^2 \tag{3.54}$$

where $\hat{y}_{(-i)t} = \hat{\mathbf{b}}_{(-i)}^T \mathbf{x}_{(-i)t}$ and $\hat{\mathbf{b}}_{(-i)}$ is the estimated parameter after removing λT pairs of dependent and independent variables.

By comparing CV_i, CV_i^2, RCV_i and RCV_i^2 values for each unit i, the model that corresponds to the smallest value among these measures, will be selected as the best fitting model.

3.5 Discussion

In this chapter, an alternative approach of the CMG is proposed in order to limit the influence of outliers and leverage observations in the panel model. The Generalized-

M estimator is introduced in the model together with the modification of the variance covariance matrix which results in the RCMG procedure. The RR^2, a robust version of R^2 and RCV, a robust version of CV as the measures to evaluate the goodness-of-fit of the fitted model are discussed. The behaviour of RCMG is asymptotically normally distributed and as such the test statistics of the hypothesis testing of the parameter estimates can be derived together with the construction of the CI. Therefore, the performance of the proposed test is investigated in the next Chapter in terms of parameter estimates, size, power and also CI.

CHAPTER 4

Finite Sample Behaviour of RCMG: A Monte Carlo Simulation Study

4.1 Introduction

Several simulation studies are conducted with the aim of studying the behaviour of small sample properties of the proposed estimator in the presence of cross correlated errors in the panel. Coakley et al. (2006) designed a Monte Carlo simulation study in order to compare the properties of the estimators in several settings with cross section dependent errors. The errors are set to be either I(0) or I(1) processes[39]. Among the approaches used for the comparatives purposes, include: Pooled, Individual fixed effects, Two-way fixed effects, Fixed effects with principal components, Mean group (MG), Seemingly unrelated (SUR) mean group, Demeaned mean group, Mean group with principal components, CMG, and Between or cross section[40]. This study focuses on the analysis of the estimator through summary statistics: sample mean, sample standard deviation, standard error and bias of the parameter estimates.

Pesaran (2006) studied the small sample properties of CMG and common correlated error pooled estimators in terms of size and power of the test by means of the Monte Carlo simulation. The MG and pooled models are also considered which include the unobserved factor f_t in the regression of y_{it}, and the "naive" estimators that exclude those unobserved factors. He computed bias and root mean square errors of the parameter estimates for the cases of homogeneous and heterogeneous slope in the panels. Kapetonis et al. (2006) used a similar approach to conduct the simulation experiments. They however considered another alternative approach which used a principal component (PC) as discussed in Kapetonis and Pesaran (2004). The

[39] I(0) is a stationary process and I(1) is integrated of lagged dependent variable of order 1 (non-stationary).
[40] This estimator is defined in Appendix B.

alternatives are the pooled and mean group versions of this PC estimator. Other related studies can be found in Philips and Sul (2003) and Coakley et al. (2002).

In this chapter, the finite sample behaviour of the proposed estimator is illustrated using a Monte Carlo simulation study as discussed in Chapter 3. The performance of the estimators are illustrated using several experiments: the first two experiments (given in Subsections 4.2.1.1 and 4.2.1.2) are summarised by the sample mean, standard deviation, standard error and bias of the parameter estimates over replications; in Section 4.3, the size and power of the estimator for the hypothesis test are examined; and in the last section, the CI for parameter estimates are obtained.

4.2 Performance Study

In this section, the performances of the estimators discussed in Chapter 3 are measured (see Pesaran, 2006; Kapetonis et al., 2006; and Coakley et al., 2006). By limiting the analysis to estimation issues, the performances of the estimator are measured based on (see Coakley et al., 2006) the following:

(i) Sample mean (Mean), $\overline{\beta}$ of the slope estimates, $\hat{\beta}$.

$$\overline{\beta} = \frac{\sum_{g=1}^{nsimul} \hat{\beta}_g}{nsimul} \tag{4.1}$$

where $\hat{\beta}_g$ is the parameter estimates obtained for $g = 1, 2, \ldots, nsimul$; with *nsimul* as the number of simulations.

(ii) Sample standard deviation, $SD(\beta)$ of the parameter estimates $\hat{\beta}$, and;

$$SD(\beta) = \sqrt{\frac{\sum_{g=1}^{nsimul} \left(\hat{\beta}_g - \overline{\beta}\right)^2}{nsimul}} \tag{4.2}$$

where $\hat{\beta}_g$ and *nsimul* are in (4.1).

(iii) Bias of the parameter estimates, $\hat{\beta}$

$$Bias(\beta) = E(\hat{\beta} - \beta) \tag{4.3}$$

where bias is the expectation of the difference between an estimator and the true value of the parameter being estimated.

(iv) Mean squared error, $MSE(\beta)$ values of the parameter estimates, $\hat{\beta}$.

$$MSE(\beta) = Var(\beta) + Bias(\beta)^2 \tag{4.4}$$

where $Var(\beta) = SD(\beta)^2$, $SD(\beta)$ and $Bias(\beta)$ are given in (4.2) and (4.3), respectively. where $SD(\beta)$ and $Bias(\beta)$ are given in (4.2) and (4.3), respectively. These measures can be used to gauge the bias and variance of the estimator and the reliability of the conventional standard errors.

4.2.1 Design of Experiment

The purpose of this section is to compare the small sample properties of CMG and RCMG with error cross section dependence. Here, two types of experiments are run with different settings among the regressor (x_{it}), unobserved factor (f_t), and factor loadings (γ_i) in the DGP process under the following conditions: (i) the degrees of cross sectional dependence; (ii) the percentage of contaminations and leverage points; and (iii) the type of contaminations. The DGP of Experiment 1 and Experiment 2 are briefly discussed in Subsections 4.2.1.1 and 4.2.1.2, respectively. In Experiment 1, the common factor only affect the error while in Experiment 2, the common factors affect both the errors and regressors.

4.2.1.1 Experiment 1

Using the same design as in Pesaran (2006) and Kapetonis et al. (2006), the DGP for panel model (similar as in Section 2.5) is given as follows:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it};$$

$$\text{and } e_{it} = \gamma_i^T f_t + \sigma_{it}\varepsilon_{it}; \quad \text{for } i = 1,2,\ldots,N \text{ and } t = -49,\ldots,0,1,2,\ldots,T$$

with $\alpha_i \sim iidU(-0.5,0.5)$; $\sigma_{it} = 1$; $\beta_i = \beta = 1$;

$x_{it} \sim iidN(0,1)$ $\varepsilon_{it} \sim iidN(0,1)$; $f_t \sim iidN(0,1)$.

In this experiment, only f_t (unobserved factor) drives the errors and the degree of cross sectional dependence (γ_i) takes the following value as in (2.32);

(i) $\gamma_i = 0$ for no cross dependency;

(ii) $\gamma_i \sim iidU(0.1,0.3)$ for mild cross dependency and ;

(iii) $\gamma_i \sim iidU(0.5,1.5)$ for strong effect of cross dependency;

with outliers defined as in (2.33)

$$e_{it} = \begin{cases} e_{it} & \text{for } t \neq \tau_i \\ e_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N.$$

In the presence of LP, m_{it} at time $t = \tau_j$, the following x_{it} is set as:

$$x_{it} = \begin{cases} x_{it} & \text{for } t \neq \tau_j \\ x_{it} + m_{it} & \text{for } t = \tau_j \end{cases} \quad \text{for } i = 1,2,\ldots,N.$$

The contaminations[41] (m_{it}) for e_{it} and x_{it} are derived from χ^2_{30}, $N(4,4)$, $N(0,4)$, $LN(1,2)$, Cauchy(0,16).

4.2.1.2 Experiment 2

In this study, the Monte Carlo design (with reference to Coakley et al., 2006) which allows for one or two common factors, f_{lt} is given as:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}; \quad \text{for } i = 1,2,\ldots,N \text{ and } t = -49,\ldots,0,1,2,\ldots,T. \tag{4.4}$$

$$e_{it} = \gamma_{li}^T f_{lt} + \varepsilon_{e,it} \quad , \varepsilon_{e,it} \sim iidN(0,1); \tag{4.5}$$

[41] The performances of the estimators are measured under various types of outliers.

$$x_{it} = \phi_{li}^T f_{lt} + \psi_i \chi_t + \varepsilon_{x,it}, \varepsilon_{x,it} \sim iidN(0, \sigma_{xi}^2); \sigma_{xi} \sim iidU[0.5, 1.5] \qquad (4.6)$$

The unobserved factors, f_{lt} and χ_t are set as:

$$f_{lt} \sim iidN(0,1); \qquad l = 1,2 \text{ and } \chi_t \sim iidN(0,1)$$

The parameters of unobserved factors are generated as:

$$\psi_i \sim iidU[0.5,1.5]; \quad \gamma_{li} \sim iidU[0.5,1.5], \text{ and } \quad \phi_{li} \sim iidU[0.5,1.5]; l = 1,2. \qquad (4.7)$$

In the presence of outliers, the DGP for the errors, e_{it} is set at time $t = \tau_i$;

$$e_{it} = \begin{cases} e_{it} & \text{for } t \neq \tau_i \\ e_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N;$$

and in the presence of leverage point at time $t = \tau_i$, as follows:

$$x_{it} = \begin{cases} x_{it} & \text{for } t \neq \tau_i \\ x_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N;$$

where $m_{it} \sim \chi_{30}^2$ [42].

Briefly, it is observed that all variance $\sigma_{ei}^2 = \sigma_{fl}^2 = \sigma_\chi^2 = 1$ but the regressor variances differ randomly across units, that is $;\sigma_{xi} \sim iidU[0.5,1.5]$. The specifications are considered here (1) ϕ_i is drawn independently from γ_i for each i ; and (2) dependence is introduced for each i if $\gamma_{li} = \phi_{li}$. In this experiment, the factor loadings (ψ_i, γ_{li}, and ϕ_{li}; $l = 1,2$) in (4.7) are set to $U[0.5,1.5]$ which imply strong CD within residuals in the panel. The details specifications under certain conditions of CD are given in Table (4.1).

For each experiment, we generate heterogeneous slope, $\alpha_i \sim iidU[-0.5,0.5]$ with $\alpha = E(\alpha_i) = 0$. $\beta_i = \beta = 1$ is set to provide a homogeneous slope for all cross sectional units. Here, how close the slope estimates value to the true β using the stated statistical measures will be investigated. Only evaluating the performance of the

[42] Only one type of LP is chosen from Experiment 1 in order to investigate the sample small properties of the procedure when LP drawn from where χ_{30}^2 (asymmetric and larger size of contamination).

estimators based on the slope coefficient will be considered since the concept of slope is highly important in the economics view. This is simply because it is used to measure the rate of change in the dependent variable as the independent variable changes.

Table 4.1 – Settings in DGP for Experiment 2

Case	**Settings**	**Definition**
A	$\psi_i = \gamma_i = \phi_i = 0$	• No cross sectional dependence
B	$\gamma_{2i} = \phi_i = 0$	• Factor f_{1t} and χ_t drives the errors and regressors respectively. (The errors and regressors are independent)
C	$\psi_i = \gamma_{2i} = \phi_{2i} = 0$	• Factor f_{1t} drives both the errors and regressors. (The errors and regressors are dependent)
C’	$\psi_i = \gamma_{2i} = \phi_{2i} = 0$ and $\gamma_{1i} = \phi_{1i}$	• Factor f_{1t} drives both the errors and regressors. • Factor-loading dependence
D	$\gamma_{2i} = \phi_{2i} = 0$	• Factor f_{1t} drives both the errors and regressors. (The errors and regressors are dependent) • Factor χ_t drives the regressors
E	$\psi_i = 0$	• Factor $f_t = (f_{1t,} f_{2t})'$ drives both the errors and regressors (The errors and regressors are dependent)
E’	$\psi_i = 0$ and $\gamma_i = \phi_i$	• Factor $f_t = (f_{1t,} f_{2t})'$ drives both the errors and regressors. (Factor-loading dependence)
F	Original setting in DGP	• Factor $f_t = (f_{1t,} f_{2t})'$ drives both the errors and regressors (The errors and regressors are dependent) • Factor χ_t drives the regressors

In both experiments, 500 replications are performed for $N = 20, T = 100$ to investigate the performance of the estimator under the various degrees and conditions of CD and various types of outliers. The first 50 observations will be removed in order to eliminate the initial effect of random generators.

4.2.2 Results and Discussion

4.2.2.1 Results - Experiment 1

Tables 4.2 to 4.7 provide the results of CMG and RCMG with and without the presence of outliers and leverage point. In Table 4.2, the value of the mean, standard deviation, bias and MSE of $\hat{\beta}$ are reported with 1% contamination present in the panel. Both the CMG and RCMG methods perform well with small MSE of 0.001 under the various degrees of CD in the uncontaminated panel (that is when $e_{it} \sim N(0,1)$). Under the cross sectional independence in the panel, the average of the CMG is comparable to the RCMG in the presence of contaminations such as $\chi^2_{30}, N(4,4)$ and $N(0,4)$. However, the RCMG yields unbiased estimates than CMG in the presence of heavy tailed outliers (drawn from Lognormal and Cauchy distributions) with small MSE in both estimators. These results hold for both cases of mild and strong CD in the panel. Thus, it is observed that CMG is quite robust and not much affected by outliers less than 5%.

As the percentage of outliers increases to 5% (see Table 4.3), the SD of CMG inflate in the presence of outliers drawn from Lognormal and Cauchy distributions and this result worsen for the case of strong CD effect. The RCMG continues to yield smaller MSE compared to CMG and these results hold as we increase the percentage of outliers to 10% (see Table 4.4). Thus, RCMG is robust even in the presence of a strong CD effect in the contaminated panel. As expected, the MSE for CMG in all cases of CD will always be large in the presence of more than 5% outliers drawn from heavy tailed distribution.

Tables 4.5 to 4.7 report the performance results of CMG and RCMG in the presence of outliers in input variable (X): the leverage point. With 1% leverage points in the panel, CMG estimator is more bias than RCMG even with a smaller value of MSE. RCMG performs better in all the CD cases with and without the presence of CD. As the percentage of leverage point is increased to 5% (see Table 4.6), the CMG continues to be bias and this subsequently affects the value of MSE. The RCMG however is slightly bias with smaller value of MSE for all CD cases. In the presence of 10% LP (see Table 4.7), $\hat{\beta}_{CMG}$ is far from the true value of $\beta = 1.0$ and intends to contribute to the huge bias to the estimates. The $\hat{\beta}_{RCMG}$ is slightly bias as the percentage of leverage points increases; however, it still outperforms CMG with a smaller value of MSE. In general, $\hat{\beta}_{RCMG}$ provides stable/consistent estimates in the presence of up to 10% contaminations.

In general, CMG is quite robust in the presence of outliers in the response variable (Y-direction) in most cases apart from when outliers drawn from Lognormal and Cauchy distribution. In the presence of LP (X-directions), the estimates of CMG will be affected and differs significantly from the true estimates as the percentage of LP increases but the RCMG less affected in all cases in the presence of outliers in X and Y directions.

The behaviour of this estimator reported in Tables 4.4 and 4.7 are illustrated via box plots in Figures 4.1 to 4.3 and 4.4 to 4.6 for the case of the contaminated panel in Y and X directions, respectively. In Figures 4.1 to 4.3, the proposed estimator provides a stable estimate of β compared to CMG in the presence of outliers for all cases of CD. In the presence of leverage points, $\hat{\beta}_{RCMG}$ is unbiased while $\hat{\beta}_{CMG}$ is consistently bias.

4.2.2.2 Results - Experiment 2

Tables 4.8 to 4.12 present the results of the simulation study under the various scenarios of the cross section dependence with the strong CD in the DGP setting given in Table 4.1. The summary of the statistics of the estimators based on 500 simulated data is computed: sample mean, sample standard deviation, bias and MSE value with / without the presence of outliers and leverage points (drawn from χ^2_{30}). In the uncontaminated panel (refer Table 4.8), the CMG performed better than the RCMG in all cases. For E’ case, both estimators give a bias estimate of β. This is due to: (1) the factor loadings, γ_i and ϕ_i are dependent, and (2) the unobserved factors f_t derive the regressors and errors, subsequently resulting in the dependency between the regressors and error. Likewise, in the presence of 5% contamination (shown in Table 4.9), the CMG outperforms the RCMG estimator in all cases. Similar results are obtained when the percentage of outliers increases by 10% (refer Table 4.10). Although the RCMG gives us a slightly bias estimate, generally the performance is comparable with small MSE. Since the outliers drawn from χ^2_{30}, it is observed that the result of CMG and RCMG are comparable with the result in Experiment 1 (see column 4 Tables 4.2 to 4.4, for strong CD effect) for cases A and B (no CD and dependency between errors and regressors). The results in Experiment 2 however conflicts the Experiment 1 for other cases (C, C’, D, E, E’ and F) in the presence of outliers. The outliers worsen the conditions of cross dependency between the factor loadings, regressors and errors and therefore results in the bias estimates of RCMG.

The results of $\hat{\beta}$ are reported in Tables 4.11 to 4.12 in the presence of LP. The CMG provides a poor estimate in all cases and it is expected since the results in Experiment 1 are comparable with the result in Experiment 2 (see column 4 Tables 4.2 to 4.4, for strong CD effect). The proposed estimator however yields reasonably good parameter estimates with a small bias and MSE) except in the case of B and D, that is when the unobserved factors, f_t and χ_t drive the errors and regressors to indicate the presence of CD among the errors and regressors. The RCMG estimate is stable even with a larger percentage of LP while the CMG continues to yield a bias estimate. It can be concluded that RCMG is not much affected by the presence of LP while the CMG fails with bias and larger MSE. As expected, RCMG outperform CMG in the presence of LP since the RCMG are developed to limit the effects of outliers especially in X-space because outliers in X-space are more influential.

Table 4.2: The Results of $\hat{\beta}$ with 1% Contamination (Experiment 1)

Estimation Method	**Performance Measure**	e_{it}: $N(0,1)$	$0.99N(0,1)+0.01\chi^2_{30}$	$0.99N(0,1)+0.01N(4,4)$	$0.99N(0,1)+0.01N(0,4)$	$0.99\ N(0,1)+0.01\ LN(1,2)$	$0.99\ N(0,1)+$ 0. 01 Cauchy (0,16)
				$\gamma_i = 0$			
CMG	Mean	0.993	1.000	0.993	0.999	1.005	1.007
	SD	0.025	0.029	0.041	0.022	0.151	0.224
	Bias	-0.007	0.000	-0.007	-0.001	0.005	0.007
	MSE	*0.001*	*0.001*	*0.002*	*0.000*	*0.023*	*0.050*
RCMG	Mean	0.996	1.000	0.992	0.999	1.000	1.000
	SD	0.027	0.027	0.027	0.025	0.027	0.032
	Bias	-0.004	0.000	-0.008	-0.001	0.000	0.000
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	1.000	1.001	0.998	1.037	1.019
	SD	0.025	0.028	0.042	0.026	0.201	0.193
	Bias	-0.002	0.000	0.001	-0.002	0.037	0.019
	MSE	*0.001*	*0.001*	*0.002*	*0.001*	*0.042*	*0.037*
RCMG	Mean	0.997	0.999	1.005	0.997	1.000	1.001
	SD	0.028	0.026	0.028	0.029	0.029	0.033
	Bias	-0.003	-0.001	0.005	-0.003	0.000	0.001
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.000	0.997	1.006	1.000	0.992	0.996
	SD	0.027	0.030	0.043	0.027	0.169	0.258
	Bias	0.000	-0.003	0.006	0.000	-0.008	-0.004
	MSE	*0.001*	*0.001*	*0.002*	*0.001*	*0.029*	*0.067*
RCMG	Mean	1.000	0.996	0.998	1.000	1.007	1.018
	SD	0.029	0.032	0.026	0.029	0.028	0.019
	Bias	0.000	-0.004	-0.002	0.000	0.007	0.018
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.002*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.3: The Results of $\hat{\beta}$ with 5% Contamination (Experiment 1)

Estimation Method	Performance Measure	$N(0,1)$	$0.95N(0,1)+0.05\chi^2_{30}$	$0.95N(0,1)+0.05N(4,4)$	$0.95N(0,1)+0.05N(0,4)$	$0.95\ N(0,1)+0.05\ LN(1,2)$	$0.95\ N(0,1)+$ 0. 05 Cauchy (0,16)
				e_{it}			
				$\gamma_i = 0$			
CMG	Mean	0.993	1.000	1.007	1.000	1.182	1.000
	SD	0.025	0.041	0.089	0.031	0.779	1.155
	Bias	-0.007	0.000	0.007	0.000	0.182	0.000
	MSE	*0.001*	*0.002*	*0.008*	*0.001*	*0.640*	*1.335*
RCMG	Mean	0.996	1.004	1.002	0.999	1.002	1.003
	SD	0.027	0.032	0.029	0.027	0.034	0.049
	Bias	-0.004	0.004	0.002	-0.001	0.002	0.003
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.002*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	0.996	0.982	1.002	1.088	0.933
	SD	0.025	0.034	0.097	0.033	0.725	1.367
	Bias	-0.002	-0.004	-0.018	0.002	0.088	-0.067
	MSE	*0.001*	*0.001*	*0.010*	*0.001*	*0.533*	*1.872*
RCMG	Mean	0.997	0.995	0.994	1.000	1.002	1.002
	SD	0.028	0.027	0.031	0.031	0.047	0.035
	Bias	-0.003	-0.005	-0.006	0.000	0.002	0.002
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.002*	*0.001*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.000	1.001	1.024	1.007	1.003	0.823
	SD	0.027	0.040	0.084	0.032	1.603	1.175
	Bias	0.000	0.001	0.024	0.007	0.003	-0.177
	MSE	*0.001*	*0.002*	*0.008*	*0.001*	*2.568*	*1.412*
RCMG	Mean	1.000	1.000	1.001	1.005	1.002	0.999
	SD	0.029	0.032	0.032	0.031	0.037	0.041
	Bias	0.000	0.000	0.001	0.005	0.002	-0.001
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.002*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.4: The Results of $\hat{\beta}$ with 10% Contamination (Experiment 1)

Estimation Method	**Performance Measure**	e_{it}: $N(0,1)$	$0.90N(0,1)+0.10\chi^2_{30}$	$0.90N(0,1)+0.10N(4,4)$	$0.90N(0,1)+0.10N(0,4)$	$0.90\ N(0,1)+0.10\ LN(1,2)$	$0.90\ N(0,1)+0.10$ Cauchy (0,16)
				$\gamma_i = 0$			
CMG	Mean	0.993	1.002	1.002	0.996	1.063	1.034
	SD	0.025	0.045	0.130	0.041	2.248	2.807
	Bias	-0.007	0.002	0.002	-0.004	0.063	0.034
	MSE	*0.001*	*0.002*	*0.017*	*0.002*	*5.059*	*7.881*
RCMG	Mean	0.996	1.001	1.000	0.999	0.989	0.999
	SD	0.027	0.031	0.038	0.034	0.098	0.060
	Bias	-0.004	0.001	0.000	-0.001	-0.011	-0.001
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.010*	*0.004*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	0.999	1.007	0.991	0.512	0.845
	SD	0.025	0.049	0.123	0.036	4.540	1.960
	Bias	-0.002	-0.001	0.007	-0.009	-0.488	-0.155
	MSE	*0.001*	*0.002*	*0.015*	*0.001*	*20.854*	*3.868*
RCMG	Mean	0.997	1.003	1.000	0.998	0.990	0.989
	SD	0.028	0.034	0.037	0.034	0.049	0.070
	Bias	-0.003	0.003	0.000	-0.002	-0.010	-0.011
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.002*	*0.005*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.000	0.993	0.991	0.998	1.186	1.083
	SD	0.027	0.045	0.120	0.037	2.054	2.092
	Bias	0.000	-0.007	-0.009	-0.002	0.186	0.083
	MSE	*0.001*	*0.002*	*0.014*	*0.001*	*4.252*	*4.385*
RCMG	Mean	1.000	0.992	0.998	1.000	0.979	1.005
	SD	0.029	0.034	0.039	0.031	0.240	0.058
	Bias	0.000	-0.008	-0.002	0.000	-0.021	0.005
	MSE	*0.001*	*0.001*	*0.002*	*0.001*	*0.058*	*0.003*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.5: The Results of $\hat{\beta}$ with 1% Leverage Points (Experiment 1)

Estimation Method	Performance Measure	X_{it}: $N(0,1)$	$0.99N(0,1)+0.01\chi^2_{30}$	$0.99N(0,1)+0.01N(4,4)$	$0.99N(0,1)+0.01N(0,4)$	$0.99\ N(0,1)+0.01\ LN(1,2)$	$0.99\ N(0,1)+$ 0. 01 Cauchy (0,16)
				$\gamma_i = 0$			
CMG	Mean	0.997	0.822	0.598	0.888	0.749	0.535
	SD	0.023	0.058	0.093	0.040	0.094	0.094
	Bias	-0.003	-0.178	-0.402	-0.112	-0.251	-0.465
	MSE	*0.001*	*0.035*	*0.170*	*0.014*	*0.072*	*0.225*
RCMG	Mean	0.996	0.982	0.983	0.986	0.991	0.981
	SD	0.027	0.026	0.028	0.030	0.030	0.027
	Bias	-0.004	-0.018	-0.017	-0.014	-0.009	-0.019
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	0.813	0.614	0.897	0.741	0.537
	SD	0.023	0.052	0.084	0.040	0.083	0.098
	Bias	-0.002	-0.187	-0.386	-0.103	-0.259	-0.463
	MSE	*0.001*	*0.038*	*0.156*	*0.012*	*0.074*	*0.224*
RCMG	Mean	0.998	0.978	0.979	0.985	0.981	0.978
	SD	0.030	0.026	0.024	0.031	0.027	0.029
	Bias	-0.002	-0.022	-0.021	-0.015	-0.019	-0.022
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.002	0.819	0.604	0.887	0.726	0.530
	SD	0.024	0.051	0.096	0.035	0.090	0.100
	Bias	0.002	-0.181	-0.396	-0.113	-0.274	-0.470
	MSE	*0.001*	*0.035*	*0.166*	*0.014*	*0.083*	*0.231*
RCMG	Mean	1.002	0.981	0.973	0.976	0.987	0.978
	SD	0.026	0.029	0.028	0.027	0.023	0.026
	Bias	0.002	-0.019	-0.027	-0.024	-0.013	-0.022
	MSE	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*	*0.001*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$

Table 4.6: The Results of $\hat{\beta}$ with 5% Leverage Points (Experiment 1)

Estimation Method	Performance Measure	X_{it}: $N(0,1)$	$0.95N(0,1)+0.05\chi^2_{30}$	$0.95N(0,1)+0.05N(4,4)$	$0.95N(0,1)+0.05N(0,4)$	$0.95\ N(0,1)+0.05\ LN(1,2)$	$0.95\ N(0,1)+0.05$ Cauchy (0,16)
				$\gamma_i = 0$			
CMG	Mean	0.997	0.439	0.132	0.611	0.259	0.066
	SD	0.023	0.048	0.041	0.047	0.066	0.034
	Bias	-0.003	-0.561	-0.868	-0.389	-0.741	-0.934
	MSE	*0.001*	*0.317*	*0.755*	*0.153*	*0.554*	*0.873*
RCMG	Mean	0.996	0.933	0.949	0.943	0.945	0.948
	SD	0.027	0.031	0.030	0.025	0.027	0.027
	Bias	-0.004	-0.067	-0.051	-0.057	-0.055	-0.052
	MSE	*0.001*	*0.005*	*0.004*	*0.004*	*0.004*	*0.003*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	0.446	0.132	0.605	0.272	0.056
	SD	0.023	0.047	0.035	0.046	0.071	0.026
	Bias	-0.002	-0.554	-0.868	-0.395	-0.728	-0.944
	MSE	*0.001*	*0.309*	*0.755*	*0.158*	*0.536*	*0.892*
RCMG	Mean	0.998	0.930	0.938	0.938	0.940	0.940
	SD	0.030	0.027	0.029	0.027	0.026	0.025
	Bias	-0.002	-0.070	-0.062	-0.062	-0.060	-0.060
	MSE	*0.001*	*0.006*	*0.005*	*0.005*	*0.004*	*0.004*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.002	0.451	0.138	0.620	0.264	0.063
	SD	0.024	0.044	0.039	0.043	0.070	0.036
	Bias	0.002	-0.549	-0.862	-0.380	-0.736	-0.937
	MSE	*0.001*	*0.303*	*0.744*	*0.146*	*0.546*	*0.879*
RCMG	Mean	1.002	0.909	0.921	0.929	0.931	0.925
	SD	0.026	0.035	0.027	0.032	0.024	0.025
	Bias	0.002	-0.091	-0.079	-0.071	-0.069	-0.075
	MSE	*0.001*	*0.009*	*0.007*	*0.006*	*0.005*	*0.006*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.7: The Results of $\hat{\beta}$ with 10% Leverage Points (Experiment 1)

Estimation Method	Performance Measure	X_{it}: $N(0,1)$	$0.90N(0,1)+0.10\chi^2_{30}$	$0.90N(0,1)+0.10N(4,4)$	$0.90N(0,1)+0.10N(0,4)$	$0.90\ N(0,1)+0.10\ LN(1,2)$	$0.90\ N(0,1)+0.10$ Cauchy (0,16)
				$\gamma_i = 0$			
CMG	Mean	0.997	0.272	0.048	0.422	0.088	0.010
	SD	0.023	0.031	0.011	0.036	0.038	0.007
	Bias	-0.003	-0.728	-0.952	-0.578	-0.912	-0.990
	MSE	*0.001*	*0.531*	*0.907*	*0.336*	*0.833*	*0.980*
RCMG	Mean	0.996	0.871	0.915	0.881	0.910	0.928
	SD	0.027	0.031	0.030	0.029	0.027	0.026
	Bias	-0.004	-0.129	-0.085	-0.119	-0.090	-0.072
	MSE	*0.001*	*0.018*	*0.008*	*0.015*	*0.009*	*0.006*
				$\gamma_i \sim iidU(0.1,0.3)$			
CMG	Mean	0.998	0.269	0.051	0.427	0.098	0.010
	SD	0.023	0.029	0.011	0.039	0.036	0.006
	Bias	-0.002	-0.731	-0.949	-0.573	-0.902	-0.990
	MSE	*0.001*	*0.535*	*0.901*	*0.330*	*0.814*	*0.979*
RCMG	Mean	0.998	0.872	0.912	0.880	0.912	0.913
	SD	0.030	0.031	0.027	0.031	0.028	0.027
	Bias	-0.002	-0.128	-0.088	-0.120	-0.088	-0.087
	MSE	*0.001*	*0.017*	*0.008*	*0.015*	*0.008*	*0.008*
				$\gamma_i \sim iidU(0.5,1.5)$			
CMG	Mean	1.002	0.266	0.049	0.418	0.093	0.009
	SD	0.024	0.026	0.012	0.039	0.037	0.005
	Bias	0.002	-0.734	-0.951	-0.582	-0.907	-0.991
	MSE	*0.001*	*0.539*	*0.904*	*0.340*	*0.824*	*0.983*
RCMG	Mean	1.002	0.826	0.873	0.838	0.881	0.887
	SD	0.026	0.036	0.030	0.032	0.030	0.025
	Bias	0.002	-0.174	-0.127	-0.162	-0.119	-0.113
	MSE	*0.001*	*0.032*	*0.017*	*0.027*	*0.015*	*0.013*

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$

Table 4.8: The Results of $\hat{\beta}$ in the Uncontaminated Panel (Experiment 2)

Case	Method	Mean	SD	Bias	MSE
A	**CMG**	1.0004	0.0272	0.0004	0.0007
	RCMG	0.9940	0.0315	-0.0060	0.0010
B	**CMG**	1.0001	0.0284	0.0001	0.0008
	RCMG	0.9801	0.0353	-0.0199	0.0016
C	**CMG**	0.9976	0.0276	-0.0024	0.0008
	RCMG	1.0197	0.0375	0.0197	0.0018
C’	**CMG**	1.0002	0.0269	0.0002	0.0007
	RCMG	1.0208	0.0378	0.0208	0.0019
D	**CMG**	0.9990	0.0249	-0.0010	0.0006
	RCMG	0.9937	0.0330	-0.0063	0.0011
E	**CMG**	0.9974	0.0309	-0.0026	0.0010
	RCMG	1.0241	0.0397	0.0241	0.0022
E’	**CMG**	1.0806	0.0317	0.0806	0.0075
	RCMG	1.1106	0.0403	0.1106	0.0139
F	**CMG**	0.9995	0.0319	-0.0005	0.0010
	RCMG	1.0043	0.0353	0.0043	0.0013

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.9: The Results of $\hat{\beta}$ with 5% Contamination in the Panel (Experiment 2)

Case	Method	Mean	SD	Bias	MSE
A	**CMG**	1.0003	0.0457	0.0003	0.0021
	RCMG	0.9988	0.0643	-0.0012	0.0041
B	**CMG**	0.9996	0.0408	-0.0004	0.0017
	RCMG	0.9961	0.0570	-0.0039	0.0033
C	**CMG**	0.9967	0.0449	-0.0033	0.0020
	RCMG	1.1278	0.0652	0.1278	0.0206
C’	**CMG**	1.0006	0.0446	0.0006	0.0020
	RCMG	1.1378	0.0641	0.1378	0.0231
D	**CMG**	0.9977	0.0463	-0.0023	0.0021
	RCMG	1.1081	0.0677	0.1081	0.0163
E	**CMG**	0.9965	0.0430	-0.0035	0.0019
	RCMG	1.1828	0.0683	0.1828	0.0381
E’	**CMG**	1.0871	0.0458	0.0871	0.0097
	RCMG	1.2722	0.0667	0.2722	0.0785
F	**CMG**	0.9960	0.0451	-0.0040	0.0020
	RCMG	1.1641	0.0754	0.1641	0.0326

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.10: The Results of $\hat{\beta}$ with 10% Contamination in the Panel (Experiment 2)

Case	Method	Mean	SD	Bias	MSE
A	**CMG**	0.9994	0.0563	-0.0006	0.0032
	RCMG	0.9956	0.0766	-0.0044	0.0059
B	**CMG**	0.9995	0.0519	-0.0005	0.0027
	RCMG	0.9976	0.0712	-0.0024	0.0051
C	**CMG**	0.9937	0.0505	-0.0063	0.0026
	RCMG	1.1430	0.0671	0.1430	0.0249
C'	**CMG**	0.9979	0.0501	-0.0021	0.0025
	RCMG	1.1563	0.0680	0.1563	0.0290
D	**CMG**	1.0051	0.0571	0.0051	0.0033
	RCMG	1.1173	0.0818	0.1173	0.0204
E	**CMG**	0.9929	0.0492	-0.0071	0.0025
	RCMG	1.2344	0.0852	0.2344	0.0622
E'	**CMG**	1.0897	0.0552	0.0897	0.0111
	RCMG	1.3111	0.0786	0.3111	0.1029
F	**CMG**	1.0027	0.0554	0.0027	0.0031
	RCMG	1.1641	0.0754	0.1641	0.0326

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

Table 4.11: The Results of $\hat{\beta}$ with 5% Leverage Points in the Panel (Experiment 2)

Case	Method	Mean	SD	Bias	MSE
A	**CMG**	0.4274	0.0513	-0.5726	0.3305
	RCMG	0.9804	0.0634	-0.0196	0.0044
B	**CMG**	0.4292	0.0575	-0.5708	0.3291
	RCMG	0.9336	0.0681	-0.0664	0.0090
C	**CMG**	0.4258	0.0489	-0.5742	0.3321
	RCMG	1.0139	0.0781	0.0139	0.0063
C'	**CMG**	0.4270	0.0489	-0.5730	0.3307
	RCMG	1.0439	0.0919	0.0439	0.0104
D	**CMG**	0.4325	0.0457	-0.5675	0.3242
	RCMG	0.9967	0.0769	-0.0033	0.0059
E	**CMG**	0.4445	0.0487	-0.5555	0.3110
	RCMG	1.0162	0.0968	0.0162	0.0096
E'	**CMG**	0.4788	0.0532	-0.5212	0.2745
	RCMG	1.1265	0.0964	0.1265	0.0253
F	**CMG**	0.4484	0.0477	-0.5516	0.3065
	RCMG	1.0045	0.0903	0.0045	0.0082

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$

Table 4.12: The Results of $\hat{\beta}$ with 10% Leverage Points in the Panel

Case	Method	Mean	SD	Bias	MSE
A	**CMG**	0.2669	0.0351	-0.7331	0.5387
	RCMG	1.0174	0.0389	0.0174	0.0018
B	**CMG**	0.2733	0.0388	-0.7267	0.5296
	RCMG	0.8868	0.0480	-0.1132	0.0151
C	**CMG**	0.2648	0.0332	-0.7352	0.5416
	RCMG	0.9666	0.0648	-0.0334	0.0053
C’	**CMG**	0.2655	0.0330	-0.7345	0.5406
	RCMG	0.9832	0.0644	-0.0168	0.0044
D	**CMG**	0.2725	0.0312	-0.7275	0.5302
	RCMG	0.9029	0.0741	-0.0971	0.0149
E	**CMG**	0.2804	0.0331	-0.7196	0.5190
	RCMG	0.9120	0.0841	-0.0880	0.0148
E’	**CMG**	0.2975	0.0326	-0.7025	0.4945
	RCMG	1.0133	0.0971	0.0133	0.0096
F	**CMG**	0.2852	0.0325	-0.7148	0.5120
	RCMG	0.8281	0.0994	-0.1719	0.0394

Mean and SD are the sample mean and standard deviations of $\hat{\beta}$ over 500 replications respectively. MSE is the mean squared error for $\hat{\beta}$.

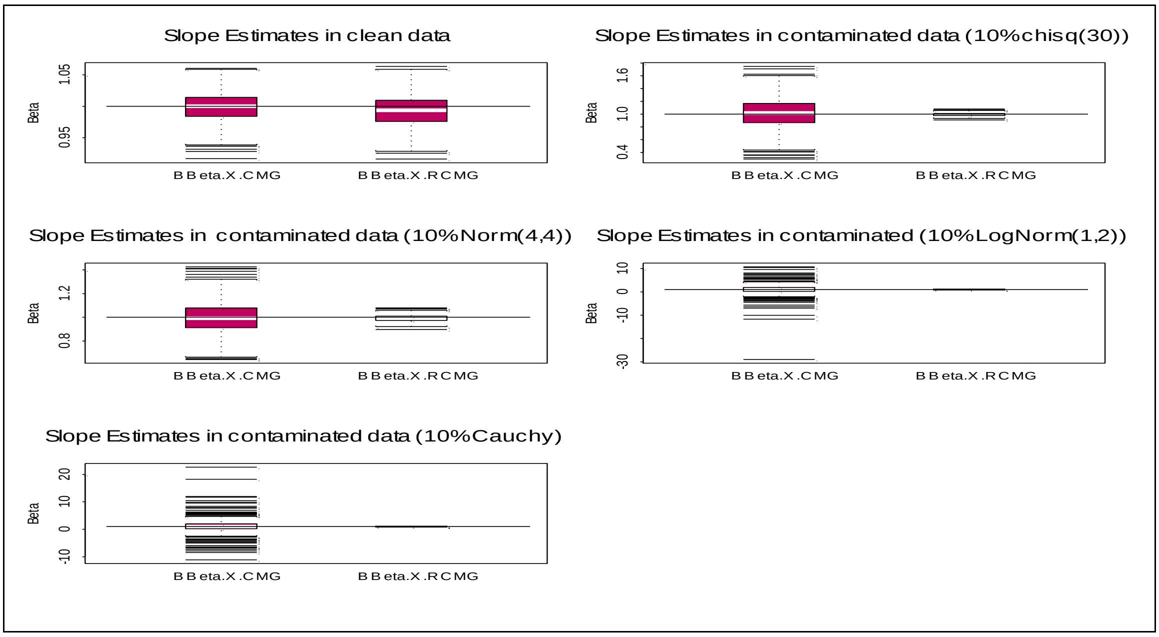

Figure 4.1: Box plots of the Parameter Estimates $\hat{\beta}$
with 10% Contamination ($\gamma_i = 0$)

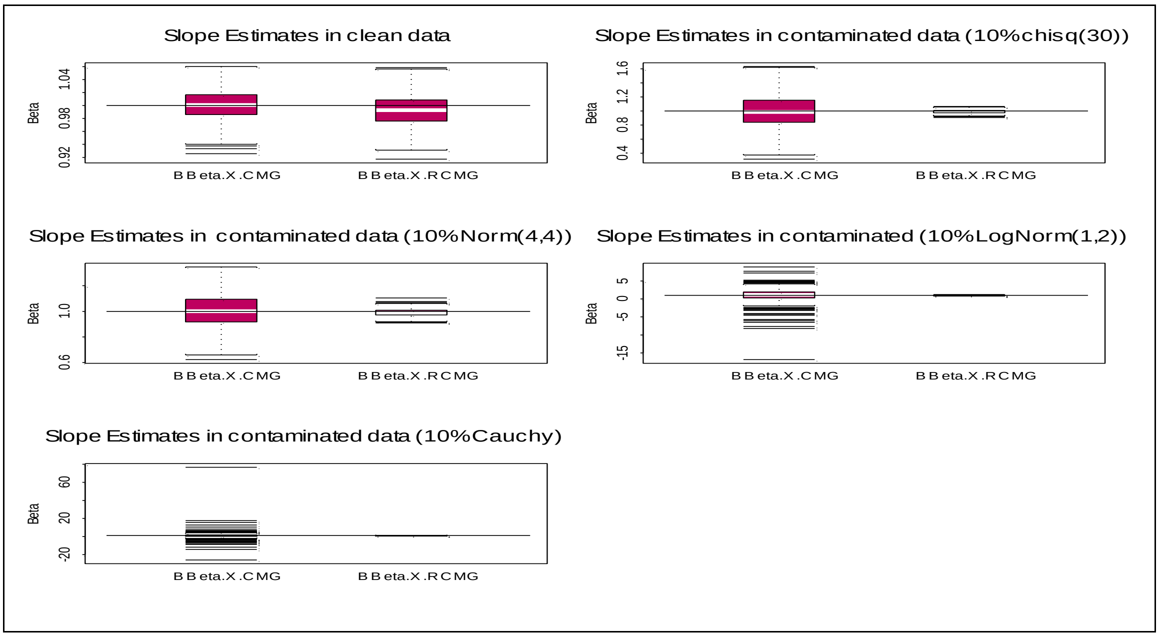

Figure 4.2: Box plots of the Parameter Estimates $\hat{\beta}$
with 10% Contamination ($\gamma_i \sim iidU(0.1, 0.3)$)

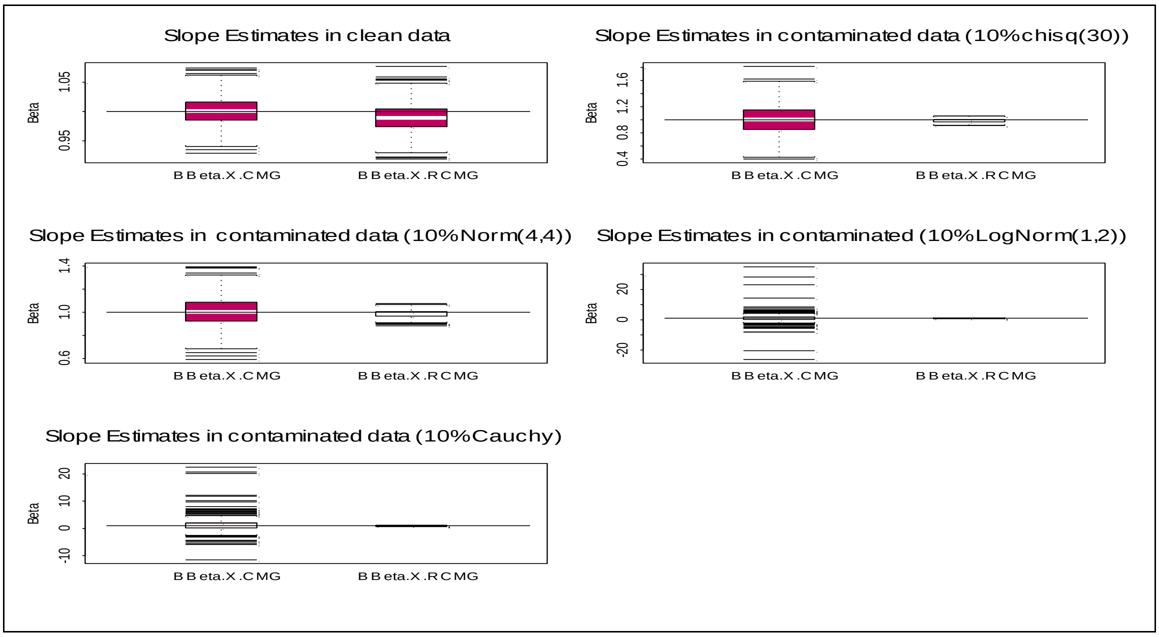

Figure 4.3: Box plots of the Parameter Estimates $\hat{\beta}$
with 10% Contamination ($\gamma_i \sim iidU(0.5,1.5)$)

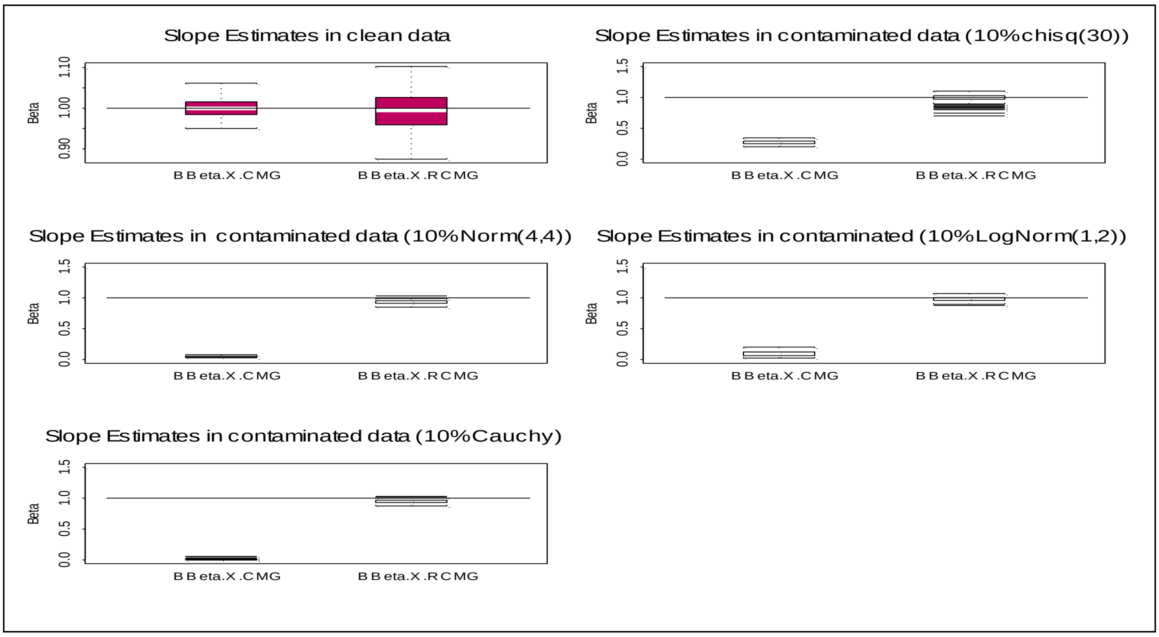

Figure 4.4: Box plots of the Parameter Estimates $\hat{\beta}$
with 10% Leverage Points ($\gamma_i = 0$)

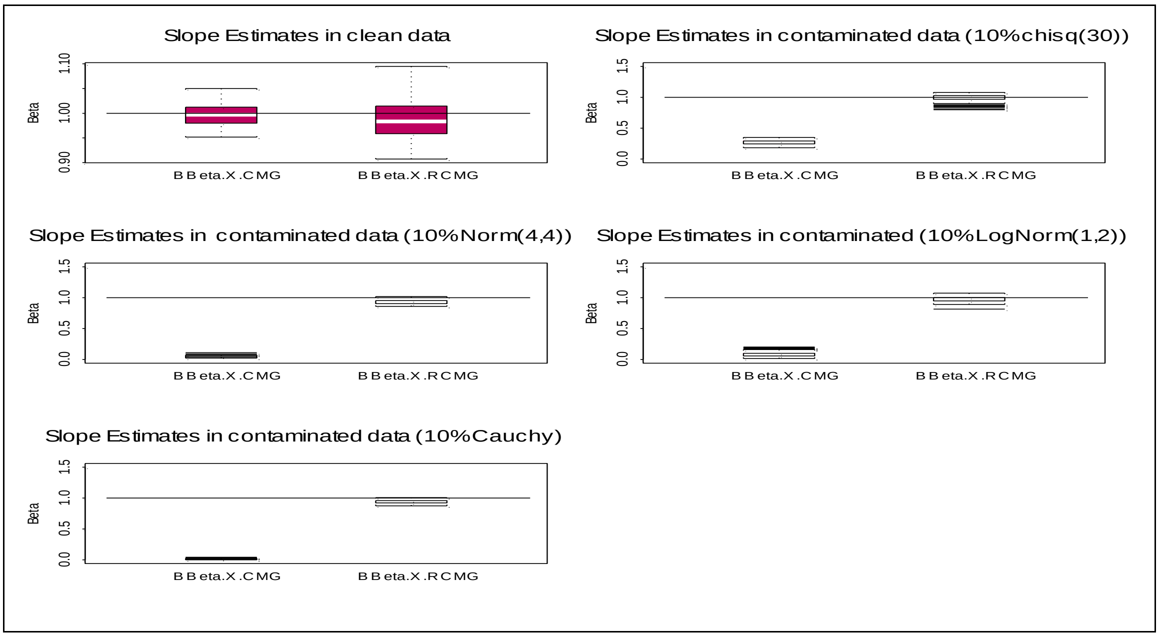

Figure 4.5: Box plots of the Parameter Estimates $\hat{\beta}$ with 10% Leverage Points ($\gamma_i \sim iidU(0.1,0.3)$)

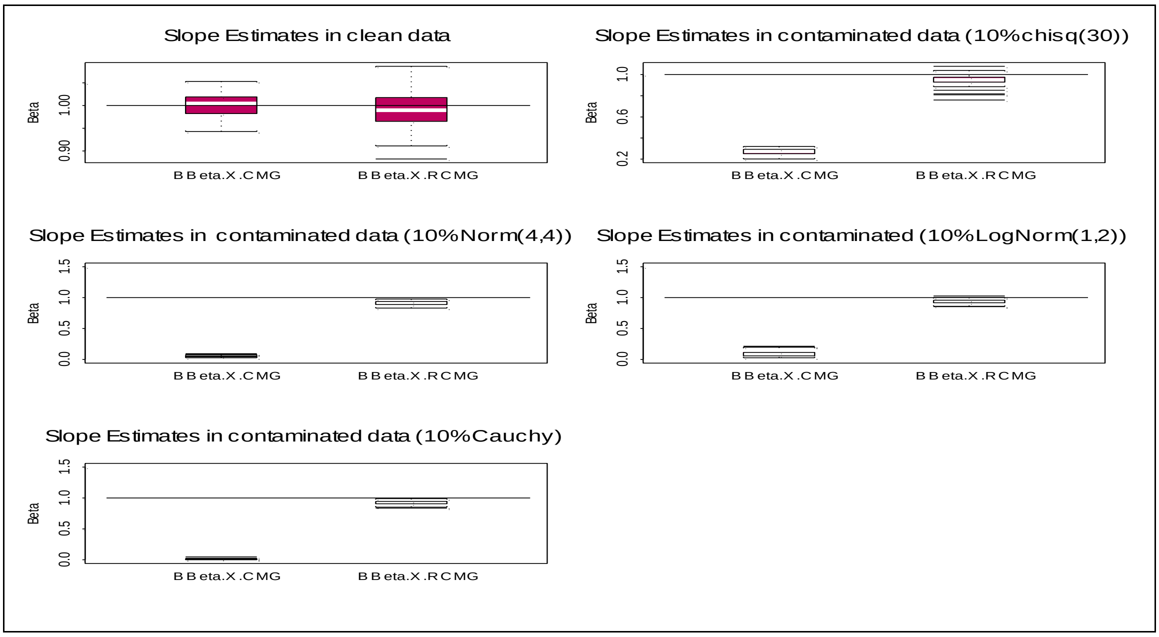

Figure 4.6: Box plots of the Parameter Estimates $\hat{\beta}$ with 10% Leverage Points ($\gamma_i \sim iidU(0.5,1.5)$)

4.3 Hypothesis Testing

In this section, the performance of the pooled, CMG and RCMG are compared in terms of size and power of the test for the parameter estimates (slope coefficient) given in hypothesis in (3.46) (refer Chapter 3). 500 runs are performed for each pair of cross sectional units and time with $N=(20,30,50)$ and $T=(20,30,50,100)$ in the uncontaminated and contaminated panels. The bias and RMSE of the slope estimates are computed as follows: (1) $\text{Bias}=E\left(\hat{\beta}-\beta\right)$as given in (4.3); and (2) $\text{RMSE}=\sqrt{Var\left(\hat{\beta}\right)+\left(Bias\left(\hat{\beta},\beta\right)\right)^2}$; where RMSE is equals to the sum of the variance and the squared bias of the estimator. Note that, RMSE is a square root of MSE in (4.3). Here, $\hat{\beta}$ is the parameter estimates and β is the true estimate.

The performances of the respective estimators are measured using the test statistics given by the following:

(1) Pooled Estimator

$$z_{Pool}=\frac{\left(\hat{\boldsymbol{\beta}}_{Pool}-\boldsymbol{\beta}\right)}{se\left(\hat{\boldsymbol{\beta}}_{Pool}\right)} \tag{4.8}$$

where $se\left(\hat{\boldsymbol{\beta}}_{pool}\right)=\sigma\left(\mathbf{X}^T\mathbf{X}\right)^{-1/2}$ with $\sigma^2=\dfrac{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}}{NT-k-1}$, k is the number of regressor and $\hat{e}_{it}$ is the estimated residuals, computed as $\hat{e}_{it}=y_{it}-\hat{y}_{it}$; with y_{it} and $\hat{y}_{it}$ the observed and fitted values of the dependent variable respectively. $\hat{\boldsymbol{\beta}}_{Pool}$ is the slope estimates obtained from the pooled model and is obtained from $\left(\hat{\boldsymbol{\alpha}}_{Pool},\hat{\boldsymbol{\beta}}_{Pool}\right)^T=\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$ (in matrix form).

(2) CMG Estimator

$$z_{CMG}=\frac{\left(\hat{\boldsymbol{\beta}}_{CMG}-\boldsymbol{\beta}\right)}{se(\hat{\boldsymbol{\beta}}_{CMG})} \tag{4.9}$$

where $se(\hat{\boldsymbol{\beta}}_{CMG}) = \frac{1}{\sqrt{N}}\left(\hat{\boldsymbol{\Sigma}}_{CMG}\right)^{1/2}$, $\hat{\boldsymbol{\Sigma}}_{CMG} = \frac{\sum_{i=1}^{N}(\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{\beta}}_{CMG})(\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{\beta}}_{CMG})^T}{N-1}$ and $\hat{\boldsymbol{\beta}}_i$ is computed in $\hat{\boldsymbol{\beta}}_i = (\mathbf{X}_i^T\mathbf{M}\mathbf{X}_i)^{-1}\mathbf{X}_i^T\mathbf{M}\mathbf{y}_i$ in matrix form, for each $i = 1,2,\ldots,N$ and $\mathbf{M}$ is given in (3.29) . The $\hat{\boldsymbol{\beta}}_{CMG}$ is the average of $\hat{\boldsymbol{\beta}}_i$, that is $\hat{\boldsymbol{\beta}}_{CMG} = \left(\sum_{i=1}^{N}\hat{\boldsymbol{\beta}}_i\right)/N$.

(3) RCMG Estimator

$$z_{RCMG} = \frac{\left(\hat{\boldsymbol{\beta}}_{RCMG} - \boldsymbol{\beta}\right)}{se\left(\hat{\boldsymbol{\beta}}_{RCMG}\right)} \tag{4.10}$$

where $se(\hat{\boldsymbol{\beta}}_{RCMG}) = \frac{1}{\sqrt{T}}\left(\hat{\boldsymbol{\Sigma}}_{RCMG}\right)^{1/2}$, $\hat{\boldsymbol{\Sigma}}_{RCMG} = \frac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$,

$$\mathbf{v}_i = \left(\mathbf{X}_i\mathbf{M}^*\mathbf{X}_i\right)^{-1}\hat{\sigma}_i^{\,2}\,\frac{E\left(\psi_i\left(\frac{e_{it}(\boldsymbol{\beta}_i)}{\sigma_i v_i(\mathbf{x}_{it})}\right)\right)^2}{\left(E\left(\psi_i'\left(\frac{e_{it}(\boldsymbol{\beta}_i)}{\sigma_i v_i(\mathbf{x}_{it})}\right)\right)\right)^2} \text{ and } v_i(\mathbf{x}_{it}) = \frac{1}{d_i(\mathbf{x}_{it})} .$$

$\hat{\boldsymbol{\beta}}_{RCMG} = \frac{\sum_{i=1}^{N}\hat{\boldsymbol{\beta}}_i}{N}$ with $\hat{\boldsymbol{\beta}}_i$ is computed as $\hat{\boldsymbol{\beta}}_i = (\mathbf{X}_i^T\mathbf{G}\mathbf{X}_i)^{-1}\mathbf{X}_i^T\mathbf{G}\mathbf{Y}_i$.

All estimators follow the standard normal distribution and therefore the z-test are used to test for $\hat{\boldsymbol{\beta}}$ (See the details in Chapter 3.)

4.3.1 Data Generating Process (DGP)

Using the same DGP in Section 4.2.1.1 (Experiment 1), we have:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}; \quad \text{and } e_{it} = \gamma_i^T f_t + \sigma_{it}\varepsilon_{it};$$

for $i = 1,2,\ldots,N.$ $t = -49,\ldots,0,1,2,\ldots,T$; where

$$\alpha_i \sim iidU(-0.5,0.5)\,;\sigma_{it} = 1$$

$$x_{it} \sim iidN(0,1);\ \varepsilon_{it} \sim iidN(0,1)\,;\ f_t \sim iidN(0,1)\,.$$

$\beta_i = \beta = 1$ is set to compute the size of the tests while the power is computed under the alternatives hypothesis of $\beta_i = \beta = 0.9$. The presence of cross dependency is set as follows:

(i) $\gamma_i = 0$ for no cross dependency;

(ii) $\gamma_i \sim iidU(0.1,0.3)$ for mild cross dependency and ;

(iii) $\gamma_i \sim iidU(0.5,1.5)$ for strong effect of cross dependency.

In the presence of contaminations at time $t = \tau_i$, the residual takes the form of the following:

$$e_{it} = \begin{cases} e_{it} & \text{for } t \neq \tau_i \\ e_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N;$$

with $m_{it} \sim LN(1,2)$. We allow a 5% contamination since $LN(1,2)$ gives the worst case scenario in our previous study.

4.3.2 Discussion

Tables 4.13 to 4.16 provide the results of the RMSE in the uncontaminated and contaminated panels for the pooled, CMG and RCMG respectively. Table 4.13 provides the results for the case when $\beta = 1$ in the uncontaminated panel. In the presence of cross sectional independence $\gamma_i = 0$, the pooled estimator yields the smallest RMSE for $(N,T) = (20,20)$. However as N increases, the RMSE decreases for all estimators and all results are comparable with the RCMG slightly bigger due to bias. The results hold in the presence of the mild CD. When the strong CD effect is observed in the panel, the RMSE of the pooled estimator are slightly larger compared to in the presence of the mild CD but not for the CMG and RCMG. Both estimators continue to yield a consistent RMSE as in the presence of the mild CD and cross sectional independence.

In the presence of contaminations (shown in Table 4.14), the pooled and CMG estimators result in larger RMSE than RCMG under the various degrees of CD. As expected, when N and T increases, the RMSE decreases for all estimators. Under CD, the RMSE for the pooled estimator is slightly smaller than when no CD is observed in the panel. The RCMG yields the smallest RMSE and outperforms all the estimators in the presence of outliers. Similar results are obtained for the case of $\beta = 0.9$ and the results are reported in Tables 4.15 to Table 4.16.

The bias of the respective estimators in Tables 4.17 to 4.20 for the case $\beta = 1$ is reported. In the uncontaminated panel (see Table 4.17), the bias value for the RCMG estimates is comparable to the bias value of the pooled and CMG for all sample size. In the presence of outliers, the estimates of β for the pooled and CMG estimators are slightly bias than in the uncontaminated panel (see Table 4.18). The $\hat{\beta}_{RCMG}$ retains a small bias and outperforms the pooled and CMG estimates in the presence of outliers for all CD cases. When $\beta = 0.9$, similar bias results are observed for both the uncontaminated and contaminated panels (See Table 4.19 and Table 4.20).

The results of the size and power[43] of the test are reported in Tables 4.21 to 4.24. The size and power of the test are computed under the null hypothesis $\beta = 1$; $\beta = 0.9$, respectively. For the uncontaminated panel (see Table 4.21), the pooled estimator gives reasonable size for $(N,T) = (20,20)$. The CMG and RCMG however are slightly oversized under the cross sectional independence. As N increases, the sizes of the study for all estimators are reasonable (close to 0.05). In the presence of the mild CD, the sizes of study are comparable for all estimators. The RCMG however is undersized when $\gamma_i \sim iidU(0.5,1.5)$ for each sample size under study. All estimators yield large powers (more than 95%) for all the CD cases and sample sizes (see Table 4.22) except for the RCMG when $T = 20$.

[43] For more details about the size and powers, refer to page 32, Chapter 2.

Reasonable sizes of the study for all estimators are obtained in the contaminated panel (see Table 4.23). The RCMG estimators however are slightly oversized for the case $\gamma_i = 0$ and $\gamma_i \sim iidU(0.1,0.3)$ but attain reasonable sizes under the strong CD effect in the panel. The RCMG also yields good powers of the tests in the presence of outliers (see Table 4.24). The two estimators of the pooled and CMG however have low powers in the presence of outliers and these powers are consistent in each sample size in all the CD cases: (1) $\gamma_i = 0$, (2) $\gamma_i \sim iidU(0.1,0.3)$, and (3) $\gamma_i \sim iidU(0.5,1.5)$. Based on these results, the proposed estimator provides a good estimate (unbiased and small RMSE) with reasonable size and high power in the presence of outliers relative to the other two estimators. The summary of the results are given in Table 4.25.

Table 4.13: RMSE of $\beta(\beta = 1)$ for the Uncontaminated Panel

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.0503	0.0432	0.0322	0.0587	0.0487	0.0364	0.0731	0.0539	0.0455
30	0.0421	0.0337	0.0286	0.0447	0.0357	0.0296	0.0475	0.0463	0.0366
50	0.0324	0.0272	0.0199	0.0338	0.0287	0.0202	0.0412	0.0325	0.0218
100	0.0228	0.0185	0.0145	0.0227	0.0188	0.0146	0.0276	0.0224	0.0172
200	0.0166	0.0131	0.0095	0.0167	0.0127	0.0091	0.0188	0.0164	0.0119
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.0529	0.0446	0.0357	0.0586	0.0461	0.0396	0.0687	0.0585	0.0506
30	0.0434	0.0351	0.0258	0.0460	0.0374	0.0274	0.0567	0.0494	0.0419
50	0.0338	0.0273	0.0218	0.0361	0.0285	0.0214	0.0406	0.0356	0.0209
100	0.0243	0.0196	0.0150	0.0238	0.0189	0.0146	0.0281	0.0240	0.0187
200	0.0161	0.0134	0.0099	0.0161	0.0127	0.0096	0.0190	0.0151	0.0110
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.0740	0.0606	0.0472	0.0589	0.0459	0.0397	0.0681	0.0595	0.0488
30	0.0612	0.0483	0.0370	0.0462	0.0372	0.0271	0.0541	0.0510	0.0410
50	0.0446	0.0375	0.0307	0.0364	0.0285	0.0212	0.0404	0.0344	0.0213
100	0.0331	0.0274	0.0207	0.0238	0.0190	0.0146	0.0276	0.0228	0.0183
200	0.0231	0.0193	0.0140	0.0162	0.0127	0.0096	0.0188	0.0148	0.0116

RMSE is computed as RMSE $= \sqrt{Var(\hat{\beta}) + (Bias(\hat{\beta}, \beta))^2}$ and Bias $= E(\hat{\beta} - \beta)$ based on 500 numbers of replications.

Table 4.14: RMSE of $\beta(\beta = 1)$ for the Contaminated Panel (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.9832	0.7680	0.6426	0.4604	0.3342	0.3742	0.0923	0.0674	0.0531
30	5.2336	0.9610	0.9119	0.3565	0.3467	0.3142	0.0673	0.0576	0.0458
50	0.5961	0.7808	0.7336	0.3367	0.3319	0.2507	0.0454	0.0376	0.0303
100	0.5487	0.4837	0.4153	0.2330	0.2433	0.1965	0.0316	0.0254	0.0181
200	0.3862	0.3784	0.3325	0.2348	0.1946	0.1765	0.0207	0.0161	0.0139
					$\gamma_i \sim iidU(0.1,0.3)$				
20	1.1278	1.0214	0.7658	0.5588	0.4171	0.4254	0.0927	0.0661	0.0560
30	1.8626	0.9775	0.5951	0.4819	0.3317	0.2835	0.0725	0.0526	0.0470
50	0.6519	0.7012	0.6315	0.2997	0.2907	0.3453	0.0480	0.0371	0.0329
100	0.5977	0.4961	0.4662	0.2576	0.2351	0.1972	0.0318	0.0262	0.0209
200	0.4038	0.3208	0.2228	0.2381	0.1860	0.1745	0.0217	0.0166	0.0138
					$\gamma_i \sim iidU(0.5,1.5)$				
20	1.1279	1.0209	0.7660	0.5919	0.4524	0.4450	0.1009	0.0725	0.0597
30	1.8633	0.9763	0.5958	0.5057	0.3545	0.3089	0.0746	0.0489	0.0507
50	0.6522	0.7025	0.6335	0.3142	0.3048	0.3518	0.0525	0.0379	0.0345
100	0.5986	0.2417	0.4669	0.2642	0.2417	0.2121	0.0342	0.0277	0.0217
200	0.4039	0.3204	0.2239	0.2400	0.1918	0.1805	0.0233	0.0184	0.0144

RMSE is computed as RMSE $= \sqrt{Var(\hat{\beta}) + (Bias(\hat{\beta}, \beta))^2}$ and Bias $= E(\hat{\beta} - \beta)$ based on 500 numbers of replications.

Table 4.15: RMSE of $\beta\left(\beta=0.9\right)$ for the Uncontaminated Panel

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i=0$				
20	0.0523	0.0437	0.0329	0.0591	0.0484	0.0371	0.0732	0.0537	0.0453
30	0.0425	0.0341	0.0287	0.0449	0.0362	0.0289	0.0472	0.0464	0.0366
50	0.0327	0.0279	0.0201	0.0336	0.0291	0.0231	0.0435	0.0326	0.0219
100	0.0233	0.0183	0.0148	0.0229	0.0193	0.0147	0.0264	0.0224	0.0173
200	0.0167	0.0135	0.0099	0.0174	0.0132	0.0098	0.0185	0.0164	0.0118
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.0531	0.0448	0.0364	0.0587	0.0469	0.0393	0.0649	0.0583	0.0508
30	0.0437	0.0356	0.0259	0.0461	0.0375	0.0273	0.0566	0.0494	0.0420
50	0.0338	0.0275	0.0222	0.0358	0.0289	0.0217	0.0419	0.0356	0.0209
100	0.0244	0.0195	0.0157	0.0234	0.0191	0.0148	0.0286	0.0240	0.0187
200	0.0162	0.0133	0.0096	0.0166	0.0128	0.0099	0.0201	0.0151	0.0110
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.0744	0.0613	0.0474	0.0588	0.0460	0.0395	0.0658	0.0596	0.0488
30	0.0609	0.0485	0.0371	0.0463	0.0377	0.0272	0.0540	0.0509	0.0410
50	0.0433	0.0377	0.0313	0.0365	0.0284	0.0218	0.0414	0.0344	0.0213
100	0.0332	0.0275	0.0212	0.0233	0.0192	0.0144	0.0278	0.0228	0.0183
200	0.0234	0.0196	0.0143	0.0165	0.0126	0.0095	0.0201	0.0148	0.0117

RMSE is computed as $\text{RMSE}=\sqrt{Var\left(\hat{\beta}\right)+\left(Bias\left(\hat{\beta},\beta\right)\right)^2}$ and $\text{Bias}=E\left(\hat{\beta}-\beta\right)$ based on 500 numbers of replications.

Table 4.16: RMSE of $\beta\left(\beta = 0.9\right)$ for the Contaminated Panel (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.9844	0.7689	0.6543	0.4651	0.3407	0.3776	0.0913	0.0653	0.0542
30	4.9776	0.9456	0.9133	0.3889	0.3532	0.3215	0.0657	0.0574	0.0457
50	0.5977	0.7843	0.7612	0.3347	0.3334	0.2566	0.0399	0.0371	0.0306
100	0.6432	0.4899	0.4231	0.2589	0.2442	0.2076	0.0311	0.0254	0.0181
200	0.3891	0.3674	0.3756	0.2379	0.1999	0.1871	0.0214	0.0160	0.0139
					$\gamma_i \sim iidU(0.1,0.3)$				
20	1.1296	1.0232	0.7669	0.5597	0.4186	0.4266	0.0941	0.0660	0.0570
30	1.8734	0.9795	0.6034	0.4910	0.3332	0.2978	0.0719	0.0513	0.0465
50	0.6662	0.7089	0.6411	0.3008	0.2981	0.3563	0.0573	0.0372	0.0328
100	0.5999	0.4816	0.4673	0.2656	0.2359	0.1986	0.0362	0.0262	0.0206
200	0.4128	0.3232	0.2234	0.2379	0.1891	0.1788	0.0223	0.0166	0.0137
					$\gamma_i \sim iidU(0.5,1.5)$				
20	1.1284	1.0221	0.7678	0.6323	0.4667	0.4578	0.1022	0.0727	0.0598
30	1.8673	0.9790	0.5982	0.5071	0.3563	0.3919	0.0750	0.0496	0.0507
50	0.6569	0.7133	0.6337	0.3243	0.3158	0.3635	0.0618	0.0379	0.0344
100	0.5778	0.4977	0.4802	0.2332	0.2424	0.2176	0.0408	0.0277	0.0217
200	0.4125	0.3210	0.2250	0.2455	0.1923	0.1817	0.0244	0.0184	0.0144

RMSE is computed as $\text{RMSE} = \sqrt{Var\left(\hat{\beta}\right) + \left(Bias\left(\hat{\beta}, \beta\right)\right)^2}$ and $\text{Bias} = E\left(\hat{\beta} - \beta\right)$ based on 500 numbers of replications.

Table 4.17: Bias of $\beta(\beta = 1)$ in the Uncontaminated Panel

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.0018	-0.0006	0.0017	0.0012	0.0010	0.0029	-0.0012	0.0082	0.0023
30	0.0002	-0.0024	0.0004	-0.0017	-0.0028	-0.0006	-0.0126	0.0010	0.0076
50	0.0013	0.0006	-0.0009	0.0003	0.0008	-0.0005	-0.0014	0.0006	-0.0026
100	-0.0010	0.0001	-0.0001	-0.0014	-0.0005	0.0000	0.0015	-0.0007	-0.0001
200	-0.0008	-0.0005	0.0005	-0.0010	-0.0005	0.0008	-0.0004	0.0022	0.0011
					$\gamma_i \sim iidU(0.1,0.3)$				
20	-0.0004	-0.0030	0.0034	-0.0009	-0.0024	0.0041	-0.0008	0.0020	0.0032
30	0.0003	-0.0009	-0.0019	-0.0007	0.0001	-0.0027	0.0069	0.0133	0.0046
50	-0.0023	-0.0005	-0.0001	-0.0026	0.0000	-0.0002	-0.0011	-0.0036	-0.0010
100	0.0008	0.0005	0.0004	0.0005	-0.0001	0.0000	0.0006	-0.0024	-0.0006
200	-0.0003	-0.0004	0.0010	-0.0001	-0.0007	0.0013	0.0001	-0.0015	0.0015
					$\gamma_i \sim iidU(0.5,1.5)$				
20	-0.0011	-0.0061	0.0028	-0.0004	-0.0021	0.0039	-0.0028	0.0019	-0.0004
30	0.0000	-0.0006	-0.0033	-0.0007	-0.0001	-0.0027	0.0103	0.0111	0.0045
50	-0.0026	-0.0011	-0.0010	-0.0026	0.0000	-0.0004	-0.0013	-0.0045	-0.0009
100	0.0002	0.0005	0.0010	0.0005	-0.0002	-0.0001	0.0008	-0.0018	-0.0002
200	-0.0006	0.0005	0.0003	-0.0001	-0.0007	0.0014	0.0005	-0.0006	0.0017

Bias is computed as $\text{Bias} = E(\hat{\beta} - \beta)$ based on 500 numbers of replications.

Table 4.18: Bias of $\beta(\beta=1)$ for the Contaminated Panel (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
	Pooled			CMG			RCMG		
				$\gamma_i = 0$					
20	-0.0631	0.0193	0.0038	-0.0329	0.0137	-0.0006	-0.0020	-0.0017	0.0006
30	0.2717	0.0865	0.0234	0.0114	0.0093	-0.0339	-0.0027	0.0045	-0.0013
50	0.0094	0.0227	-0.0742	0.0001	0.0355	-0.0105	-0.0004	0.0089	-0.0036
100	-0.0426	0.0141	0.0125	0.0019	-0.0030	-0.0168	-0.0011	-0.0016	-0.0030
200	-0.0032	0.0002	-0.0360	-0.0086	-0.0038	-0.0091	-0.0018	0.0009	0.0008
				$\gamma_i \sim iidU(0.1{,}0.3)$					
20	-0.0195	-0.0269	-0.0305	-0.0134	-0.0344	0.0190	-0.0016	-0.0065	-0.0003
30	-0.0754	-0.0356	-0.0223	0.0166	-0.0032	0.0139	-0.0039	-0.0019	-0.0015
50	-0.0197	-0.0311	0.0355	-0.0210	0.0044	-0.0298	0.0000	0.0048	-0.0047
100	0.0158	-0.0146	-0.0056	-0.0001	0.0035	-0.0057	0.0010	0.0014	-0.0021
200	0.0281	0.0235	-0.0149	0.0138	0.0011	0.0043	-0.0015	0.0008	0.0021
				$\gamma_i \sim iidU(0.5{,}1.5)$					
20	-0.0180	-0.0286	-0.0301	-0.0151	-0.0304	0.0205	-0.0023	-0.0113	0.0015
30	-0.0761	-0.0343	-0.0228	0.0189	-0.0036	0.0154	-0.0006	-0.0023	-0.0021
50	-0.0195	-0.0298	0.0339	-0.0228	0.0034	-0.0302	0.0009	0.0029	-0.0054
100	0.0156	0.0062	-0.0058	-0.0013	0.0062	-0.0042	0.0013	0.0013	-0.0020
200	0.0273	0.0236	-0.0147	0.0139	-0.0003	0.0026	-0.0005	0.0009	0.0013

Bias is computed as Bias $= E(\hat{\beta} - \beta)$ based on 500 numbers of replications.

Table 4.19: Bias of $\beta\left(\beta=0.9\right)$ for the Uncontaminated Panel

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.0021	-0.0006	0.0019	0.0015	0.0011	0.0031	-0.0010	0.0086	0.0023
30	0.0004	-0.0027	0.0005	-0.0024	-0.0028	-0.0006	-0.0121	0.0015	0.0077
50	0.0013	0.0006	-0.0009	0.0007	0.0008	-0.0005	-0.0022	0.0006	-0.0025
100	-0.0015	0.0004	-0.0001	-0.0024	-0.0004	0.0001	0.0001	-0.0007	-0.0001
200	-0.0009	-0.0006	0.0006	-0.0013	-0.0006	0.0009	-0.0011	0.0022	0.0011
					$\gamma_i \sim iidU(0.1,0.3)$				
20	-0.0005	-0.0033	0.0037	-0.0010	-0.0034	0.0032	0.0022	0.0021	0.0032
30	0.0004	-0.0011	-0.0019	-0.0009	0.0001	-0.0027	0.0070	0.0133	0.0047
50	-0.0023	-0.0005	-0.0001	-0.0025	0.0004	-0.0005	-0.0074	-0.0035	-0.0009
100	0.0009	0.0005	0.0004	0.0008	-0.0003	0.0007	0.0011	-0.0023	-0.0005
200	-0.0005	-0.0004	0.0016	-0.0003	-0.0004	0.0023	0.0012	-0.0014	0.0016
					$\gamma_i \sim iidU(0.5,1.5)$				
20	-0.0018	-0.0066	0.0038	-0.0005	-0.0033	0.0032	0.0016	0.0021	-0.0004
30	0.0001	-0.0007	-0.0043	-0.0004	-0.0003	-0.0027	0.0104	0.0111	0.0046
50	-0.0028	-0.0011	-0.0020	-0.0027	0.0001	-0.0003	-0.0056	-0.0043	-0.0009
100	0.0003	0.0005	0.0011	0.0006	-0.0003	-0.0004	0.0006	-0.0018	-0.0002
200	-0.0004	0.0006	0.0004	-0.0003	-0.0009	0.0024	0.0012	-0.0006	0.0017

Bias is computed as $\text{Bias} = E\left(\hat{\beta}-\beta\right)$ based on 500 numbers of replications.

Table 4.20: Bias of $\beta\left(\beta = 0.9\right)$ for the Contaminated Panel (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	-0.0644	0.0204	0.0049	-0.0331	0.0149	-0.0007	-0.0013	-0.0008	-0.0006
30	0.2821	0.0871	0.0239	0.0127	0.0094	-0.0349	-0.0104	0.0039	-0.0017
50	0.0095	0.0229	-0.0755	0.0005	0.0366	-0.0113	0.0011	0.0091	-0.0031
100	0.0559	0.0147	0.0123	-0.0004	-0.0039	-0.0184	0.0021	-0.0012	-0.0029
200	-0.0037	0.0012	-0.0473	-0.0088	-0.0044	-0.0111	-0.0016	0.0010	0.0008
					$\gamma_i \sim iidU(0.1,0.3)$				
20	-0.0215	-0.0321	-0.0323	-0.0153	-0.0364	0.0231	0.0011	-0.0062	-0.0006
30	-0.0862	-0.0364	-0.0271	0.0213	-0.0056	0.0154	-0.0037	-0.0015	-0.0019
50	-0.0234	-0.0332	0.0355	-0.0232	0.0099	-0.0312	0.0008	0.0049	-0.0049
100	0.0173	-0.0176	-0.0065	-0.0044	0.0065	-0.0088	0.0055	0.0014	-0.0019
200	0.0299	0.0243	-0.0158	0.0176	0.0091	0.0054	-0.0042	0.0009	0.0022
					$\gamma_i \sim iidU(0.5,1.5)$				
20	-0.0194	-0.0293	-0.0332	-0.0191	-0.0355	0.0276	0.0023	-0.0110	0.0015
30	-0.0766	-0.0362	-0.0274	0.0201	-0.0096	0.0157	-0.0007	-0.0017	-0.0021
50	-0.0222	-0.0321	0.0354	-0.0234	0.0045	-0.0371	0.0003	0.0032	-0.0052
100	-0.0467	-0.0183	-0.0098	0.0069	0.0092	-0.0083	0.0053	0.0016	-0.0019
200	0.0279	0.0255	-0.0197	0.01422	-0.0007	0.0048	-0.0034	0.0010	0.0013

Bias is computed as Bias $= E\left(\hat{\beta} - \beta\right)$ based on 500 numbers of replications.

Table 4.21: Size of the Test for the Uncontaminated Panel at 5% Significant Level

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.043	0.046	0.040	0.080	0.059	0.054	0.080	0.050	0.067
30	0.040	0.041	0.068	0.051	0.057	0.062	0.066	0.071	0.079
50	0.041	0.053	0.038	0.054	0.070	0.060	0.071	0.067	0.066
100	0.050	0.047	0.048	0.063	0.073	0.064	0.065	0.069	0.053
200	0.040	0.042	0.025	0.076	0.058	0.035	0.066	0.079	0.061
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.036	0.064	0.072	0.066	0.054	0.056	0.054	0.062	0.092
30	0.043	0.044	0.034	0.074	0.066	0.050	0.061	0.089	0.088
50	0.044	0.053	0.048	0.081	0.073	0.060	0.072	0.077	0.045
100	0.060	0.051	0.052	0.076	0.060	0.070	0.063	0.079	0.089
200	0.044	0.050	0.040	0.046	0.044	0.050	0.062	0.067	0.055
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.060	0.061	0.054	0.070	0.051	0.068	0.014	0.009	0.037
30	0.049	0.051	0.038	0.070	0.066	0.052	0.010	0.026	0.020
50	0.039	0.053	0.060	0.081	0.081	0.058	0.016	0.011	0.019
100	0.039	0.064	0.046	0.077	0.062	0.066	0.013	0.023	0.016
200	0.048	0.058	0.025	0.050	0.044	0.040	0.008	0.011	0.010

The test statistics for the respective estimation procedure is rejected at 5% significant level if the test $> |z|_{0.025} = 1.96$ and the results are based on 500 numbers of replications.

Table 4.22: Power of the Test for the Uncontaminated Panel at 5% Significant Level

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	1.000	1.000	1.000	1.000	1.000	1.000	0.976	0.978	0.981
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.1,0.3)$				
20	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.937	0.958
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	0.991
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.5,1.5)$				
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.934	0.964
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992	0.997
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The test statistics for the respective estimation procedure is rejected at 5% significant level if the test $> |z|_{0.025} = 1.96$ and the results are based on 500 numbers of replications.

Table 4.23: Size of the Test for the Contaminated Panel at 5% Significant Level (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
	Pooled			CMG			RCMG		
				$\gamma_i = 0$					
20	0.044	0.044	0.052	0.037	0.028	0.038	0.086	0.080	0.074
30	0.061	0.052	0.028	0.039	0.032	0.046	0.089	0.071	0.070
50	0.066	0.046	0.050	0.030	0.050	0.060	0.080	0.047	0.063
100	0.042	0.062	0.055	0.044	0.060	0.045	0.076	0.062	0.057
200	0.050	0.046	0.035	0.050	0.054	0.045	0.054	0.064	0.069
				$\gamma_i \sim iidU(0.1,0.3)$					
20	0.056	0.046	0.038	0.057	0.044	0.040	0.091	0.080	0.079
30	0.051	0.044	0.058	0.043	0.040	0.042	0.083	0.077	0.083
50	0.044	0.054	0.045	0.052	0.038	0.040	0.076	0.072	0.081
100	0.034	0.050	0.045	0.050	0.038	0.045	0.080	0.081	0.078
200	0.074	0.040	0.030	0.060	0.052	0.055	0.079	0.066	0.078
				$\gamma_i \sim iidU(0.5,1.5)$					
20	0.056	0.054	0.048	0.046	0.032	0.028	0.044	0.039	0.047
30	0.050	0.048	0.052	0.036	0.036	0.040	0.039	0.034	0.045
50	0.044	0.054	0.050	0.050	0.036	0.045	0.038	0.032	0.041
100	0.034	0.052	0.040	0.048	0.036	0.045	0.036	0.042	0.039
200	0.076	0.040	0.030	0.060	0.046	0.055	0.032	0.035	0.046

The test statistics for the respective estimation procedure is rejected at 5% significant level if the test $> |z|_{0.025} = 1.96$ and the results are based on 500 numbers of replications.

Table 4.24: Power of the Test for the Contaminated Panel at 5% Significant Level (5% contamination)

T / N	20	30	50	20	30	50	20	30	50
		Pooled			CMG			RCMG	
					$\gamma_i = 0$				
20	0.453	0.450	0.432	0.680	0.648	0.618	0.986	0.966	0.992
30	0.444	0.436	0.524	0.660	0.656	0.706	0.980	0.989	1.000
50	0.488	0.510	0.595	0.686	0.634	0.760	1.000	1.000	1.000
100	0.548	0.544	0.665	0.740	0.728	0.805	1.000	1.000	1.000
200	0.614	0.652	0.740	0.784	0.810	0.860	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.453	0.436	0.478	0.653	0.648	0.608	0.990	0.988	0.947
30	0.457	0.470	0.520	0.689	0.642	0.654	1.000	0.991	0.993
50	0.506	0.474	0.540	0.708	0.668	0.705	1.000	1.000	1.000
100	0.544	0.532	0.660	0.716	0.722	0.785	1.000	1.000	1.000
200	0.594	0.632	0.730	0.728	0.818	0.865	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.433	0.434	0.470	0.596	0.594	0.552	0.982	0.988	0.969
30	0.456	0.446	0.514	0.630	0.606	0.628	0.996	0.995	0.991
50	0.496	0.480	0.540	0.658	0.624	0.705	1.000	1.000	1.000
100	0.498	0.532	0.660	0.700	0.702	0.770	1.000	1.000	1.000
200	0.598	0.628	0.735	0.726	0.826	0.860	1.000	1.000	1.000

The test statistics for the respective estimation procedure is rejected at 5% significant level if the test $> |z|_{0.025} = 1.96$ and the results are based on 500 numbers of replications.

Table 4.25: Summary of the Results for Tables 4.13 to 4.24.

	Uncontaminated Panel			Contaminated Panel		
Estimator	Parameter Estimates	Size	Power	Parameter Estimates	Size	Power
			no CD			
Pooled	Unbiased (Consistent)	√	HIGH	Bias (Inconsistent)	√	LOW
CMG	Unbiased Unbiased (Consistent)	√	HIGH	Unbiased (Inconsistent)	√	MD
RCMG	Unbiased (Consistent)	√	HIGH	Unbiased (Consistent)	√	HIGH
			with CD			
Pooled	Unbiased (Consistent)	√	HIGH	Bias (Inconsistent)	√	LOW
CMG	Unbiased Unbiased (Consistent)	√	HIGH	Unbiased (Inconsistent)	√	MD
RCMG	Unbiased (Consistent)	√	HIGH	Unbiased (Consistent)	√	HIGH

The summary is based on the estimation results. The abbreviations for √=reasonable size, US = undersize, LOW = low power, MD = medium power and HIGH = high power[44].

[44] The details of definition of size and power have been discussed in Section 2.5, Chapter 2.

4.4 Confidence Interval

In this section, a Monte Carlo simulation study is run in order to obtain an approximate confidence interval for the parameter based on the pooled, CMG and RCMG estimators. The $100(1-\alpha)\%$ CI for the parameter estimates are computed as in (3.48) and given by the following:

(1) Pooled Estimator

$$\left[\hat{\beta}_{Pool} - z_{\alpha/2} se\left(\hat{\beta}_{Pool}\right), \hat{\beta}_{Pool} + z_{\alpha/2} se\left(\hat{\beta}_{Pool}\right)\right]$$

where $se\left(\hat{\beta}_{Pool}\right) = \sigma\left(\mathbf{X}^T\mathbf{X}\right)^{-1/2}$ and $\sigma^2 = \dfrac{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}}{NT-k-1}$ with k is the number of independent variable.

(2) CMG Estimator

$$\left[\hat{\beta}_{CMG} - z_{\alpha/2} se\left(\hat{\beta}_{CMG}\right), \hat{\beta}_{CMG} + z_{\alpha/2} se\left(\hat{\beta}_{CMG}\right)\right]$$

where $se\left(\hat{\beta}_{CMG}\right) = \dfrac{1}{\sqrt{N}}\left(\hat{\Sigma}_{CMG}\right)^{1/2}$ and $\hat{\Sigma}_{CMG} = \dfrac{\sum_{i=1}^{N}(\hat{\beta}_i - \hat{\beta}_{CMG})(\hat{\beta}_i - \hat{\beta}_{CMG})^T}{N-1}$.

(3) RCMG Estimator

$$\left[\hat{\beta}_{RCMG} - z_{\alpha/2} se\left(\hat{\beta}_{RCMG}\right), \hat{\beta}_{RCMG} + z_{\alpha/2} se\left(\hat{\beta}_{RCMG}\right)\right]$$

where $se\left(\hat{\beta}_{RCMG}\right) = \dfrac{1}{\sqrt{T}}\left(\hat{\Sigma}_{RCMG}\right)^{1/2}$, $\hat{\Sigma}_{RCMG} = \dfrac{1}{N^2}\sum_{i=1}^{N}\mathbf{v}_i$ and $\mathbf{v}_i$ is defined in **Theorem 3.1.**

Here α is chosen as 10% and 5% respectively and $z_{\alpha/2}$ is a standard normal. The data are generated as follows:

$$y_{it} = \alpha_i + \beta_i^T x_{it} + e_{it}\text{; and } e_{it} = \gamma_i^T f_t + \sigma_{it}\varepsilon_{it}\text{;}$$

for $i = 1, 2, \ldots, N.\quad t = -49, \ldots, 0, 1, 2, \ldots, T$

with $\alpha_i \sim iidU(-0.5, 0.5)$; $\sigma_{it} = 1$

$x_{it} \sim iidN(0,1)$ $\varepsilon_{it} \sim iidN(0,1)$; $f_t \sim iidN(0,1)$;

and $\beta_i = \beta = 1.0$. Here, γ_i is set as follows:

(i) $\gamma_i = 0$ for no cross dependency;

(ii) $\gamma_i \sim iidU(0.1,0.3)$ for mild cross dependency and ;

(iii) $\gamma_i \sim iidU(0.5,1.5)$ for strong effect of cross dependency.

In the presence of contaminations at time $t = \tau_i$, the residual takes a similar form as before, that is:

$$e_{it} = \begin{cases} e_{it} & \text{for } t \neq \tau_i \\ e_{it} + m_{it} & \text{for } t = \tau_i \end{cases} \quad \text{for } i = 1,2,\ldots,N;$$

with $m_{it} \sim LN(1,2)$ and 5% contamination are chosen.

In this experiment, 500 samples for each of size (N,T); $N = (20,30,50)$ and $T = (20,30,50,100,200)$, are simulated. The 95% and 90% CI of parameter estimates $\hat{\beta}_{RCMG}$ are computed with the respective length of CI.

4.4.1 Results and Discussion

The CI of the parameter estimates for the respective estimator (Pooled, CMG and RCMG) is reported in Tables 4.26 to 4.31. In the uncontaminated panel (Table 4.26), the length of the CI for the pooled estimates attains a small value as N and T increases when no CD is present. Similar results are observed for the case of the mild CD. The length however increases in the presence of the strong CD. The length of CI for $\hat{\beta}_{CMG}$ however is comparable with and without the presence of CD (Table 4.27). For $\hat{\beta}_{RCMG}$, similar findings are observed and these are reported in Table 4.28. Based on these results, the pooled estimates yields the shorter length for the case of the no and mild CD in the panel but not for the strong CD case. The CMG however provides the shortest length of CI in the presence of the strong CD effect.

For the contaminated panel (Table 4.29) and no CD (row 1), the pooled estimates yields a larger length of CI for $(N,T)=(20,20)$, but this results decrease as N and T increase. The CI of the pooled estimator however worsens under the CD. Similar results are obtained for CMG (Table 4.30) with a slightly shorter length of CI than the pooled estimates. The parameter estimates $\hat{\beta}_{RCMG}$ (Table 4.31) however provides better results for small N and T, and these values improve with shorter length of CI as N and T increase.

Table 4.26: CI of $\hat{\beta}_{Pool}$ in the Uncontaminated Panel

	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
T/N	20				30				50			
					$\gamma_i = 0$							
20	(0.9174,1.0822)	(0.9018,1.0960)	0.1648	0.1942	(0.9279,1.0701)	(0.9132,1.0840)	0.1422	0.1708	(0.9399,1.0556)	(0.9301,1.0627)	0.1158	0.1326
30	(0.9355,1.0686)	(0.9240,1.0779)	0.1342	0.1539	(0.9415,1.0539)	(0.9325,1.0622)	0.1124	0.1296	(0.9575,1.0438)	(0.9425,1.0539)	0.0863	0.1114
50	(0.9484,1.0551)	(0.9390,1.0618)	0.1067	0.1228	(0.9564,1.0451)	(0.9456,1.0520)	0.0887	0.1065	(0.9627,1.0358)	(0.9576,1.0417)	0.0732	0.0841
100	(0.9631,1.0359)	(0.9558,1.0431)	0.0729	0.0873	(0.9693,1.0304)	(0.9639,1.0338)	0.0612	0.0699	(0.9741,1.0252)	(0.9711,1.0291)	0.0511	0.0579
200	(0.9707,1.0268)	(0.9650,1.0309)	0.0561	0.0659	(0.9785,1.0220)	(0.9751,1.0241)	0.0435	0.0490	(0.9814,1.0166)	(0.9783,1.0200)	0.0352	0.0417
					$\gamma_i \sim iidU(0.1,0.3)$							
20	(0.9195,1.0878)	(0.9044,1.1017)	0.1683	0.1973	(0.9244,1.0684)	(0.9120,1.0836)	0.1440	0.1716	(0.9444,1.0563)	(0.9332,1.0661)	0.1119	0.1329
30	(0.9310,1.0711)	(0.9145,1.0851)	0.1401	0.1706	(0.9439,1.0561)	(0.9308,1.0615)	0.1122	0.1307	(0.9544,1.0490)	(0.9411,1.0552)	0.0946	0.1141
50	(0.9398,1.0547)	(0.9305,1.0674)	0.1149	0.1368	(0.9533,1.0453)	(0.9447,1.0493)	0.0920	0.1046	(0.9665,1.0333)	(0.9572,1.0395)	0.0667	0.0823
100	(0.9633,1.0434)	(0.9571,1.0539)	0.0800	0.0967	(0.9706,1.0317)	(0.9634,1.0370)	0.0611	0.0736	(0.9759,1.0248)	(0.9729,1.0285)	0.0489	0.0556
200	(0.9691,1.0295)	(0.9638,1.0341)	0.0604	0.0703	(0.9786,1.0209)	(0.9738,1.0209)	0.0423	0.0509	(0.9834,1.0163)	(0.9802,1.0190)	0.0329	0.0389
					$\gamma_i \sim iidU(0.5,1.5)$							
20	(0.8795,1.1147)	(0.8578,1.1254)	0.2352	0.2675	(0.8948,1.0980)	(0.8835,1.1170)	0.2032	0.2335	(0.9342,1.0785)	(0.9186,1.0958)	0.1442	0.1772
30	(0.8971,1.1010)	(0.8783,1.1251)	0.2040	0.2468	(0.9209,1.0804)	(0.9032,1.0906)	0.1594	0.1874	(0.9352,1.0609)	(0.9232,1.0772)	0.1257	0.1540
50	(0.9197,1.0715)	(0.9067,1.0830)	0.1518	0.1763	(0.9375,1.0689)	(0.9258,1.0836)	0.1314	0.1579	(0.9560,1.0483)	(0.9434,1.0579)	0.0924	0.1145
100	(0.9489,1.0556)	(0.9376,1.0657)	0.1068	0.1281	(0.9545,1.0449)	(0.9426,1.0523)	0.0905	0.1097	(0.9667,1.0328)	(0.9624,1.0418)	0.0661	0.0793
200	(0.9583,1.0376)	(0.9499,1.0451)	0.0793	0.0951	(0.9685,1.0313)	(0.9621,1.0362)	0.0628	0.0740	(0.9766,1.0234)	(0.9734,1.0259)	0.0468	0.0524

The length of CI is computed as length of CI =upper CI – lower CI

Table 4.27: CI of $\hat{\beta}_{CMG}$ in the Uncontaminated Panel

	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
T/N		20				30				50		
					$\gamma_i = 0$							
20	(0.9051,1.1015)	(0.8788,1.1133)	0.1964	0.2345	(0.9228,1.0840)	(0.9102,1.0956)	0.1612	0.1854	(0.9377,1.0595)	(0.9245,1.0686)	0.1218	0.1440
30	(0.9233,1.0710)	(0.9162,1.0808)	0.1477	0.1646	(0.9415,1.0560)	(0.9308,1.0707)	0.1145	0.1399	(0.9527,1.0477)	(0.9385,1.0550)	0.0950	0.1164
50	(0.9455,1.0544)	(0.9318,1.0626)	0.1090	0.1308	(0.9507,1.0469)	(0.9437,1.0566)	0.0962	0.1129	(0.9621,1.0373)	(0.9552,1.0476)	0.0751	0.0924
100	(0.9614,1.0332)	(0.9548,1.0414)	0.0719	0.0866	(0.9707,1.0313)	(0.9662,1.0395)	0.0606	0.0733	(0.9731,1.0235)	(0.9686,1.0283)	0.0504	0.0597
200	(0.9728,1.0272)	(0.9641,1.0308)	0.0544	0.0667	(0.9787,1.0217)	(0.9746,1.0258)	0.0430	0.0512	(0.9810,1.0155)	(0.9781,1.0184)	0.0345	0.0403
					$\gamma_i \sim iidU(0.1,0.3)$							
20	(0.9080,1.1011)	(0.8897,1.1160)	0.1931	0.2262	(0.9237,1.0748)	(0.9073,1.0888)	0.1511	0.1815	(0.9345,1.0597)	(0.9215,1.0708)	0.1252	0.1493
30	(0.9213,1.0787)	(0.9041,1.0787)	0.1574	0.1840	(0.9367,1.0549)	(0.9244,1.0724)	0.1182	0.1480	(0.9531,1.0464)	(0.9417,1.0578)	0.0932	0.1162
50	(0.9366,1.0615)	(0.9228,1.0701)	0.1249	0.1472	(0.9507,1.0399)	(0.9396,1.0448)	0.0892	0.1052	(0.9645,1.0350)	(0.9572,1.0411)	0.0705	0.0838
100	(0.9628,1.0403)	(0.9543,1.0498)	0.0775	0.0955	(0.9686,1.0322)	(0.9644,1.0360)	0.0635	0.0716	(0.9779,1.0231)	(0.9713,1.0265)	0.0452	0.0552
200	(0.9711,1.0277)	(0.9668,1.0318)	0.0566	0.0649	(0.9780,1.0208)	(0.9739,1.0247)	0.0428	0.0508	(0.9829,1.0167)	(0.9801,1.0199)	0.0338	0.0399
					$\gamma_i \sim iidU(0.5,1.5)$							
20	(0.9099,1.1024)	(0.8908,1.1168)	0.1925	0.2260	(0.9252,1.0778)	(0.9104,1.0864)	0.1526	0.1760	(0.9373,1.0553)	(0.9226,1.0682)	0.1180	0.1456
30	(0.9206,1.0804)	(0.9079,1.0918)	0.1598	0.1839	(0.9372,1.0563)	(0.9239,1.0708)	0.1190	0.1469	(0.9504,1.0461)	(0.9428,1.0548)	0.0957	0.1120
50	(0.9346,1.0586)	(0.9222,1.0680)	0.1240	0.1458	(0.9537,1.0479)	(0.9447,1.0557)	0.0941	0.1110	(0.9670,1.0345)	(0.9612,1.0447)	0.0674	0.0835
100	(0.9622,1.0409)	(0.9561,1.0505)	0.0787	0.0944	(0.9691,1.0334)	(0.9606,1.0404)	0.0643	0.0798	(0.9762,1.0239)	(0.9721,1.0262)	0.0477	0.0541
200	(0.9706,1.0315)	(0.9662,1.0315)	0.0576	0.0653	(0.9778,1.0204)	(0.9735,1.0234)	0.0426	0.0499	(0.9835,1.0176)	(0.9802,1.0215)	0.0341	0.0413

The length of CI is computed as length of CI =upper CI – lower CI

Table 4.28: CI of $\hat{\beta}_{RCMG}$ in the Uncontaminated Panel

T/N	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
	20				30				50			
$\gamma_i = 0$												
20	(0.8756,1.1231)	(0.8589,1.1431)	0.2475	0.2842	(0.9042,1.0991)	(0.8829,1.1157)	0.1949	0.2327	(0.9393,1.0806)	(0.9250,1.0960)	0.1413	0.1710
30	(0.9068,1.0938)	(0.8836,1.1044)	0.1870	0.2207	(0.9302,1.0734)	(0.9104,1.0904)	0.1432	0.1800	(0.9407,1.0584)	(0.9177,1.0707)	0.1177	0.1530
50	(0.9388,1.0938)	(0.9231,1.1044)	0.1550	0.1813	(0.9432,1.0527)	(0.9317,1.0711)	0.1095	0.1393	(0.9547,1.0312)	(0.9481,1.0376)	0.0765	0.0895
100	(0.9555,1.0465)	(0.9484,1.0570)	0.0910	0.1086	(0.9576,1.0299)	(0.9560,1.0383)	0.0723	0.0823	(0.9710,1.0267)	(0.9653,1.0294)	0.0557	0.0641
200	(0.9649,1.0286)	(0.9619,1.0286)	0.0594	0.0667	(0.9760,1.0282)	(0.9610,1.0356)	0.0522	0.0746	(0.9710,1.0267)	(0.9776,1.0294)	0.0557	0.0518
$\gamma_i \sim iidU(0.1,0.3)$												
20	(0.8914,1.1115)	(0.8615,1.1335)	0.2202	0.2720	(0.9022,1.0937)	(0.8866,1.1161)	0.1915	0.2294	(0.9293,1.1012)	(0.9210,1.1179)	0.1719	0.1969
30	(0.9049,1.0892)	(0.8875,1.1062)	0.1843	0.2187	(0.9297,1.0770)	(0.9091,1.0887)	0.1473	0.1795	(0.9386,1.0593)	(0.9214,1.0866)	0.1207	0.1652
50	(0.9299,1.0694)	(0.9175,1.0847)	0.1395	0.1671	(0.9416,1.0539)	(0.9352,1.0616)	0.1123	0.1264	(0.9614,1.0290)	(0.9553,1.0406)	0.0676	0.0853
100	(0.9533,1.0442)	(0.9434,1.0534)	0.0909	0.1100	(0.9538,1.0340)	(0.9436,1.0378)	0.0803	0.0942	(0.9680,1.0344)	(0.9602,1.0360)	0.0663	0.0758
200	(0.9705,1.0289)	(0.9651,1.0461)	0.0584	0.0810	(0.9733,1.0269)	(0.9664,1.0323)	0.0537	0.0659	(0.9824,1.0175)	(0.9781,1.0235)	0.0351	0.0454
$\gamma_i \sim iidU(0.5,1.5)$												
20	(0.8883,1.1097)	(0.8628,1.1385)	0.2214	0.2756	(0.9081,1.0891)	(0.8887,1.1167)	0.1811	0.2280	(0.9210,1.1005)	(0.9129,1.1190)	0.1795	0.2061
30	(0.9139,1.0908)	(0.8916,1.0993)	0.1769	0.2078	(0.9310,1.0804)	(0.9102,1.0912)	0.1494	0.1810	(0.9359,1.0643)	(0.9247,1.0782)	0.1284	0.1535
50	(0.9293,1.0658)	(0.9120,1.0838)	0.1365	0.1719	(0.9350,1.0440)	(0.9275,1.0690)	0.1090	0.1415	(0.9643,1.0357)	(0.9512,1.0432)	0.0708	0.0920
100	(0.9517,1.0458)	(0.9427,1.0541)	0.0941	0.1114	(0.9583,1.0318)	(0.9460,1.0343)	0.0736	0.0883	(0.9725,1.0337)	(0.9620,1.0366)	0.0612	0.0747
200	(0.9691,1.0341)	(0.9630,1.0415)	0.0650	0.0785	(0.9725,1.0229)	(0.9687,1.0266)	0.0504	0.0579	(0.9790,1.0189)	(0.9785,1.0227)	0.0399	0.0442

The length of CI is computed as length of CI =upper CI – lower CI

Table 4.29: CI of $\hat{\beta}_{Pool}$ in the Contaminated Panel

	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
T/*N*		20				30				50		
					$\gamma_i = 0$							
20	(0.0062,1.7465)	(-0.6119,2.0778)	1.7403	2.6898	(0.0550,1.8851)	(-0.4382,2.1613)	1.8301	2.5996	(0.2914,1.8018)	(0.0121,2.3509)	1.5104	2.3389
30	(0.1451,2.0847)	(-0.2654,2.5206)	1.9397	2.7859	(0.3336,1.7274)	(0.0483,2.3801)	1.3938	2.3318	(0.2999,1.6502)	(-0.0384,2.1510)	1.3504	2.1894
50	(0.3012,2.0107)	(-0.3281,2.3226)	1.7095	2.6507	(0.3124,1.6115)	(-0.0243,2.0508)	1.2991	2.0751	(0.4018,1.4987)	(0.0501,1.7086)	1.0969	1.6584
100	(0.2313,1.6131)	(-0.0462,1.9887)	1.3818	2.0349	(0.2614,1.5708)	(-0.1811,1.7369)	1.3094	1.9180	(0.6033,1.4855)	(0.4247,1.7491)	0.8822	1.3245
200	(0.4379,1.5361)	(0.1618,1.7327)	1.0982	1.5709	(0.5539,1.4584)	(0.3238,1.6042)	0.9045	1.2804	(0.6129,1.3823)	(0.4044,1.4867)	0.7694	1.0823
					$\gamma_i \sim iidU(0.1,0.3)$							
20	(-0.3033,1.9085)	(-1.3946,2.7744)	2.2118	4.1690	(0.0812,1.9253)	(-0.5551,2.5743)	1.8441	3.1294	(0.1520,1.6586)	(-0.3005,1.9129)	1.5066	2.2135
30	(0.0225,2.1861)	(-0.4787,2.7472)	2.1636	3.2259	(-0.0958,1.7845)	(-0.7386,2.2414)	1.8804	2.9801	(0.2676,1.8755)	(-0.1788,2.1425)	1.6079	2.3213
50	(0.1495,1.7839)	(-0.4547,2.0226)	1.6344	2.4773	(0.1405,1.6726)	(-0.2032,1.9289)	1.5321	2.1320	(0.4435,1.6636)	(0.1566,2.0296)	1.2200	1.8730
100	(0.3675,1.5994)	(0.0908,1.8920)	1.2319	1.8012	(0.4597,1.5480)	(0.1734,1.8233)	1.0882	1.6499	(0.5140,1.4609)	(0.2719,1.6490)	0.9470	1.3771
200	(0.4015,1.5176)	(0.1647,1.8083)	1.1161	1.6436	(0.5238,1.4749)	(0.3159,1.6063)	0.9510	1.2904	(0.6838,1.4246)	(0.5104,1.5905)	0.7408	1.0801
					$\gamma_i \sim iidU(0.5,1.5)$							
20	(-0.3263,1.8778)	(-1.3946,2.7377)	2.2041	4.1323	(0.0687,1.8782)	(-0.5310,2.5629)	1.8095	3.0939	(0.1389,1.6637)	(-0.3582,2.0066)	1.5248	2.3648
30	(0.0462,2.1641)	(-0.4857,2.7208)	2.1180	3.2065	(-0.1182,1.7851)	(-0.6971,2.2347)	1.9033	2.9319	(0.4469,1.7220)	(0.0364,2.1559)	1.2751	2.1195
50	(0.1321,1.7818)	(-0.4507,2.0290)	1.6497	2.4797	(0.1203,1.6745)	(-0.1847,1.9346)	1.5542	2.1194	(0.2749,1.6582)	(-0.2394,1.9366)	1.3834	2.1759
100	(0.3561,1.5924)	(0.0845,1.8824)	1.2363	1.7978	(0.3971,1.5715)	(-0.2205,1.7839)	1.1744	2.0044	(0.4272,1.5125)	(0.2708,1.8272)	1.0852	1.5564
200	(0.5069,1.5215)	(0.2613,1.6450)	1.0145	1.3837	(0.5560,1.3713)	(0.2253,1.5443)	0.8153	1.3190	(0.6204,1.3419)	(0.3866,1.4855)	0.7214	1.0989

The length of CI is computed as length of CI =upper CI – lower CI

Table 4.30: CI of $\hat{\beta}_{CMG}$ in the Contaminated Panel

	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
T/N		20				30				50		
						$\gamma_i = 0$						
20	(0.3876,1.4750)	(0.0972,1.6100)	1.0874	1.5128	(0.5338,1.5813)	(0.3382,1.8168)	1.0476	1.4786	(0.5337,1.4955)	(0.2500,1.7377)	0.9617	1.4878
30	(0.4814,1.5850)	(0.3287,1.6768)	1.1036	1.3482	(0.5425,1.4332)	(0.3547,1.6684)	0.8907	1.3137	(0.5010,1.4236)	(0.3185,1.5476)	0.9226	1.2291
50	(0.5103,1.5065)	(0.2374,1.6294)	0.9962	1.3919	(0.6436,1.4527)	(0.4923,1.5764)	0.8090	1.0841	(0.5701,1.3768)	(0.4020,1.4991)	0.8067	1.0972
100	(0.6186,1.3923)	(0.5402,1.4816)	0.7737	0.9414	(0.6306,1.3520)	(0.5008,1.4602)	0.7214	0.9594	(0.7024,1.2860)	(0.6151,1.3723)	0.5835	0.7572
200	(0.6233,1.3590)	(0.5346,1.4443)	0.7358	0.9097	(0.6869,1.2983)	(0.5868,1.3681)	0.6113	0.7813	(0.7080,1.2906)	(0.6366,1.3763)	0.5827	0.7397
						$\gamma_i \sim iidU(0.1,0.3)$						
20	(0.4272,1.5234)	(0.1548,1.8411)	1.0963	1.6862	(0.3995,1.4862)	(0.0191,1.6747)	1.0867	1.6556	(0.4497,1.5402)	(0.3095,1.6840)	1.0905	1.3745
30	(0.4386,1.5556)	(0.2454,1.9499)	1.1170	1.7045	(0.5416,1.5291)	(0.3102,1.6868)	0.9875	1.3766	(0.5766,1.4898)	(0.4560,1.6079)	0.9133	1.1519
50	(0.4886,1.4680)	(0.2656,1.5219)	0.9794	1.2562	(0.5734,1.4537)	(0.4269,1.6564)	0.8804	1.2295	(0.5979,1.3927)	(0.4247,1.4725)	0.7948	1.0479
100	(0.5792,1.4094)	(0.4830,1.5124)	0.8301	1.0294	(0.6153,1.3538)	(0.4443,1.4431)	0.7385	0.9988	(0.6598,1.3591)	(0.5541,1.4740)	0.6993	0.9199
200	(0.6519,1.3641)	(0.4694,1.4339)	0.7122	0.9645	(0.6836,1.2942)	(0.5997,1.4050)	0.6106	0.8052	(0.7441,1.2785)	(0.6987,1.3020)	0.5344	0.6034
						$\gamma_i \sim iidU(0.5,1.5)$						
20	(0.3386,1.5441)	(0.0684,1.8054)	1.2055	1.7370	(0.2555,1.6027)	(0.2555,1.6027)	1.3472	1.8309	(0.4956,1.5175)	(0.2517,1.8512)	1.0219	1.5995
30	(0.4190,1.5893)	(0.1146,1.9488)	1.1703	1.8341	(0.4656,1.5410)	(0.4656,1.5410)	1.0754	1.3913	(0.6041,1.5483)	(0.3265,1.6207)	0.9443	1.2942
50	(0.4550,1.4638)	(0.2582,1.5599)	1.0088	1.3017	(0.5695,1.4715)	(0.5695,1.4715)	0.9019	1.2232	(0.5572,1.4587)	(0.3568,1.6081)	0.9015	1.2513
100	(0.5869,1.3928)	(0.4679,1.5687)	0.8059	1.1008	(0.5942,1.4256)	(0.5942,1.4256)	0.8314	1.0996	(0.6538,1.3406)	(0.5259,1.4794)	0.6868	0.9536
200	(0.6527,1.3481)	(0.5945,1.4428)	0.6954	0.8484	(0.6873,1.3077)	(0.6873,1.3077)	0.6205	0.8348	(0.7141,1.2859)	(0.6619,1.3733)	0.5718	0.7113

The length of CI is computed as length of CI =upper CI – lower CI

Table 4.31: CI of $\hat{\beta}_{RCMG}$ in the Contaminated Panel

T/N	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI	90% CI	95% CI	length of 90% CI	length of 95% CI
		20				30				50		
					$\gamma_i = 0$							
20	(0.8578,1.1655)	(0.8133,1.1992)	0.3077	0.3859	(0.8661,1.1342)	(0.8243,1.1631)	0.2680	0.3389	(0.9031,1.1115)	(0.8777,1.1272)	0.2084	0.2495
30	(0.8864,1.1046)	(0.8611,1.1191)	0.2182	0.2580	(0.8972,1.0930)	(0.8818,1.1117)	0.1957	0.2299	(0.9349,1.0655)	(0.9224,1.0752)	0.1306	0.1528
50	(0.9188,1.0825)	(0.9052,1.0959)	0.1636	0.1907	(0.9393,1.0604)	(0.9279,1.0691)	0.1212	0.1412	(0.9532,1.0465)	(0.9462,1.0572)	0.0933	0.1110
100	(0.9470,1.0525)	(0.9354,1.0639)	0.1055	0.1285	(0.9554,1.0399)	(0.9425,1.0470)	0.0844	0.1045	(0.9664,1.0305)	(0.9595,1.0375)	0.0641	0.0781
200	(0.9660,1.0330)	(0.9616,1.0407)	0.0670	0.0792	(0.9732,1.0231)	(0.9691,1.0273)	0.0499	0.0582	(0.9795,1.0214)	(0.9751,1.0239)	0.0419	0.0488
					$\gamma_i \sim iidU(0.1,0.3)$							
20	(0.8620,1.1579)	(0.8250,1.2043)	0.2959	0.3793	(0.8744,1.1182)	(0.8351,1.1496)	0.2438	0.3145	(0.8969,1.1048)	(0.8775,1.1274)	0.2078	0.2499
30	(0.8845,1.1030)	(0.8630,1.1218)	0.2185	0.2589	(0.9101,1.0907)	(0.8828,1.1127)	0.1806	0.2299	(0.9323,1.0719)	(0.9226,1.0846)	0.1396	0.1620
50	(0.9275,1.0775)	(0.9122,1.0980)	0.1500	0.1857	(0.9356,1.0685)	(0.9177,1.0685)	0.1246	0.1508	(0.9528,1.0507)	(0.9430,1.0555)	0.0980	0.1125
100	(0.9421,1.0488)	(0.9296,1.0598)	0.1067	0.1302	(0.9556,1.0437)	(0.9482,1.0505)	0.0881	0.1023	(0.9658,1.0329)	(0.9582,1.0366)	0.0672	0.0784
200	(0.9633,1.0349)	(0.9532,1.0452)	0.0716	0.0919	(0.9698,1.0273)	(0.9649,1.0315)	0.0575	0.0666	(0.9782,1.0217)	(0.9743,1.0258)	0.0435	0.0515
					$\gamma_i \sim iidU(0.5,1.5)$							
20	(0.8333,1.1720)	(0.7886,1.2264)	0.3387	0.4378	(0.8550,1.1337)	(0.8227,1.1789)	0.2787	0.3562	(0.8944,1.1377)	(0.8595,1.1377)	0.2192	0.2782
30	(0.8803,1.1262)	(0.8544,1.1626)	0.2458	0.3082	(0.9090,1.0924)	(0.8885,1.1055)	0.1834	0.2169	(0.9216,1.0688)	(0.9075,1.0808)	0.1472	0.1733
50	(0.9130,1.0788)	(0.8863,1.0958)	0.1658	0.2095	(0.9374,1.0615)	(0.9235,1.0759)	0.1241	0.1524	(0.9491,1.0481)	(0.9344,1.0636)	0.0990	0.1292
100	(0.9446,1.0518)	(0.9335,1.0629)	0.1072	0.1294	(0.9569,1.0437)	(0.9490,1.0498)	0.0867	0.1008	(0.9664,1.0333)	(0.9605,1.0389)	0.0669	0.0784
200	(0.9619,1.0384)	(0.9549,1.0447)	0.0765	0.0899	(0.9691,1.0319)	(0.9612,1.0387)	0.0628	0.0775	(0.9745,1.0222)	(0.9676,1.0277)	0.0477	0.0598

The length of CI is computed as length of CI = upper CI – lower CI.

4.5 Conclusion

In this chapter, the finite of sample behaviour of the estimation procedures are examined via simulation experiments. By limiting the analysis to estimation issues, the performances of the estimator are measured based on the sample mean, standard deviation, bias and MSE of the parameter estimates in the presence of outliers and leverage points in the first part of the study. The proposed estimator yields unbiased estimates with small MSE under such conditions. Extensive simulation studies are run to investigate the behaviour of the proposed estimation procedure in terms of power and size of the test. The proposed estimator provides comparable results with the CMG in the uncontaminated panel; however outperforms the CMG in the contaminated panel. With small bias and RMSE, a reasonable size and high power, the RCMG retains its robustness with and without the presence of outliers. This is shown in 90% and 95% CI, where the values of $\hat{\beta}_{RCMG}$ are $\pm 10\%$ from β for large N and T.

CHAPTER 5

Panel Unit Root Tests

5.1 Introduction

There has been a considerable amount of research work related to unit root test. One of the most commonly used procedures in testing the presence of unit root is that of Dickey and Fuller (1979)[45]. The Dickey-Fuller[46] (DF) test, as it is commonly called, tests the presence of unit root in an autoregressive (AR) model under the assumption that the residuals are iid. In statistics and econometric, the augmented version of the DF test (ADF) is widely used for many empirical studies.

The panel unit root test can be found in Im et al. (2003), Levin and Lin (1992, 1993), Levin et al. (2002), Bai and Ng (2004), Philips and Sul (2003), Moon and Perron (2004), Pesaran (2007) and Choi (2001, 2002). Hurlin (2010) distinguished two generations of unit root tests on which the first generation tests relied on the assumption that all cross sectional units are independent (see Dickey and Fuller, 1979). The first generation of unit root tests are those proposed by Quah (1994), Breitung and Meyer (1994) and Levin and Lin (1992, 1993). Quah (1990, 1994) showed that the asymptotic property of the DF unit root tests is a standard normal distribution when the residuals are iid and all groups are homogeneous. Breitung and Meyer (1994) also showed the same asymptotic distribution of DF test and this is applicable for large N and small fixed T.

For the second generation of panel unit root tests, the presence of CD among the residuals is allowed within the panel. The assumption of the cross correlated errors is

[45] The test is based on the AR model; $y_t = \rho y_{t-1} + e_t$, $t = 1,2,\ldots,n.$ where y_t is the variable of interest, n is the number of observation, ρ is the coefficient and e_t is the error term. The model is non-stationary if $\rho = 1$ meaning the unit root is present in the series.

[46] The hypothesis testing for this test is under the null hypothesis, H_0: $\rho = 1$ versus the alternative; H_1: $\rho < 1$.

due to the evidence obtained on the strong co-movements among the economic variables (Barbieri, 2009). The assumption that the individual time series in the panel are cross sectional independent is not practical in the context of cross country regressions. The presence of such CD may affect the finite sample behaviour of the panel unit root test (O'Connell, 1998). Those who proposed the tests which incorporated the CD are: Pesaran (2007), Philips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004) and Choi (2002).

Pesaran (2004), Bai and Ng (2002), Moon and Perron (2004) and Philips and Sul (2003) used the factor approach to explain the CD. In their study, the CD is allowed to have different effects on different cross sectional units. Pesaran (2007) introduced a new unit root test in which the standard DF regressions are augmented with the cross sectional averages of lagged levels and first differences (hereafter called CADF)[47]. The test is then generalized based on the t-ratio of the average of the individual CADF-test statistics. Moon and Perron (2004) proposed a pooled panel unit root test based on "de-factored" observations in which the factor loadings were estimated by using the principal component approach. This test has good asymptotic power properties if the model does not contain deterministic trends. Bai and Ng (2004) used similar orthogonalisation procedures as in Moon and Perron. They however specified the model by allowing the possibility of unit roots in the common factors and then used the first differences of the model and applied the principal components. The unit root test is then computed by taking the partial sums of the estimated first differences on both factor loadings and the individual de-factored series.

The existence of outliers implies that some shocks will only have temporary effects and thus, providing that they are sufficiently large or sufficiently frequent indicated that the series is stationary (Franses and Haldrup, 1994). Martin and Yohai (1986) showed via the simulation experiment that an AO biases the OLS estimator

[47] Refer equation (5.7) on page 136.

downward for the parameter in a stationary first order autoregressive process. Hence, in some situations it could be expected that the AOs will establish the wrong impression that a time series is stationary when it is actually non-stationary. In addition, the presence of a cross dependency may deteriorate the asymptotic distribution of the standard unit root test which is normally distributed (Philips and Sul, 2003; Banerjee, 1999). Due to such interest, in this chapter, we propose a robust unit root test in the panel data model which aims at reducing the effects of outliers in the presence of the CD. Specifically, the presence of the unit root will be tested for when both the CD and outliers exist in the panel. The finite sample behaviour of the proposed test is studied and its performance is evaluated through the Monte Carlo simulation study.

5.2 The Unit Root Test

5.2.1 Augmented Dickey-Fuller (ADF) Test

The following model is considered in the ADF test:

$$\Delta y_{it} = \alpha_i + b_i y_{it-1} - \sum_{j=1}^{p_i} \delta_{ij} \Delta y_{it-j} + e_{it}; \quad i = 1,2,\ldots,N;\ t = 1,2,\ldots,T \tag{5.1}$$

where y_{it} is generated according to a finite order AR($p_i + 1$), p_i is the number of augmenting lags, α_i is the intercept, and b_i, δ_{ij} are parameters for the respective variables of y_{it-1} and Δy_{it-j}.

This test can be employed for larger and complicated set of time series. In the absence of the unit root, negative values for b_i are expected. Specifically, H_0 and H_1 are defined as follows:

H_0: $b_i = 0$ for all i; $i = 1,2,\ldots,N$; $t = 1,2,\ldots,T$;

H_1: $b_i < 0$ for some i; $i = 1,2,\ldots,N$; $t = 1,2,\ldots,T$ (5.2) where under H0: y_{it-1} has a unit root and under H$_1$: $y_{it-1} \sim I(0)$[48].

Following the work of Dickey and Fuller (1979), the t-ratio statistics for testing the presence of a unit root is given by the following:

$$t_i = \frac{\hat{b}_i - b_i}{\sqrt{\sigma_i^2 \left(y_{i,-1}^T y_{i,-1}\right)^{-1}}} \text{ for each cross sectional units } i = 1,2,.....,N \qquad (5.3)$$

where $\mathbf{y}_{i,-1} = \left(y_{i0}, y_{i1}, \ldots, y_{iT-1}\right)^T$; and $\sigma_i^2 = \frac{\sum_{t=1}^{T} \hat{e}_{it}^2}{T - k_i - 2}$, with $\hat{e}_{it} = \Delta y_{it} - \Delta \hat{y}_{it}$.

Here $\hat{b}_i$ is the estimates of b_i obtained from the OLS estimator. The asymptotic distribution for the ADF test is a Wiener process under certain conditions[49]. The average of t_i is computed which is given by $\bar{t} = \frac{\sum_{i=1}^{N} t_i}{N}$ (follows the work of Im et al. (2003)).

5.2.2 Pesaran's Unit Root Test (2007)

In the presence of the CD, Pesaran (2007) proposed an alternative approach to the ADF where the standard ADF is augmented with the cross section averages of the lagged levels and first-differences of the individual series. The methodology is introduced by Pesaran (2004) in which the unobserved common factors are used to explain the CD. Specifically, Pesaran considers the following model to test the presence of the unit root:

$$\Delta y_{it} = \alpha_i + b_i y_{it-1} + \gamma_i f_t + \varepsilon_{it}; \quad i = 1,2,\ldots,N. \;\; t = 1,2,\ldots,T \qquad (5.4)$$

where $\Delta y_{it} = y_{it} - y_{it-1}$, $b_i = (1 - \rho_i)$[50] with the following unit root hypothesis.

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[48]This means y_{it-1} are integrated with order 0 (the data is stationary).

[49] See Dickey and Fuller (1979) for more details of the properties of the test.

[50] Consider this model, $y_{it} = \rho_i y_{it-1} + e_{it}$, $i = 1,2,\ldots,N$; $t = 1,2,\ldots,T$ where y_{it} is the variable of interest for each cross sectional unit i at time t. Here, ρ_i is the autoregressive coefficient for each i and e_{it} is the error term.

$$\mathrm{H}_0\!: \; b_i = 0; \qquad \text{for all } i = 1,2,\ldots,N \qquad \text{(this is equivalent to testing } \rho_i = 1\text{)} \quad (5.5)$$

against the possible alternative:

$$\mathrm{H}_1: \begin{cases} b_i < 0, & i = 1,2,\ldots,N_1 \\ b_i = 0, & i = N_1 + 1,\ldots,N \end{cases} \quad (5.6)$$

where the fraction of the stationary individuals is such that $N_1 / N \to \delta$ such that $0 < \delta \le 1$ as $N \to \infty$. Under the alternative hypothesis, the panel can be a combination of stationary and non-stationary model.

Model in (5.4) can be expressed as cross-sectional augmented ADF (CADF) model:

$$\Delta y_{it} = \alpha_i + b_i y_{it-1} + c_i \bar{y}_{t-1} + d_i \Delta \bar{y}_t + e_{it}; \quad i = 1,2,\ldots,N. \;\; t = 1,2,\ldots,T; \quad (5.7)$$

that is (5.7) a DF (or ADF) regression which is augmented with the cross section averages of lagged levels and first differences of the individual series, i.

Let CADF$_i$ be the ADF statistics for the i^{th} cross sectional unit given by the t-ratio of the OLS estimate $\hat{b}_i$ of b_i in the CADF regression (5.7). Then, the Pesaran unit root test is given by

$$\mathrm{CIPS} = \frac{\sum_{i=1}^{N} \mathrm{CADF}_i}{N} \quad (5.8)$$

where CIPS stands for cross-sectional augmented IPS (Im et al. (1997) unit root test) . This CADF$_i$ is given by

$$\mathrm{CADF}_i = t_i(N,T) = \frac{\left(\mathbf{y}_{i,-1}^T \overline{\mathbf{M}} \mathbf{y}_{i,-1}\right)^{-1} \left(\mathbf{y}_{i,-1}^T \overline{\mathbf{M}} \Delta \mathbf{y}_i\right)}{\sqrt{\sigma_i^2 \left(\mathbf{y}_{i,-1}^T \overline{\mathbf{M}} \mathbf{y}_{i,-1}\right)^{-1}}} \quad (5.9)$$

where $\mathbf{y}_{i,-1} = \left(y_{i1},\ldots,y_{iT-1}\right)^T$, $\Delta \mathbf{y}_i = \left(\Delta y_{i2}, \Delta y_{i3}, \ldots, \Delta y_{iT}\right)^T$; $\sigma_i^2 = \frac{\sum_{t=1}^{T} \hat{e}_{it}^2}{T-4}$, with $\hat{e}_{it} = \Delta y_{it} - \Delta \hat{y}_{it}$ and $\overline{\mathbf{M}}$ is defined as $\overline{\mathbf{M}} = \mathbf{I}_t - \overline{\mathbf{H}}\left(\overline{\mathbf{H}}^T \overline{\mathbf{H}}\right)^{-1} \overline{\mathbf{H}}^T$ with $\overline{\mathbf{H}} = (\mathbf{1}, \Delta \overline{\mathbf{y}}_t, \overline{\mathbf{y}}_{t-1})$ **.**

$\mathbf{I}_t$ is a unit matrix of order $T \times T$ and $\overline{\mathbf{H}}$ is the combination of the dummy variables, average of cross section of the first difference of y_{it} and its first lagged value y_{it-1} .

The asymptotic distribution of this distribution is more skewed compared to the ADF (asymptotically normal) distribution in the presence of the CD (Philips and Sul, 2003). The critical value of the test statistics in (5.9) is given in Table C1, Appendix C[51] and those are obtained from the simulation experiment based on the cross-sectional augmented DF statistics (denoted by CADF) model[52].

Although Pesaran (2007) proposes a truncated version of CIPS, termed as CIPS* which tends to standard normal distribution for large N and T, the critical value for CIPS* obtained by him coincides with CIPS[53]. Thus, for comparing the models, the CIPS is used.

5.2.3 The Proposed Unit Root Test

The Pesaran's unit root test uses the OLS estimator that is non-robust. It has been known in the literature that the OLS is sensitive to the influence of outliers in the data. To limit the influence of outliers in the data, the estimation procedure is adopted in Chapter 3 by using the Generalized M-estimator to investigate the presence of the unit root in the model. Recall that[54] the Generalized M-estimator is obtained by solving the following equation:

$$\sum_{t=1}^{T} u_i(y_{it-1}) v_i(y_{it-1}) \psi_i\left(\frac{\hat{e}_{it}(b_i)}{\hat{\sigma}_i v_i(y_{it-1})}\right) y_{it-1} = 0; \qquad \text{for } i = 1,2,\ldots,N. \tag{5.10}$$

where $u_i(y_{it-1}) = 1$ and $v_i(y_{it-1}) = \dfrac{1}{d(y_{it-1})}$.

To test for a unit root, the following hypothesis test is considered:

H_0: $b_i = 0$ for all i; $i = 1,2,\ldots,N$; $t = 1,2,\ldots,T$;

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[51] Quoted from Pesaran (2007).

[52] The critical values are obtained from the estimates of $\Delta Y_{it} = \alpha_i + b_i Y_{it-1} + c_i \Delta \overline{Y}_t + d_i \overline{Y}_{t-1} + e_{it}$ regression based on 10,000 runs.

[53] for more details, see Pesaran, 2007.

[54] Note that, the details of the objective function for Generalized M- estimator has been given in Chapter 3.

$$\text{H}_1: \quad b_i < 0 \text{ for some } i; \qquad i = 1,2,\ldots,N;\ t = 1,2,\ldots,T \tag{5.11}$$

where under H_0: y_{it-1} is not stationary and under H_1 : y_{it-1} is stationary[55].

Under H_0 of no unit root, the generalisation of the test is given by:

$$t_i^* = \frac{\hat{b}_i^* - b_i}{\sqrt{Var(\hat{b}_i^*)}} \tag{5.12}$$

where $\hat{b}_i^*$ is the Generalized M-estimator and it is computed as follows:

$$\hat{b}_i^* = \left(\mathbf{y}_{i,-1}^T \mathbf{G}_i \mathbf{y}_{i,-1}\right)^{-1}\left(\mathbf{y}_{i,-1}^T \mathbf{G}_i \Delta\mathbf{y}_i\right)$$

where $\mathbf{y}_{i,-1} = \left(y_{i1},\ldots,y_{iT-1}\right)^T$, $\Delta\mathbf{y}_i = \left(\Delta y_{i2}, \Delta y_{i3},\ldots,\Delta y_{iT}\right)^T$ and $\mathbf{G}_i = \overline{\mathbf{M}}^* \mathbf{W}_i\left(z_{it}\right)$

and

$$Var(\hat{b}_i^*) = \left(\mathbf{y}_{i,-1}^T \overline{\mathbf{M}}^* \mathbf{y}_{i,-1}\right)^{-1} \hat{\sigma}_i^{\,2} \, \frac{E\left(\psi_i\left(\dfrac{\hat{e}_{it}(b_i)}{\hat{\sigma}_i v_i\left(y_{it-1}\right)}\right)\right)^2}{\left(E\left(\psi_i'\left(\dfrac{\hat{e}_{it}(b_i)}{\hat{\sigma}_i v_i\left(y_{it-1}\right)}\right)\right)\right)^2}. \tag{5.13}$$

The $\overline{\mathbf{M}}^*$ is computed as $\overline{\mathbf{M}}^* = \mathbf{I}_t - \overline{\mathbf{H}}^*\left(\overline{\mathbf{H}}^{*T}\overline{\mathbf{H}}^*\right)^{-1}\overline{\mathbf{H}}^{*T}$; $\mathbf{I}_t$ is an identity T by T matrix and $\overline{\mathbf{H}}^* = \left(\mathbf{1}, \psi\left(\bar{y}_{t-1}\right), \psi\left(\Delta\bar{y}_t\right)\right)$. The value of $\psi(.)$ takes the form

$$\psi\left(\bar{y}_{t-1}\right) = \begin{cases} \bar{y}_{t-1} & ,\text{if } \left|\bar{y}_{t-1}\right| \le c \\ sign\left(\bar{y}_{t-1}\right) \times \left|\underset{i}{\text{median}}\left(y_{1t-1},\ldots,y_{Nt-1}\right)\right| & ,\text{elsewhere} \end{cases}$$

and

$$\psi\left(\Delta\bar{y}_t\right) = \begin{cases} \Delta\bar{y}_t & ,\text{if } \left|\Delta\bar{y}_t\right| \le d \\ sign\left(\Delta\bar{y}_t\right) \times \left|\underset{i}{\text{median}}\left(\Delta y_{1t},\ldots,\Delta y_{Nt}\right)\right| & ,\text{elsewhere} \end{cases} \tag{5.14}$$

where c and d[56] are the critical values and are computed as $3\hat{\sigma}_{\bar{y}_{t-1}}$ and $3\hat{\sigma}_{\Delta\bar{y}_t}$, respectively. $\hat{\sigma}_{\bar{y}_{t-1}}$ and $\hat{\sigma}_{\Delta\bar{y}_t}$ are a robust scale and this is given by $\hat{\sigma}_{\bar{y}_{t-1}} = 1.4825\,\underset{t}{\text{median}}\left|\bar{y}_{t-1} - \underset{t}{\text{median}}\left(\bar{y}_{t-1}\right)\right|$, $\hat{\sigma}_{\Delta\bar{y}_t} = 1.4825\,\underset{t}{\text{median}}\left|\Delta\bar{y}_t - \underset{t}{\text{median}}\left(\Delta\bar{y}_t\right)\right|$, respectively.

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[55] Similar hypothesis tests as in (5.2).

[56] c and d is the critical value, chosen to achieve specified level of efficiency

The proposed unit root test is the average of t_i^* which is given by

$$\text{RCIPS} = \bar{t}_i^* = \frac{\sum_{i=1}^{N} t_i^*}{N} \tag{5.15}$$

where t_i^* is given in (5.12) .

The asymptotic distribution of the test statistics given in (5.12) is obtained through the extensive simulation experiment. Based on Figure 5.2, the RCIPS unit root test tends to have an approximate t-distribution with a mean μ and a standard deviation, σ. As the sample size increase, it is believed that the RCIPS will tend to a standard normal distribution. This result is comparable with Pesaran (2007) under conditions where e_{it} is normally distributed.

To investigate the performance of the RCIPS, the critical region of test statistics is required. Therefore, the critical region of RCIPS test is obtained through simulation experiment at the 0.05 level of significance and it is given in Table (5.13). The DGP and results are given in the next section.

5.3 Simulation Experiment

First, the critical values for the ADF and RCIPS test are computed. Following the work of Im et al. (2003), the DGP for computing critical values for ADF and RCIPS test is given by

$$y_{it} = y_{it-1} + e_{it}, \quad \text{with } e_{it} \sim iidN(0,1)\text{; for } i = 1, 2, \ldots, N. \quad t = -49, \ldots, 0, 1, 2, \ldots, T$$

The value of the test statistics at 1%, 5% and 10% quartiles are computed based on the formula given in (5.3) and (5.12).

Next, for computing the size and power of the unit root tests in the presence of the CD, Monte Carlo simulations are run with the following DGP setup:

$$y_{it} = \mu_i(1-\phi_i) + \phi_i y_{it-1} + e_{it}; \qquad \text{and}$$

$$e_{it} = \gamma_i^T f_t + \varepsilon_{it}; \mu_i \sim iidN(0,1);$$

$$f_t \sim iidN(0,1)\,;\, \varepsilon_{it} \sim iidN(0,\sigma_i^2)\,;\, \sigma_i^2 \sim iidU[0.5,1.5].$$

The performance of the tests is measured by setting: 1) $\phi_i = 1$ and 2) $\phi_i \sim U[0.75,0.95]$ for computing the size and power of the test, respectively. The setup of γ_i is similar to the other experiments in the previous chapter, which is as follows:

(i) $\gamma_i = 0$ for no cross dependency;

(ii) $\gamma_i \sim iidU(0.1,0.3)$ for mild cross dependency and ;

(iii) $\gamma_i \sim iidU(0.5,1.5)$ for strong effect of cross dependency.

A panel contaminated by outliers is represented by

$$y^*_{it} = y_{it} + \xi(L)\omega I_{it}(\tau) \qquad \text{for } i = 1,2,\ldots,N. \quad t = -49,\ldots,0,1,2,\ldots,T$$

where

y^*_{it} is the observed contaminated series

y_{it} is the uncontaminated series

$\xi(L)$ is the dynamic pattern of the outliers

ω is the magnitude of outliers

$I_{it}(\tau)$ is the indicator function of the presence of outlier at time $t = \tau$ which takes the following form:

$$I_{it}(\tau) = \begin{cases} 1 & \text{if } t = \tau \\ 0 & \text{elsewhere} \end{cases}$$

In the presence of AO, $\xi(L) = 1$ while TC takes the form of $\xi(L) = \dfrac{1}{1-\delta L}$ where δ represents the velocity of the dynamic effect and is bound by $[0,1]$ (Tsay, 1998).

The distribution of the test can be computed in the traditional way (see Fuller, 1976; and Herce, 1993). However, the computation of RCIPS is similar to the CADF statistics under the null hypothesis of absence of unit root based on 5,000 replications. In order to compute the critical values of the test statistics at 1%, 5% and 10% nominal levels, the pair of sample size $N = (20,30,50)$ and $T = (20,30,50,100,200)$ are chosen. The size and power of the tests are investigated at the 5% significant level using the same sample size with 500 replications.

5.3.1 Results and Discussion

The critical values for the ADF and RCIPS tests are provided in Tables 5.1 to 5.3. The nominal levels chosen are at 1%, 5% and 10%, respectively. The values of the mean and standard deviation of the respective unit root tests are also reported. For $N = 20$, the critical values for the ADF test decreases at each respective nominal level, as T increases. Similar results are obtained for the mean and standard deviation of this test with a large T. The average and standard deviation of the ADF test statistics are closer to -1.5 and 0.12, respectively, for large N and T, and this is expected based on Figure 5.1. Note that, the average of the ADF test is comparable with those obtained in Im et al. (2003). The t-statistics of the RCIPS is slightly larger than the ADF unit root tests (Tables 5.3 to 5.4). It is observed that these values approximated -1.28 (mean) and 0.2 (standard deviation for large N and T) (see Figure 5.2). As we would expect in theory, as the sample size increase, the absolute of critical value will also increase. The results at 5% nominal levels are used as critical values in order to compute the size and power of the tests which are reported in Tables 5.5 to 5.10.

The results at 5% nominal levels are used as critical values in order to compute the size and power of the tests which are reported in Tables 5.5 to 5.10.

The size and power of the unit root tests are investigated for the uncontaminated panel, the panel with additive outliers (AO) and the panel with temporary change (TC) which is based on 500 replications. The results are tabulated in Tables 5.5 to 5.10. For the uncontaminated panel, the size of the ADF unit root is 0 for the small N and T under the cross sectional independence ($\gamma_i = 0$). Similar results hold when the number of N and T increases. The CIPS gives a similar size for a small sample but attains a reasonable size as T increases. The t-test of the RCIPS however is slightly oversized even when N and T are large. In the presence of the CD ($\gamma_i \sim iidU(0.1,0.3)$ and $\gamma_i \sim iidU(0.5,1.5)$), the size of the ADF, CIPS and RCIPS are comparable with those in the absence CD in the panel. The behaviour of these tests can be observed in Figure 5.3.

In the presence of the AO and TC (see Tables 5.6 to 5.7) in the panel, the sizes for the ADF test are 0 for all CD cases. The CIPS also provides similar results for all sample sizes and CD cases. The size for the RCIPS however is small for $(N,T) = (20,20)$. The results hold as N increases in the presence of AO. The RCIPS does not perform well in the presence of AO in the panel. In the presence of the TC, the RCIPS achieves good size of the test compared to the others and comparable with the uncontaminated panel. The behaviour of these tests (ADF, CIPS and RCIPS) is illustrated in Figures 5.4 to 5.5 in the presence of AO and TC.

In investigating the power of the test in the uncontaminated panel (See Table 5.8 and Figure 5.6), the ADF test surprisingly yields a stronger power compared to the CIPS for $T \leq 50$ under the CD even though the ADF test relaxes the assumption of the CD. The CIPS however provides good power as T increases and the result is comparable to those obtained in Pesaran (2007). The RCIPS outperforms the ADF and CIPS even for small sample and all CD cases. In the presence of the AO and TC (See Tables 5.9 to 5.10 and Figures 5.7 to 5.8) in the panel, the powers for the ADF and CIPS tests are poor when $T \leq 50$. The power however increases for $T \geq 100$ with an increasing N. The powers for all tests are good as N increases in the presence of TC in the panel. The RCIPS provides a sensible power when $T \leq 30$ in the presence of the AO but outperforms the ADF and CIPS in the presence of the TC. Based on these results, the RCIPS provides a good reasonable size[57] (close to 0.05) and power (greater than

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[57] The definition of size and power of the tests have been discussed in Section 2.5, Chapter 2.

0.95) in the presence of the AO and TC relative to the other two estimators especially when N and T are small.

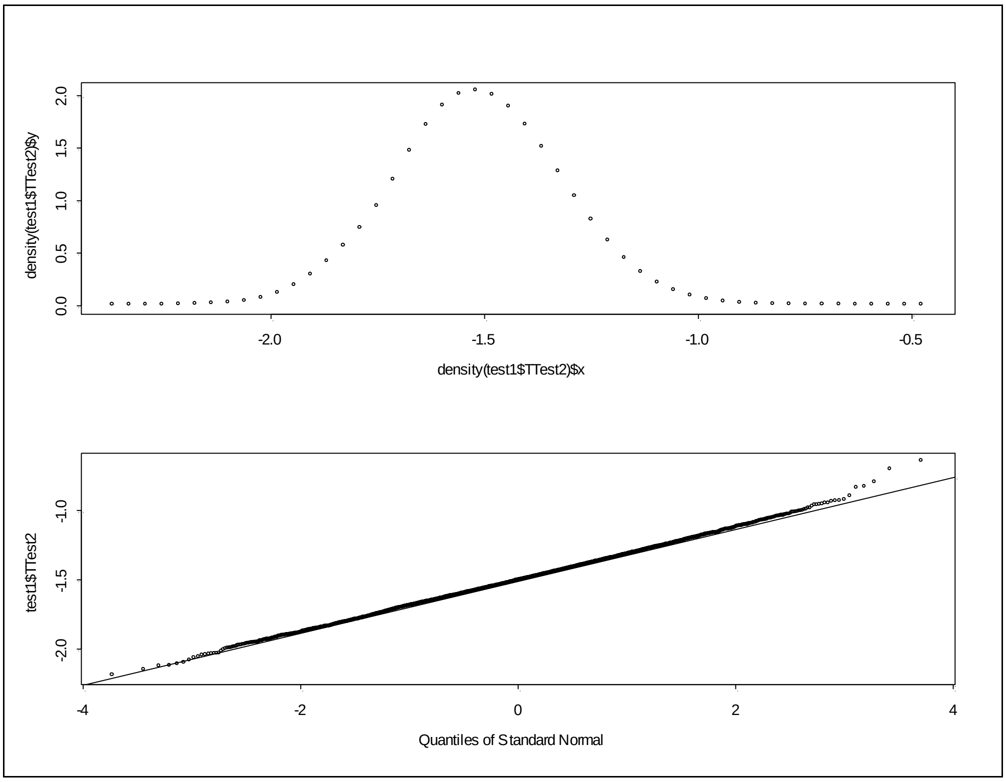

Figure 5.1 : The Density and QQ plots of t-statistics (ADF unit root test)

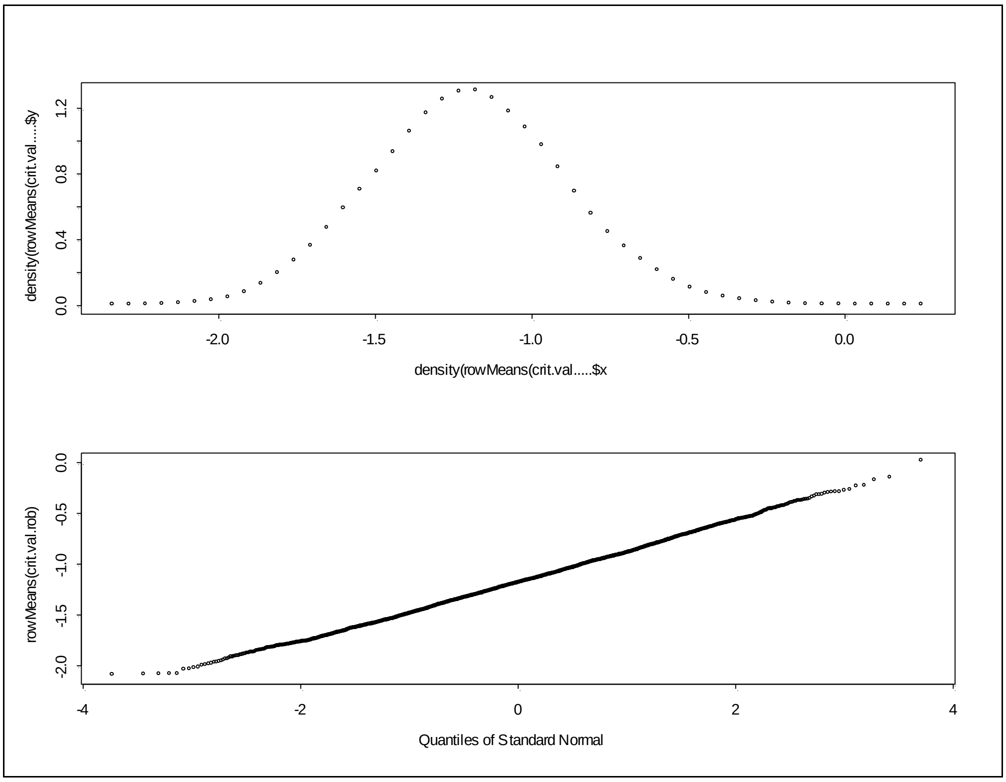

Figure 5.2: The Density and QQ plots of t-statistics (RCIPS unit root test)

Table 5.1: Critical Values for the ADF Unit Root Tests

T/*N*	20			30			50		
Quartile	1%	5%	10%	1%	5%	10%	1%	5%	10%
20	-1.8719	-1.7451	-1.6746	-1.8021	-1.6848	-1.6249	-1.7071	-1.6117	-1.5702
30	-1.9041	-1.7858	-1.7167	-1.8443	-1.7182	-1.6575	-1.7348	-1.6591	-1.6164
50	-1.9276	-1.7562	-1.7340	-1.8473	-1.7445	-1.6861	-1.7847	-1.6946	-1.6522
100	-1.9429	-1.8244	-1.7531	-1.8621	-1.7671	-1.7132	-1.7893	-1.7092	-1.6637
200	-1.9661	-1.8365	-1.7679	-1.8751	-1.7714	-1.7178	-1.7983	-1.7184	-1.6762

The table contains the 1%, 5% and 10% critical values of the proposed test and it is based on 5,000 numbers of runs.

Table 5.2: Summary Statistics of Average $\left(\bar{t}^*\right)$ ADF

Mean

T/*N*	20	30	50
20	-1.4067	-1.4039	-1.4062
30	-1.4536	-1.4528	-1.4558
50	-1.4895	-1.4847	-1.4906
100	-1.5120	-1.5145	-1.5100
200	-1.5241	-1.5208	-1.5221

Standard Deviation

T/*N*	20	30	50
20	0.2045	0.1699	0.1291
30	0.1992	0.1611	0.1271
50	0.1933	0.1605	0.1255
100	0.1886	0.1539	0.1217
200	0.1911	0.1551	0.1197

Table 5.3: Critical Values for the Proposed Unit Root Tests (RCIPS)

T/*N*		20			30			50	
Quartile	1%	5%	10%	1%	5%	10%	1%	5%	10%
20	-1.6240	-1.3834	-1.2711	-1.5179	-1.3423	-1.2458	-1.4291	-1.2888	-1.2124
30	-1.6565	-1.4592	-1.3637	-1.6139	-1.4300	-1.3319	-1.5264	-1.3843	-1.2931
50	-1.7569	-1.5555	-1.4484	-1.6979	-1.4987	-1.4138	-1.6126	-1.4483	-1.3692
100	-1.8267	-1.6090	-1.5238	-1.7662	-1.5894	-1.6866	-1.6866	-1.5242	-1.4575
200	-1.8983	-1.6946	-1.5992	-1.8397	-1.6613	-1.5646	-1.7706	-1.6182	-1.5319

The table contains the 1%, 5% and 10% critical values of the proposed test and it is based on 5,000 numbers of runs.

Table 5.4: Summary Statistics of Average $\left(\bar{t}^*\right)$ RCIPS

Mean

T/*N*	20	30	50
20	-0.9418	-0.9465	-0.9576
30	-1.0274	-1.0365	-1.0375
50	-1.1204	-1.1165	-1.1273
100	-1.2015	-1.2130	-1.2077
200	-1.2717	-1.2753	-1.2794

Standard Deviation

T/*N*	20	30	50
20	0.2569	0.2269	0.1916
30	0.2488	0.2253	0.1920
50	0.2503	0.2225	0.1885
100	0.2448	0.2186	0.1922
200	0.2483	0.2227	0.2010

Table 5.5: Size of the Unit Root Tests in the Uncontaminated Panel

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.000	0.004	0.042	0.000	0.000	0.074	0.000	0.000	0.082
30	0.000	0.014	0.080	0.000	0.008	0.050	0.000	0.008	0.066
50	0.000	0.024	0.062	0.000	0.014	0.056	0.000	0.014	0.076
100	0.000	0.035	0.068	0.000	0.028	0.080	0.000	0.012	0.052
200	0.000	0.036	0.080	0.000	0.052	0.080	0.000	0.036	0.080
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.000	0.008	0.040	0.000	0.004	0.074	0.000	0.002	0.068
30	0.000	0.012	0.082	0.000	0.002	0.052	0.000	0.004	0.066
50	0.000	0.012	0.070	0.000	0.012	0.056	0.000	0.014	0.080
100	0.000	0.029	0.072	0.000	0.028	0.080	0.000	0.036	0.076
200	0.000	0.050	0.074	0.000	0.034	0.064	0.000	0.024	0.080
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.002	0.006	0.040	0.000	0.002	0.058	0.000	0.006	0.056
30	0.000	0.010	0.074	0.000	0.004	0.042	0.000	0.002	0.062
50	0.000	0.012	0.052	0.000	0.014	0.048	0.000	0.008	0.058
100	0.000	0.046	0.076	0.000	0.022	0.078	0.000	0.028	0.074
200	0.000	0.034	0.068	0.000	0.034	0.058	0.000	0.024	0.080

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

Table 5.6: Size of the Unit Root Tests in the Presence of the AO

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.000	0.000	0.016	0.000	0.000	0.022	0.000	0.000	0.006
30	0.000	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.002
50	0.000	0.000	0.008	0.000	0.000	0.018	0.000	0.000	0.000
100	0.000	0.000	0.026	0.000	0.000	0.020	0.000	0.000	0.008
200	0.000	0.000	0.046	0.000	0.000	0.012	0.000	0.000	0.016
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.000	0.000	0.006	0.000	0.000	0.014	0.000	0.000	0.002
30	0.000	0.000	0.016	0.000	0.000	0.004	0.000	0.000	0.002
50	0.000	0.000	0.001	0.000	0.000	0.008	0.000	0.000	0.010
100	0.000	0.000	0.034	0.000	0.000	0.012	0.000	0.000	0.016
200	0.000	0.000	0.044	0.000	0.000	0.034	0.000	0.000	0.038
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.000	0.000	0.008	0.000	0.002	0.006	0.000	0.000	0.004
30	0.000	0.000	0.012	0.000	0.000	0.010	0.000	0.000	0.002
50	0.000	0.000	0.004	0.000	0.000	0.026	0.000	0.000	0.020
100	0.000	0.000	0.048	0.000	0.000	0.070	0.000	0.000	0.052
200	0.000	0.000	0.080	0.000	0.000	0.076	0.000	0.000	0.070

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

Table 5.7: Size of the Unit Root Tests in the Presence of TC

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.000	0.000	0.066	0.000	0.000	0.036	0.000	0.000	0.048
30	0.000	0.000	0.044	0.000	0.000	0.026	0.000	0.000	0.032
50	0.000	0.000	0.038	0.000	0.000	0.040	0.000	0.000	0.046
100	0.000	0.000	0.064	0.000	0.000	0.054	0.000	0.000	0.046
200	0.000	0.006	0.054	0.000	0.000	0.052	0.000	0.000	0.042
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.000	0.000	0.054	0.000	0.000	0.064	0.000	0.000	0.062
30	0.000	0.000	0.048	0.000	0.000	0.040	0.000	0.000	0.038
50	0.000	0.000	0.048	0.000	0.000	0.048	0.000	0.000	0.060
100	0.000	0.002	0.066	0.000	0.000	0.046	0.000	0.000	0.064
200	0.000	0.006	0.076	0.000	0.000	0.042	0.000	0.000	0.074
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.002	0.000	0.038	0.002	0.000	0.038	0.000	0.000	0.056
30	0.000	0.000	0.042	0.000	0.000	0.022	0.000	0.000	0.044
50	0.000	0.000	0.030	0.000	0.000	0.032	0.000	0.000	0.054
100	0.002	0.000	0.050	0.000	0.000	0.044	0.000	0.000	0.038
200	0.000	0.008	0.042	0.000	0.000	0.052	0.000	0.000	0.044

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

Table 5.8: Power of the Unit Root Tests in the Uncontaminated Panel

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.516	0.000	0.740	0.668	0.034	0.914	0.878	0.028	0.974
30	0.852	0.206	0.916	0.958	0.250	0.974	0.998	0.274	0.982
50	1.000	0.866	0.982	1.000	0.952	0.994	1.000	0.994	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.548	0.000	0.762	0.662	0.024	0.926	0.884	0.014	0.956
30	0.862	0.216	0.926	0.972	0.256	0.978	0.998	0.236	0.990
50	1.000	0.852	0.992	1.000	0.940	0.998	1.000	0.992	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.518	0.000	0.792	0.610	0.022	0.912	0.708	0.018	0.952
30	0.744	0.206	0.920	0.770	0.240	0.964	0.812	0.282	0.982
50	0.950	0.862	0.994	0.954	0.952	1.000	0.964	0.998	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

Table 5.9: Power of the Unit Root Tests in the Presence of AO

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.028	0.000	0.504	0.024	0.000	0.558	0.012	0.000	0.680
30	0.050	0.002	0.592	0.072	0.000	0.788	0.040	0.000	0.842
50	0.164	0.014	0.840	0.134	0.002	0.940	0.204	0.000	0.988
100	0.558	0.876	0.998	0.822	0.266	1.000	0.954	0.342	1.000
200	0.998	0.952	1.000	1.000	0.994	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.020	0.004	0.474	0.024	0.000	0.532	0.026	0.000	0.706
30	0.060	0.000	0.628	0.068	0.000	0.780	0.064	0.000	0.798
50	0.178	0.010	0.870	0.164	0.002	0.936	0.214	0.000	0.994
100	0.672	0.882	1.000	0.836	0.240	1.000	0.962	0.306	1.000
200	1.000	0.966	1.000	1.000	0.990	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.188	0.000	0.422	0.210	0.000	0.480	0.304	0.000	0.682
30	0.276	0.000	0.616	0.310	0.000	0.752	0.372	0.000	0.804
50	0.482	0.010	0.834	0.554	0.004	0.922	0.644	0.000	0.986
100	0.938	0.918	1.000	0.958	0.282	1.000	0.986	0.354	1.000
200	1.000	0.980	1.000	1.000	0.994	1.000	1.000	1.000	1.000

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

Table 5.10: Power of the Unit Root Tests in the Presence of TC

	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS	ADF	CIPS	RCIPS
N		20			30			50	
T					$\gamma_i = 0$				
20	0.014	0.000	0.796	0.030	0.000	0.854	0.014	0.000	0.956
30	0.050	0.000	0.910	0.086	0.000	0.964	0.080	0.000	0.988
50	0.350	0.010	0.982	0.406	0.006	0.996	0.660	0.010	1.000
100	0.978	0.746	1.000	1.000	0.906	1.000	1.000	0.954	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.1,0.3)$				
20	0.014	0.000	0.772	0.012	0.000	0.884	0.016	0.000	0.970
30	0.064	0.000	0.874	0.096	0.000	0.976	0.094	0.000	0.988
50	0.294	0.014	0.968	0.440	0.010	0.994	0.670	0.008	1.000
100	0.980	0.752	1.000	1.000	0.916	1.000	1.000	0.948	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
					$\gamma_i \sim iidU(0.5,1.5)$				
20	0.150	0.000	0.788	0.152	0.000	0.832	0.268	0.000	0.960
30	0.288	0.000	0.864	0.384	0.000	0.962	0.440	0.000	0.980
50	0.642	0.026	0.968	0.698	0.022	0.988	0.828	0.022	1.000
100	0.992	0.836	1.000	0.994	0.952	1.000	0.998	0.978	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The results given are the proportion of rejecting the null if there is a unit root in panel based on 500 runs. The presence of the unit root is investigated in (1) ADF unit root test, (2) CIPS of Pesaran's unit root test, and (3) the proposed test, RCIPS. The test is significant based on its corresponding critical values given in Tables 5.1, C1 and 5.2, respectively. The critical value is set at the 5% significant levels.

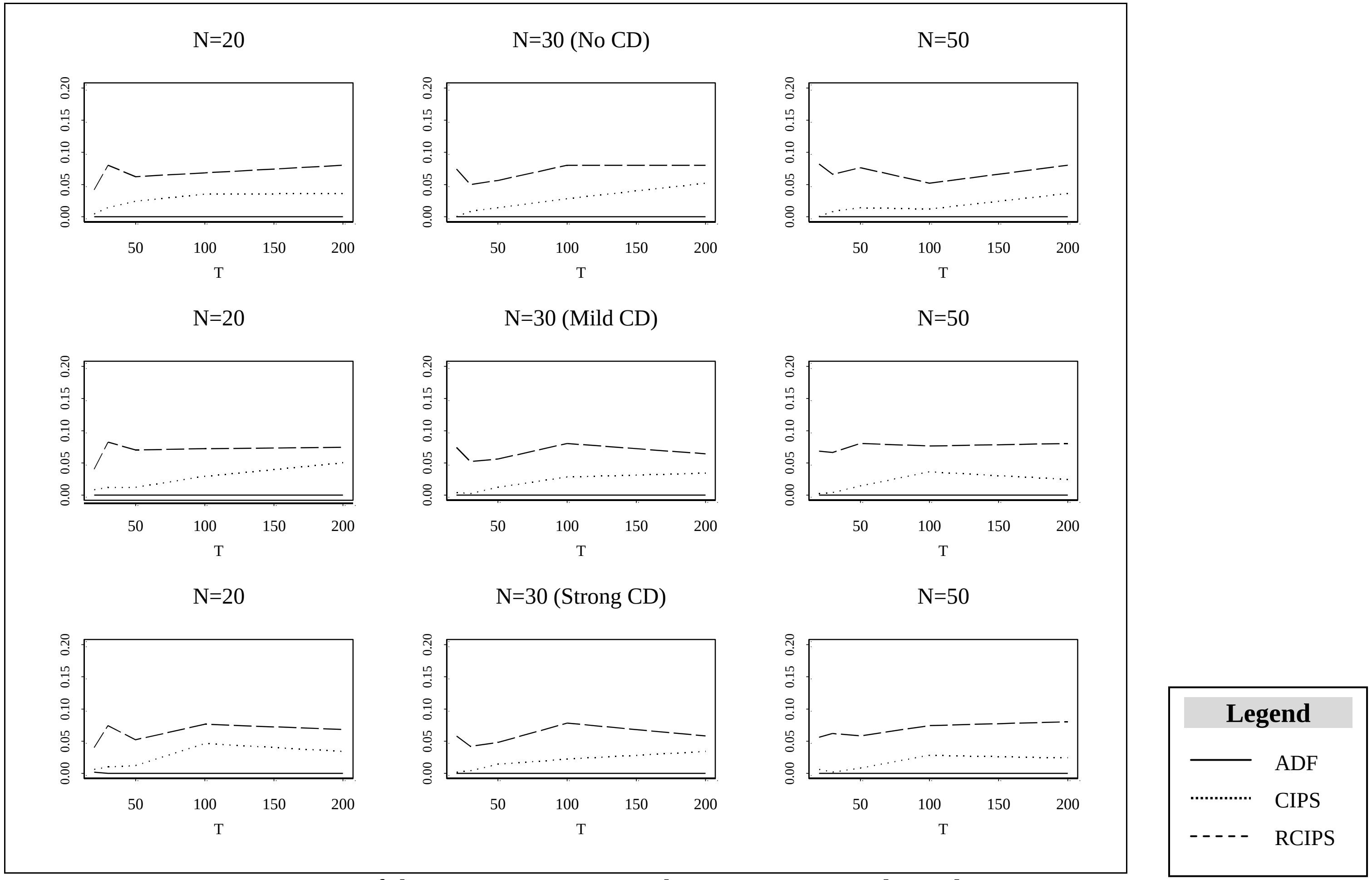

Figure 5.3: Size of the Unit Root Tests in the Uncontaminated Panel

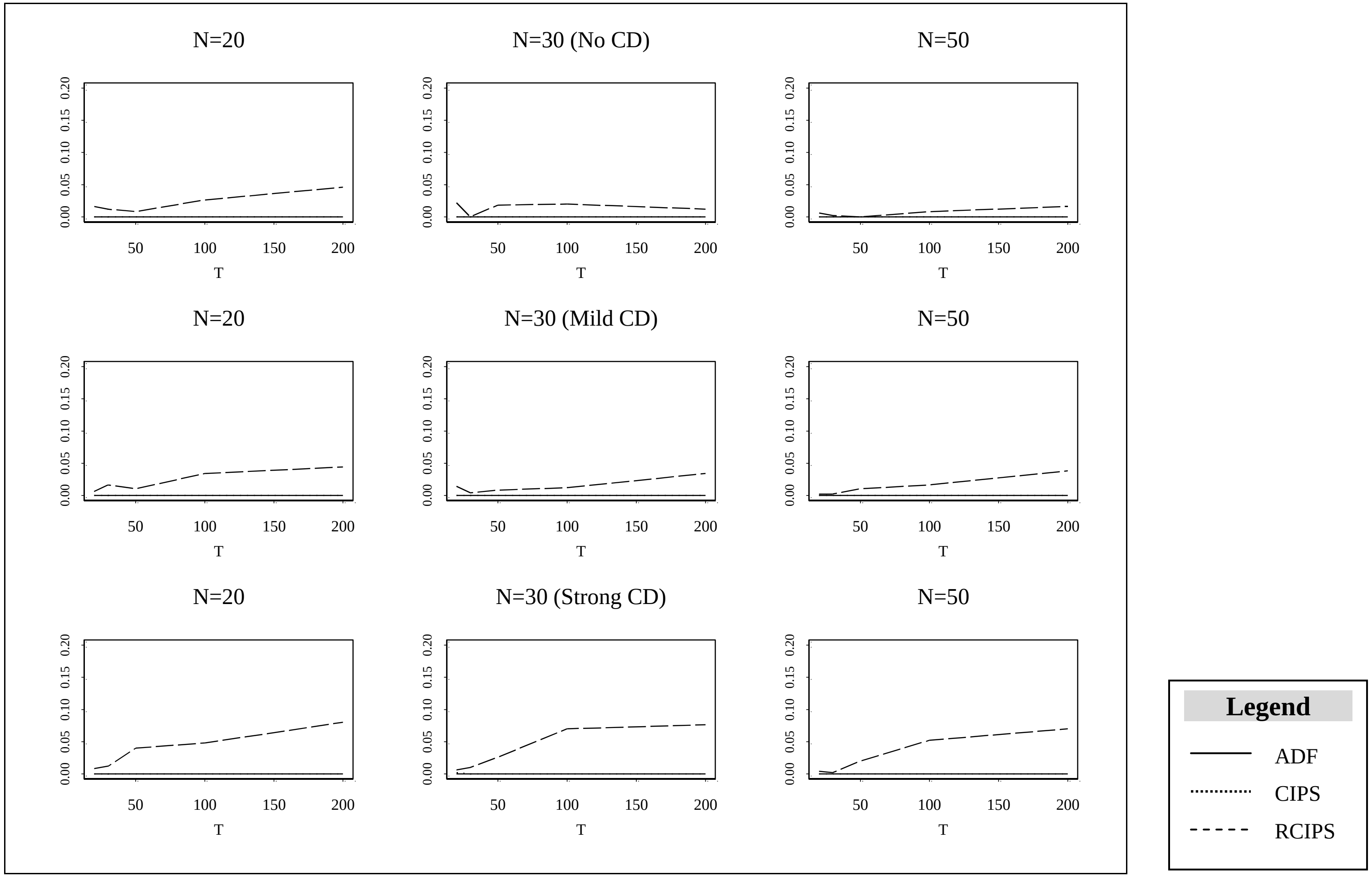

Figure 5.4: Size of the Unit Root Tests in the Presence of the AO in the Panel

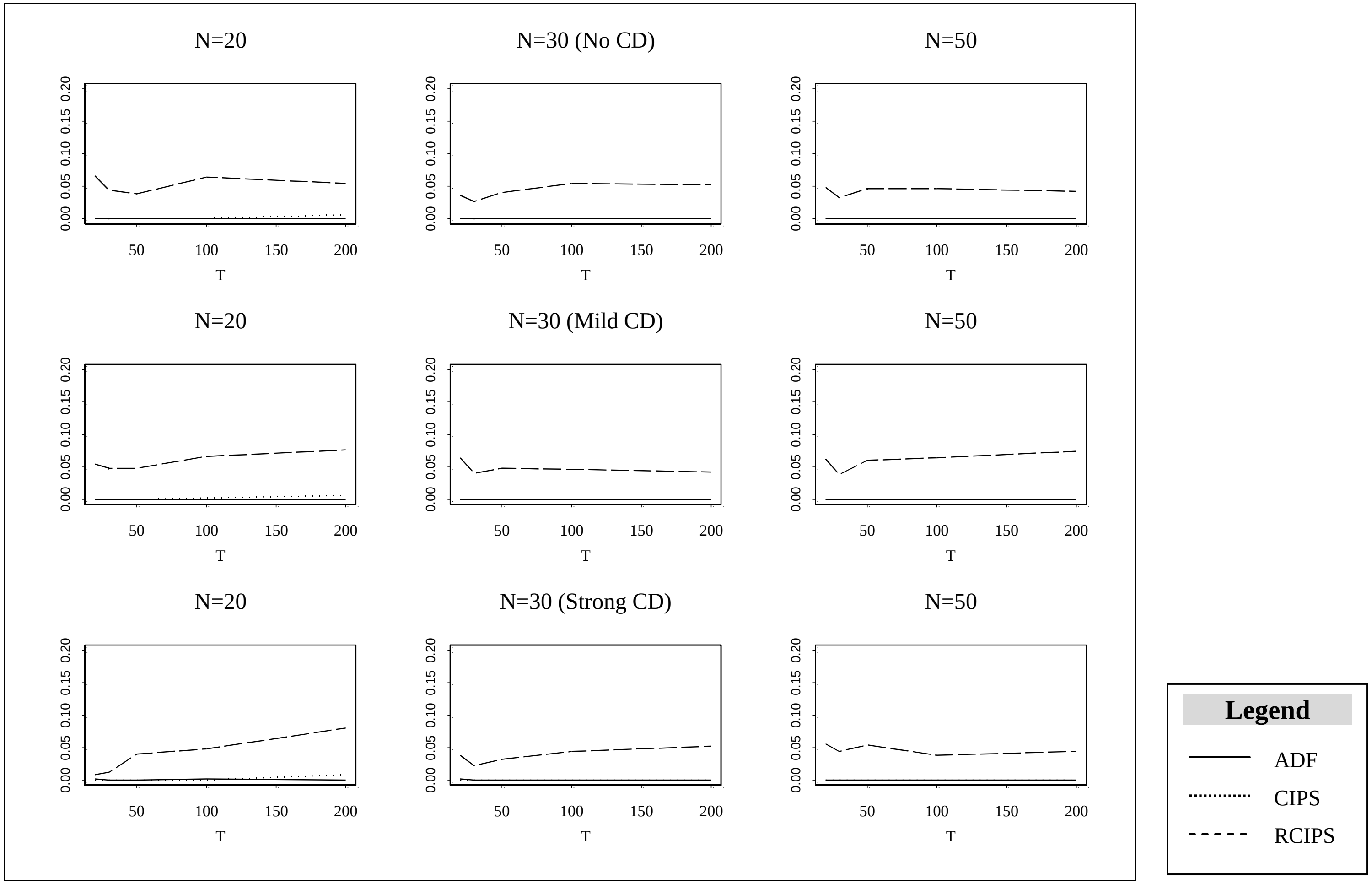

Figure 5.5: Size of the Unit Root Tests in the Presence of the TC in the Panel.

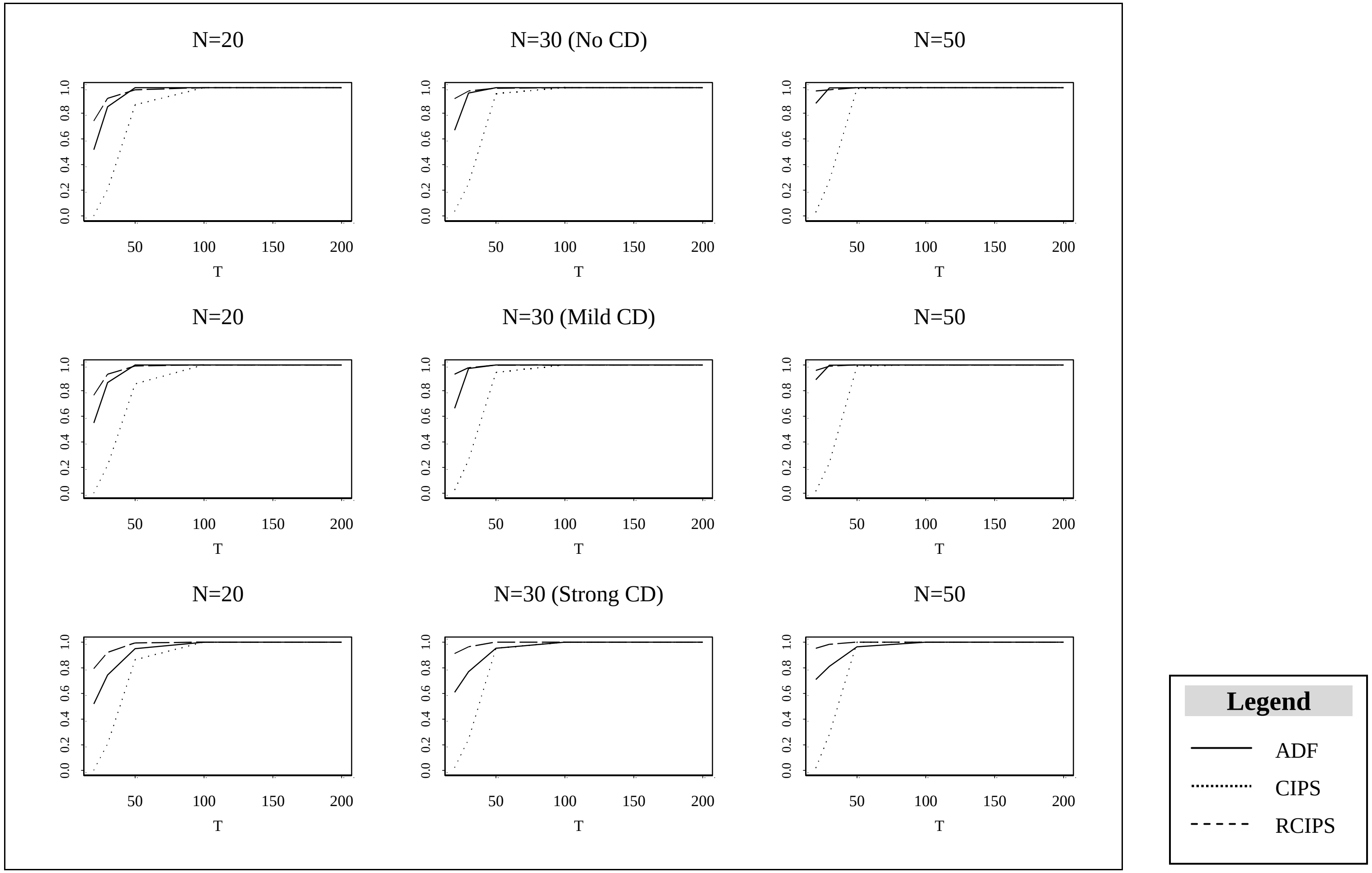

Figure 5.6: Power of the Unit Root Tests in the Uncontaminated Panel

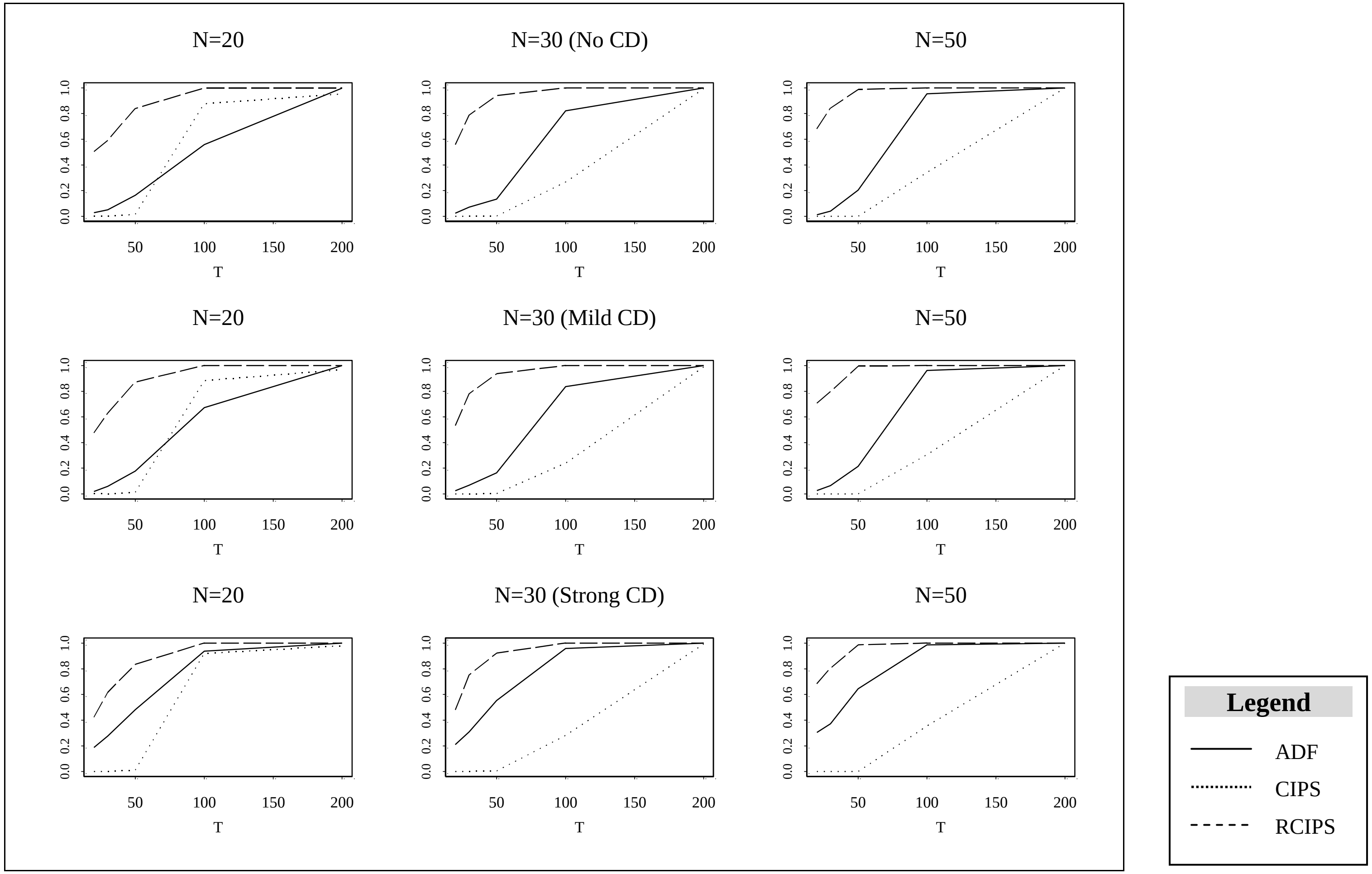

Figure 5.7: Power of the Unit Root Tests in the Presence of the AO in the Panel

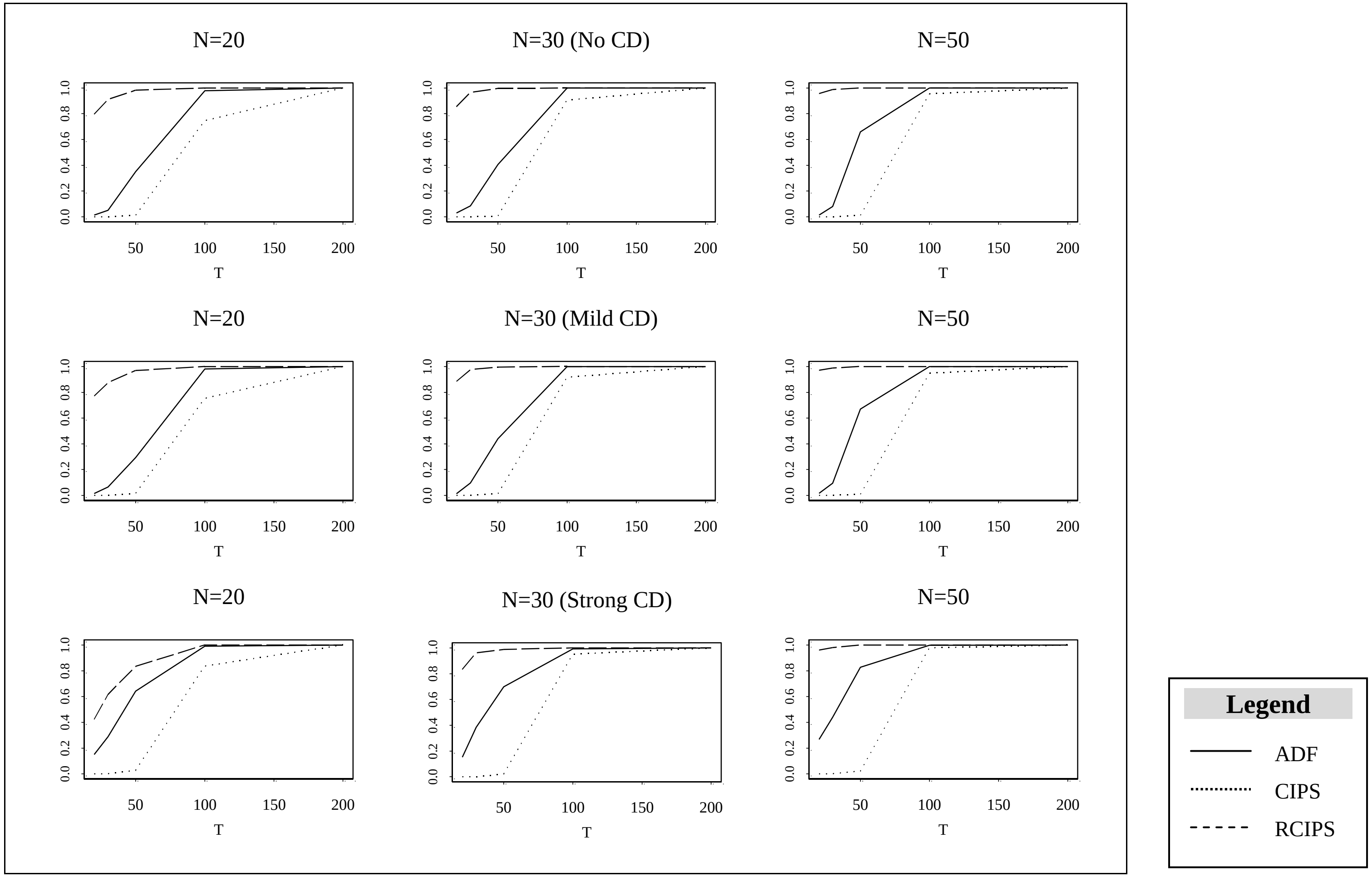

Figure 5.8: Power of the Unit Root Tests in the Presence of the TC in the Panel

5.4 Discussion

An alternative approach to the ADF and Pesaran unit root test is proposed in order to investigate the presence of the unit root when outliers occur in the panel. Two types of outliers are considered: (1) Additive outliers, and (2) Temporary change[58]. The proposed unit root test is based on the Generalized M-estimator which has been discussed in Chapter 3. The finite sample behaviour of the tests is studied and compared via the Monte Carlo experiments. The results show that the proposed unit root test outperforms the ADF and Pesaran's unit root test in terms of power with/ without the presence of outliers and CD in the panel especially for the small pair of sample size. The proposed test however suffers slightly from size distortion compared to the ADF in the uncontaminated panel.

[58] The definition of these outliers has been given in Chapter 1.

CHAPTER 6

Empirical Illustrations

6.1 Introduction

There are considerable amount of empirical studies conducted to illustrate the behaviour of the tests and estimation procedure in the panel framework (see Philips and Sul, 2003; Coakley et al., 2002; Pesaran, 2004; Noman, 2008; and Serlenga and Shin, 2004). Kapetanios and Pesaran (2004) compared two alternative methods in modeling a panel set of company returns. One uses the CMG technique and the other uses the principal component of Stock and Watson (2002). Serlenga and Shin (2004) generalized the Hausman-Taylor estimation methodology and used the factor structure as a proxy on the unobserved factor. The proposed method is applied to a gravity equation of bilateral trade flows amongst the 15 European countries over the period of 1960 to 2001. The results show that the proposed approach fit the data reasonably well and provide much more sensible results than the conventional methods such as pooled, FE, RE and between or cross section regression methods.

Empirical applications on the CD tests recently have been studied in Pesaran (2004), Cerrato and Sarantis (2007), Hoyos and Sarafidis (2006), and Sarafidis et al. (2009). Pesaran used the PCD test to investigate the presence of the CD for per capita output of innovations across countries within a given region, and across countries in different regions. The results show a significant evidence of CD in output innovations across many countries and regions in the world. Cerrato and Sarantis tested for the presence of CD prior to applying the panel unit root tests in the PPP data of 20 OECD countries. The LM test results strongly reject the null hypothesis of the cross sectional independence among the group of innovations. Hoyos and Sarafidis described the new command of the Stata routine to test for the presence of the CD. The example of the Cobb-Douglas production relationship taken from Baltagi (2001) is used to illustrate the command and the results show evidence of the presence of CD in the data. In a recent study, Sarafidis et al. investigated the presence of CD in a linear dynamic model with regressor for employment equations of the United Kingdom (UK) firms. The results show that there is enough evidence to reject the null hypothesis of the cross sectional independence in the data.

Several studies have been conducted to investigate the presence of the unit root in the PPP data. MacDonald (1996) implemented the ADF unit root test using the real exchange rate (RER) data sets. The ADF test statistics is able to reject the null hypothesis of a unit root both for high frequency data (monthly) and annual data. Coakley and Fuertes (1997) used the Im et al. (1995) unit root test to analyse the stationary of the RER for the G10[59] economies and Switzerland. The results show that there is a mean reversion in the RER in the period of the monthly data of 1973 -1996 and a half life of less than three years for a one-off shock. Caporale and Cerrato (2006) highlighted various drawbacks of the standard panel data methods, which represents important challenges for future research, such as: (i) the existing unit root tests suffer from severe size distortions in the presence of negative moving average errors, (ii) the common de-meaning procedure used to correct the bias however has a non effective result in the presence of CD; and (iii) co-integration between cross sectional units could also lead to size distortion however it is concluded that the panel approach is unlikely to solve the PPP puzzle. Harris et al. (2005) investigated the PPP hypothesis in the case when the CD is present. The data of a group of 17 countries are investigated using a new panel based test of stationary that allows for arbitrary CD. The results of unit root tests indicate that there is significant evidence against the PPP.

[59] G-10 refers to the group of countries that have agreed to participate in the General Arrangements to Borrow (GAD).

In this chapter, real examples of the panel dataset through a proper estimation procedure are examined. The aim of this chapter is to estimate the appropriate model and provide a reasonable explanation about the relationship of the economic variables. In this estimation process, the CD tests are evaluated first. The chapter is divided into two parts: (1) first, for the pure static model, the presence of the CD is investigated and then the parameter of the model is estimated, (2) second, for the dynamic model, the CD tests are considered first and then the presence of a unit root is investigated in the panel.

6.2 Estimation Procedure

The example of the Gasoline data is considered in modeling the panel dataset for the pure static case. Prior to estimating the model, the presence of the CD is examined. In the estimation procedure, the parameters of the model are estimated using the methods of: (1) Pooled, (2) CMG, and (3) RCMG, as discussed in Chapter 3. The fitted model is evaluated using the criteria of: (1) the coefficient of determination $\left(R^2\right)$; (2) a robust version of R^2, $\left(RR^2\right)$; (3) cross-validation (CV) and CV^2; together with (4) a robust version of CV and CV^2, RCV and RCV^2 as discussed in Section 3.4, Chapter 3.

6.2.1 Gasoline Data

The gasoline data are annual demands (observations) of 18 OECD countries for the sample period of 1960 until 1978. These data are available at http://www.wiley.com/legacy/wileychi/baltagi/supp/Gasoline.dat (Baltagi, 2001). It consist information of wide variations of per capita income, relative gasoline process, and cars per capita, both over time and across countries.

The approach is to model a gasoline consumption based on three variables: (1) utilisation of the typical auto, (2) gasoline efficiency, and (3) stock of cars on the road, which form the following consumption identity (Baltagi and Griffin, 1983):

Gasoline = Miles driven × Gasoline consumption × Cars consumption per car per mile

$$= \text{Utilisation} \quad \times \quad \frac{1}{\text{efficiency}} \quad \text{Stock of cars} \qquad (6.1)$$

The advantage of analysing gasoline consumption in these three components is that the effects of short and long run consumption can be separated. For example, changes in the utilisation of cars may be achieved in the short run with the existing stock of cars, while changes in auto efficiency need a longer period to turn over the car stock. Due to the lack of data on the miles driven and gasoline consumption per mile, the utilisation and efficiency factors are combined, thus leaving gasoline consumption per car $\left(Gas/Car\right)$ to be explained by variables of utilisation (*U*) and efficiency (*E*) as follows:

$$Gas/Car = U/E \qquad (6.2)$$

The empirical determination of (6.2) is based on the variables that reflect the utilisation of cars, such as per capita income $\left(Y/N\right)$ and gasoline price $\left(P_{MG}/P_{GDP}\right)$. On the other hand, the rising stock of cars per capita $\left(Car/N\right)$ is likely to lead to reduced car utilisation. Specifically, the model describes how gasoline price and income affect the utilisation and the efficiency of the car fleet. Thus, the following relationship which is the model of Baltagi and Griffin (1983) is considered:

$$\ln\frac{Gas}{Car} = \beta_1 \ln\frac{Y}{N} + \beta_2 \ln\frac{P_{MG}}{P_{GDP}} + \beta_3 \ln\frac{Car}{N} + u \qquad (6.3)$$

where $\ln\frac{Gas}{Car}$ is the logarithm of motor gasoline consumption per car; $\ln\frac{Y}{N}$ is the logarithm of real per capita income; $\ln\frac{P_{MG}}{P_{GDP}}$ is the logarithm of real motor gasoline price; $\ln \frac{Car}{N}$ is the logarithm of the stock of cars per capita; $\beta_1, \beta_2, \beta_3$ are parameters to be estimated for the respective independent variable; and u is the residual.

Table 6.1 provides a statistical summary of the gasoline consumption per car for all 18 OECD countries. Turkey records the highest average annual gasoline consumption (5.8) followed by Canada, Greece and the US (4.8-4.9). Italy exhibits the lowest gasoline consumption. With small kurtosis for all countries, it is expected that gasoline consumption per car value is closer to the mean, with no fat tails, but slightly skewed.

Table 6.1: Summary Statistics of $\ln \frac{Gas}{Car}$ for 18 OECD Countries

	Min	**Max**	**Mean**	**Median**	**Std. Dev**	**Skewness**	**Kurtosis**
Austria	3.9227	4.1994	4.0565	4.0475	0.0693	0.0592	0.5685
Belgium	3.8182	4.1640	3.9223	3.8778	0.1034	1.2468	0.5826
Canada	4.8110	4.8997	4.8624	4.8558	0.0262	-0.0949	-0.8663
Denmark	4.0005	4.5020	4.1899	4.1617	0.1582	0.6495	-0.5593
France	3.7495	3.9081	3.8152	3.8080	0.0499	0.7487	-0.5537
Germany	3.8488	3.9324	3.8934	3.8894	0.0239	0.1215	-0.6616
Greece	4.4800	5.3815	4.8787	4.8948	0.2547	-0.0028	-0.6499
Ireland	4.1649	4.3256	4.2256	4.2211	0.0437	0.4306	-0.3332
Italy	3.3802	4.0507	3.7296	3.7374	0.2200	-0.0867	-1.0947
Japan	3.9487	5.9953	4.6996	4.5183	0.6841	0.5318	-1.0926
Netherland	3.7114	4.6463	4.0803	3.9877	0.2864	0.5780	-0.7250
Norway	3.9603	4.4350	4.1098	4.0846	0.1231	1.2787	1.6262
Spain	3.6204	4.7494	4.0553	3.9941	0.3170	0.6445	-0.2065
Sweden	3.9132	4.0674	4.0061	4.0026	0.0364	-0.3338	1.3780
Switzerland	4.0500	4.4413	4.2376	4.2592	0.1018	-0.0767	-0.0388
Turkey	5.1413	6.1566	5.7664	5.7221	0.3290	-0.4013	-1.1804
UK	3.9126	4.1002	3.9847	3.9768	0.0479	1.3011	1.5555
US	4.7879	4.8603	4.8191	4.8110	0.0219	0.5910	-0.9557

Table 6.2: The Correlation Results among the $(i, j)^{\text{th}}$ $(\hat{\rho}_{ij})$ Cross Sectional Units of 18 OECD Countries

(i, j)	**Austria**	**Belgium**	**Canada**	**Den-mark**	**France**	**Germany**	**Greece**	**Ireland**	**Italy**	**Japan**	**Nether-land**	**Norway**	**Spain**	**Sweden**	**Switzer-land**	**Turkey**	**UK**	**US**
Austria	1																	
Belgium	0.2232	1																
Canada	-0.1216	0.0062	1															
Denmark	0.1078	0.2657	0.0875	1														
France	0.1702	0.1568	-0.0339	-0.1117	1													
Germany	0.5281	0.5635	0.1212	-0.0725	0.1222	1												
Greece	-0.1496	-0.2590	-0.1384	0.1721	-0.1420	-0.1469	1											
Ireland	0.3410	0.1872	0.3348	-0.4431	0.0524	0.4785	-0.1229	1										
Italy	0.4136	0.6413	0.3324	0.3262	0.3296	0.6425	-0.3221	0.0765	1									
Japan	-0.2323	0.0008	-0.1187	0.0710	-0.1539	-0.0992	-0.2817	-0.4674	0.1326	1								
Netherland	0.1106	0.4795	0.4487	0.4933	0.0726	0.1409	-0.0280	-0.0751	0.4736	0.2036	1							
Norway	0.2021	0.5797	0.4091	0.3571	-0.1781	0.3193	-0.2481	0.4025	0.3523	-0.0587	0.6133	1						
Spain	0.4263	0.3728	-0.0203	0.0892	0.2151	0.2016	-0.3466	0.5053	0.0304	-0.5250	-0.0042	0.4923	1					
Sweden	0.0018	0.4639	-0.1285	0.3808	-0.0341	0.1530	0.0710	-0.3909	0.3468	0.1198	0.3102	-0.0259	-0.0994	1				
Switzerland	0.4248	0.4128	-0.1441	0.1920	0.4633	0.5549	0.2016	0.1405	0.5324	-0.0700	0.3545	0.2406	0.0294	0.2347	1			
Turkey	0.3558	-0.4073	0.0270	0.1141	-0.0195	0.1207	0.3349	-0.1759	0.1130	0.0454	-0.0685	-0.4880	-0.3366	0.2617	0.1208	1		
UK	0.1176	0.5108	0.5874	0.0367	0.1542	0.2463	-0.2279	0.6155	0.2976	-0.3293	0.4553	0.7352	0.5798	-0.1431	0.0195	-0.4878	1	
US	-0.0044	0.6397	0.4320	0.1789	0.1938	0.4333	-0.4969	0.3221	0.5451	0.0056	0.3272	0.5258	0.3841	0.3356	0.1560	-0.3288	0.5938	1

$\hat{\rho}_{ij}$ is computed as $\hat{\rho}_{ij} = \hat{\rho}_{ji} = \dfrac{\sum_{t=1}^{T} \hat{e}_{it}\hat{e}_{jt}}{\left(\sum_{t=1}^{T} \hat{e}_{it}^2\right)^{1/2} \left(\sum_{t=1}^{T} \hat{e}_{jt}^2\right)^{1/2}}$

Before estimating the model in (6.3), the presence of CD is investigated among the innovations in the model. The results are reported in Tables 6.2 and 6.3. From Table 6.2, some countries provide very small correlations, for example: between Belgium and Canada, and Belgium and Japan, indicating that these countries are independent to each other. The US, UK and Italy yield strong cross dependency within most of these 18 OECD countries. The LM, RLM1, RLM2, PCD, RPCD1 and RPCD2 (showed in Table 6.3) are compared at 5% significant levels based on the respective critical values. All tests provide significant evidence of the presence of the CD in model (6.3) and this result is comparable with the value of the absolute correlation $|\hat{\rho}| = 0.266$. In view of the presence of the CD, the parameter estimation procedure that relaxes the independence assumption should really be considered.

Table 6.3: Cross Dependency Test Results of Gasoline Data

Test	Test statistics
$\lvert\hat{\rho}\rvert$[60]	0.266
LM	303.296*
RLM1	314.660*
RLM2	282.630*
PCD	7.795*
RPCD1	7.382*
RPCD2	5.925*

*The test is significant when $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 182.865$ and $|\mathrm{PCD}|, |\mathrm{RPCD1}|, |\mathrm{RPCD2}| > N(0,1) = 1.96$ at 5% significant levels.

[60] $|\hat{\rho}|$ is computed as $|\hat{\rho}| = \dfrac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}|\hat{\rho}_{ij}|}{N(N-1)}$

Table 6.4: Estimation Results of Gasoline Data

	Methods	Pooled	CMG	RCMG
Parameter Estimates	$\hat{\beta}_1$	**0.195**	**0.123**	**-0.059**
	t-stats	7.654* (0.000)	1.386 (0.917)	-0.602 (0.274)
	$\hat{\beta}_2$	**-0.512**	**-0.214**	**-0.316**
	t-stats	-15.619* (0.000)	-4.716* (0.000)	-7.604* (0.000)
	$\hat{\beta}_3$	**-0.575**	**-0.593**	**-0.410**
	t-stats	-35.691* (0.000)	-6.221* (0.000)	-5.986* (0.000)
Goodness-of-fit	R^2	0.685	0.986	0.977
	RR^2	0.645	0.999	0.999

Note: * significant at 5% significant level

The estimators used to estimate the model parameters are (1) Pooled, (2) CMG, and (3) RCMG. The results are reported in Table 6.4. For the pooled estimator, $\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$ show significant test statistics with low $R^2 = 0.685$. These results are consistent with the fact that the cross sectional independence is violated since the CD test results provides evidence on the presence of the CD. On the other hand, the CMG and RCMG estimators which relax the independent assumption show that only $\hat{\beta}_2$ and $\hat{\beta}_3$ are significant. This suggests the importance of the real motor gasoline price and the stock of cars per capita variables (predictor) in predicting the gasoline consumption per car (response). The coefficient of determination R^2 and the robust version RR^2 indicate a good fit for both the CMG and RCMG estimator. Further checks on the residual plots illustrate the bell-shaped residuals distributed around 0 (see Figure 6.1) with slightly deviates from the normal distribution.

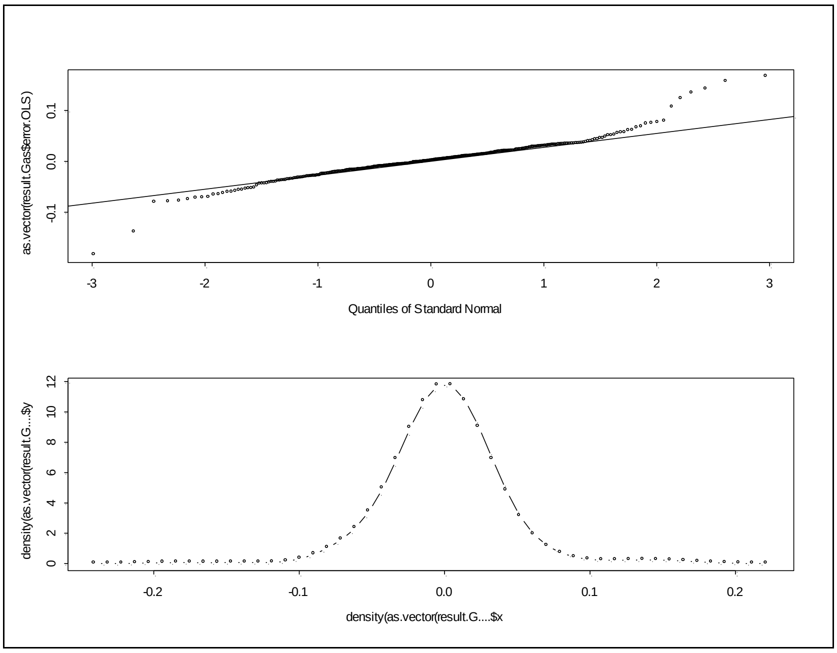

Figure 6.1: QQ and Density Plots of Gasoline Data

The CV and CV^2 for the respective estimation procedure are reported in Table 6.5. Both CMG and RCMG provide a comparable value of CV and CV^2 when the i^{th} pair of the dependent and independent variables are omitted.

Table 6.5: Cross-Validation Results of Gasoline Data
(CV)

i	Pooled	CMG	RCMG
1	0.23365	0.01677	0.02323
2	0.19195	0.01657	0.02299
3	0.17799	0.01656	0.02264
4	0.20511	0.01419	0.01993
5	0.20131	0.01304	0.01668
6	0.19827	0.01155	0.01555
7	0.19226	0.01024	0.01409
8	0.11023	0.01051	0.01479
9	0.09632	0.00793	0.01277
10	0.09914	0.00823	0.01391
11	0.11094	0.00898	0.01488
12	0.11628	0.01010	0.01610
13	0.12064	0.01075	0.01686
14	0.13380	0.01188	0.01842
15	0.17073	0.01289	0.01944
16	0.17767	0.01385	0.02037
17	0.18521	0.01689	0.02358
18	0.19094	0.01749	0.02434

$\left(CV^2\right)$

i	Pooled	CMG	RCMG
1	0.11603	0.00071	0.00110
2	0.07434	0.00071	0.00115
3	0.07064	0.00073	0.00115
4	0.10965	0.00052	0.00090
5	0.10692	0.00048	0.00069
6	0.10695	0.00044	0.00066
7	0.10485	0.00040	0.00060
8	0.03090	0.00055	0.00079
9	0.02674	0.00034	0.00063
10	0.02779	0.00035	0.00074
11	0.03170	0.00037	0.00077
12	0.03244	0.00040	0.00081
13	0.03297	0.00041	0.00082
14	0.03889	0.00044	0.00089
15	0.07349	0.00047	0.00092
16	0.07445	0.00050	0.00094
17	0.07587	0.00073	0.00118
18	0.07655	0.00074	0.00120

To illustrate the usefulness of the proposed method in presence of contamination, 5% contamination is allowed in $\frac{Gas}{Car}$ in model (6.2)[61]. The CD tests and parameter estimates are computed based on the three techniques mentioned earlier as well as the value of R^2, RR^2, CV and the results are reported Table 6.7.

In the presence of outliers (see Figure 6.3), the LM and PCD tests provide insignificant test statistics indicating that there is no cross dependency in the panel whereas the cross section dependence exists among the residuals (see the value $|\hat{\rho}|$ in Table 6.7). This shows that the cross dependency effect is masked by the presence of outliers. The robust versions of these tests however provide a significant result of there is cross dependency in the panel.

[61] The contamination chosen is from $\chi^2_{(20)}$ so that the band of this value will not exceed some points of *d*.

Table 6.6: Robust Cross-Validation Results of Gasoline Data

(RCV)

i	Pooled	CMG	RCMG
1	0.1463	0.0106	0.0167
2	0.1522	0.0113	0.0168
3	0.1422	0.0121	0.0178
4	0.1348	0.0118	0.0173
5	0.1541	0.0118	0.0162
6	0.1724	0.0112	0.0159
7	0.1878	0.0103	0.0153
8	0.1604	0.0098	0.0172
9	0.1515	0.0094	0.0175
10	0.1474	0.0098	0.0192
11	0.1468	0.0097	0.0178
12	0.1401	0.0099	0.0180
13	0.1350	0.0098	0.0176
14	0.1359	0.0102	0.0177
15	0.1436	0.0105	0.0171
16	0.1411	0.0106	0.0168
17	0.1436	0.0113	0.0177
18	0.1347	0.0112	0.0172

(RCV^2)

i	Pooled	CMG	RCMG
1	0.00602	0.00003	0.00009
2	0.00636	0.00004	0.00009
3	0.00550	0.00005	0.00011
4	0.00526	0.00004	0.00010
5	0.00685	0.00004	0.00008
6	0.00947	0.00003	0.00008
7	0.01128	0.00003	0.00007
8	0.01014	0.00003	0.00009
9	0.00886	0.00003	0.00010
10	0.00901	0.00003	0.00011
11	0.00827	0.00003	0.00010
12	0.00726	0.00003	0.00010
13	0.00619	0.00003	0.00010
14	0.00603	0.00003	0.00010
15	0.00636	0.00003	0.00009
16	0.00598	0.00004	0.00009
17	0.00619	0.00004	0.00010
18	0.00575	0.00004	0.00009

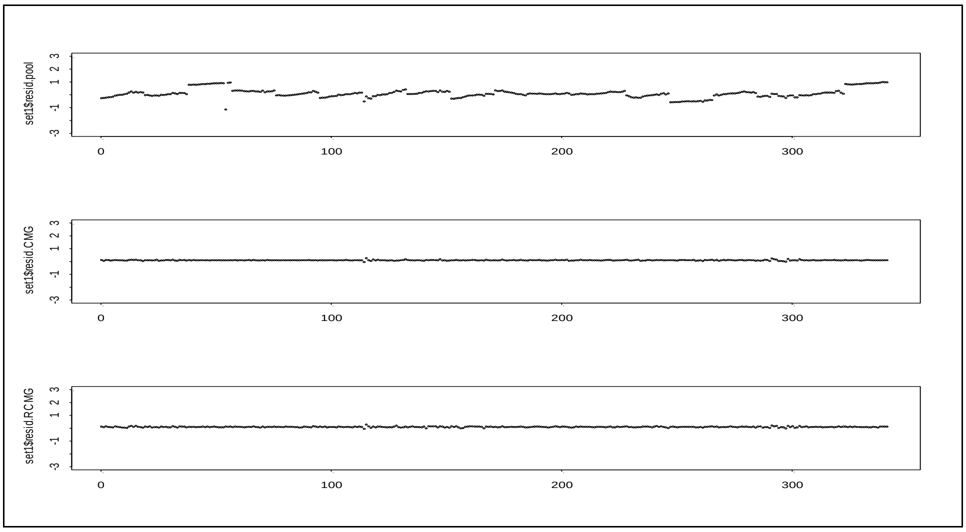

Figure 6.2: Residual Plots of Gasoline Data;
(1)Pooled model; (2) CMG; (3) RCMG

Table 6.7: Cross Dependency Test Results of Gasoline Data with 5% contamination

Test	Test statistics
$\lvert\hat{\rho}\rvert$ [62]	0.143
LM	139.253
RLM1	239.520*
RLM2	278.440*
PCD	1.028
RPCD1	8.023*
RPCD2	7.631*

*The test is significant when $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 182.865$ and $\lvert\mathrm{PCD}\rvert, \lvert\mathrm{RPCD1}\rvert, \lvert\mathrm{RPCD2}\rvert > N(0,1) = 1.96$ at 5% significant levels.

62 $\lvert\hat{\rho}\rvert$ is computed as $\lvert\hat{\rho}\rvert = \dfrac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\lvert\hat{\rho}_{ij}\rvert}{N(N-1)}$

Table 6.8: Estimation Results of Gasoline Data with 5% contamination

	Methods	Pooled	CMG	RCMG
Parameter Estimates	$\hat{\beta}_1$	**0.246**	**0.331**	**0.141**
	t-stats	7.642* (0.000)	0.463 (0.678)	0.416 (0.661)
	$\hat{\beta}_2$	**-0.461**	**-0.617**	**-0.346**
	t-stats	-11.141* (0.000)	-1.290 (0.099)	-1.993* (0.023)
	$\hat{\beta}_3$	**-0.484**	**-1.279**	**-0.535**
	t-stats	-19.603* (0.000)	-2.152* (0.016)	-2.029* (0.021)
Goodness-of-fit	R^2	0.549	0.580	0.492
	RR^2	0.575	0.999	0.999

Note: * significant at 5% significant level

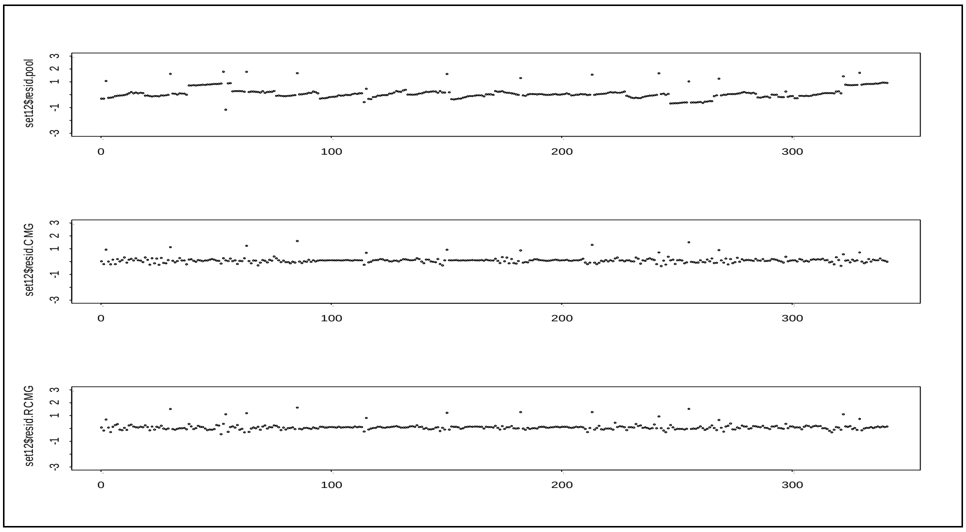

Figure 6.3: Residual Plots of Gasoline Data
with 5% contamination; (1)Pooled model; (2) CMG; (3)RCMG.

Table 6.9: Cross-Validation Results of Gasoline Data with 5% contamination

(CV)

i	Pooled	CMG	RCMG
1	0.28895	0.12092	0.13144
2	0.23529	0.11436	0.11705
3	0.21772	0.10577	0.10756
4	0.25276	0.10343	0.11300
5	0.24811	0.10419	0.11213
6	0.24562	0.10355	0.11197
7	0.23017	0.08373	0.09289
8	0.13545	0.07032	0.07932
9	0.12139	0.06720	0.07417
10	0.12626	0.06191	0.07038
11	0.14170	0.07177	0.07932
12	0.14846	0.07307	0.08057
13	0.15748	0.08361	0.09138
14	0.17900	0.09333	0.10078
15	0.21827	0.10371	0.11175
16	0.22741	0.11265	0.12017
17	0.23653	0.11677	0.12443
18	0.24663	0.12436	0.13424

(CV^2)

i	Pooled	CMG	RCMG
1	0.19140	0.05080	0.06313
2	0.14241	0.04856	0.05303
3	0.13030	0.04062	0.04718
4	0.17074	0.03941	0.05059
5	0.16745	0.04144	0.05089
6	0.17060	0.04601	0.05379
7	0.15525	0.03705	0.04451
8	0.06689	0.02853	0.03731
9	0.06394	0.02938	0.03711
10	0.06436	0.02768	0.03541
11	0.07173	0.03089	0.04028
12	0.07310	0.03093	0.04032
13	0.08036	0.03671	0.04606
14	0.09696	0.03990	0.04963
15	0.13990	0.04697	0.05679
16	0.14490	0.05010	0.05897
17	0.14755	0.05061	0.05948
18	0.15428	0.05264	0.06391

Table 6.10: Robust Cross-Validation Results of Gasoline Data with 5% contamination

(RCV)

i	Pooled	CMG	RCMG
1	0.15811	0.06785	0.07478
2	0.14563	0.07029	0.06674
3	0.13439	0.06871	0.06657
4	0.15829	0.07617	0.08221
5	0.16648	0.08121	0.09067
6	0.22535	0.09010	0.10357
7	0.25329	0.07983	0.08575
8	0.16867	0.06986	0.06980
9	0.15890	0.07004	0.06680
10	0.18326	0.05150	0.06358
11	0.19528	0.06458	0.07233
12	0.18251	0.04905	0.05736
13	0.17708	0.05196	0.06365
14	0.15656	0.05905	0.06680
15	0.15109	0.06784	0.07487
16	0.14308	0.07004	0.07670
17	0.15052	0.06986	0.07487
18	0.14796	0.07054	0.07670

(RCV^2)

i	Pooled	CMG	RCMG
1	0.00839	0.00079	0.00142
2	0.00623	0.00081	0.00119
3	0.00546	0.00075	0.00115
4	0.00788	0.00113	0.00187
5	0.00968	0.00158	0.00239
6	0.01333	0.00218	0.00321
7	0.01624	0.00119	0.00204
8	0.01025	0.00099	0.00137
9	0.00782	0.00095	0.00121
10	0.01186	0.00049	0.00091
11	0.01226	0.00071	0.00116
12	0.01053	0.00048	0.00075
13	0.00854	0.00057	0.00092
14	0.00856	0.00063	0.00103
15	0.00800	0.00073	0.00123
16	0.00719	0.00081	0.00136
17	0.00756	0.00082	0.00135
18	0.00707	0.00088	0.00146

For the estimation results (see Table 6.8), the RCMG estimator yields significant values of $\hat{\beta}_2$ and $\hat{\beta}_3$ in the presence of outliers whereas the CMG reports a significant $\hat{\beta}_3$. $\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$ are significant for the pooled model due of the presence of positive and negative large residuals which are averaged out (see residual plot for pooled model in Figure 6.3). These results however are unreliable due to the smaller values of R^2 and RR^2, indicating a poor fit. Both the CMG and RCMG provide a large value of RR^2, indicating a good fit. The values of CV and CV^2 (see Table 6.9) for all the estimators are slightly large compared to the uncontaminated panel. The robust RCV is also large but not the RCV^2 (see Table 6.10). Based on these results, the RCMG provides comparable estimates with its uncontaminated panel, which illustrates its robustness.

6.3 Panel Unit Root Tests

To investigate the presence of the CD and unit root in the dynamic panel framework, the PPP data are suggested. The panel unit root tests used include: (1) ADF unit root test, (2) CIPS and (3) RCIPS. The CD is first examined prior to testing the unit root.

6.3.1Purchasing Power Parity (PPP)

The PPP is a criterion for an appropriate exchange rate between currencies. It is a rate that a representative basket of goods in country A costs the same as in country B if the currencies are exchanged at that rate. This PPP is related to the exchange rates where the equilibrium exchange rates are often defined in terms of the PPP (see Gökcan, 2002; Haw and Baharumshah, 2002). A constant real exchange is required for PPP to hold. One should expect PPP to hold in a world where the transportation and transaction costs are negligible, consumption basket is identical and no arbitrage profit existed. The real exchange rate will vary without these conditions. So, the most common way to test for PPP is investigating the presence of unit roots in RERs.

In the time series model, the most common definition of the RER is the nominal exchange rate (q_t) adjusted by the price levels (p_t) and is given as follows:

$$q_t = s_t + p_t^* - p_t \qquad (6.4)$$

where the PPP condition is defined by

$$s_t = p_t - p_t^* \qquad (6.5)$$

Here, s_t is the log exchange rate, p_t and p_t^* are the log domestic and foreign price levels respectively.

Note that, if relative PPP holds, then q_t in (6.4) is a constant. This is clearly not true in the short run, but it could happen in the long run empirically. The PPP holds when it achieves mean stationary in the long run and it can be evaluated by application of the unit root tests. Generally, the panel methods with long time spans provide more evidence in favour of a trend reverting q than the pure time series methods.

6.3.2 Data and Model

Two examples are considered on PPP real data set: (1) Asian and (2) Central and Eastern Europe (CEEC) countries. The data employed are on the monthly RER of 13 countries in both examples[63]. The first covers the period M1 1986 to M4 2010 in which the time dimension is equals to 290 while in the second it consists the data from M1 1996 to M4 2010 which is equivalent to $T = 170$.

The following equation is used to model RER:

$$q_{it} = s_{it} + p_{it}^* - p_{it} \qquad (6.6)$$

where q_{it} is the logarithm nominal exchange rate at country i th currency in terms of US dollar, p_{it} and p_{it}^* are the logarithm of consumer price indices in the US and country i, respectively.

[63] The data is available upon request.

For a panel of countries $(i = 1,2,\ldots,N)$ the RER can be represented by,

$$\Delta s_{it} = \alpha_i + b_i s_{it-1} + \varepsilon_{it};$$

where $\Delta s_{it} = s_{it} - s_{it-1}$; is the first difference of RER; α_i is the intercept which varies across i. Accordingly, if $b_i < 0$, then s_i is stationary and if $b_i = 0$ then the series is nonstationary or has a unit root. For the ADF test, the following model is used;

$$\Delta s_{it} = \alpha_i + b_i s_{it-1} + \rho_i \Delta s_{it-1} + \varepsilon_{it};$$

while for the CIPS and RCIPS unit root tests, the following model is considered:

$$\Delta s_{it} = \alpha_i + b_i s_{it-1} + \gamma_i f_t + \varepsilon_{it};$$

Here, γ_i is the common factor, f_t is unobserved factor and ε_{it} is the disturbances and the CD is explained by the f_t has been derived in (3.10) to (3.14) in Chapter 3.

6.3.3 Critical values

Critical values are computed based on two procedures: (1) DGP in Chapter 5, Section 5.4 and (2) the idea in Breuer et al. (1999). The simulated critical values of the second procedure are based on the exact values of the parameter estimates in PPP for each group of countries using Pesaran's (2007) method of estimation. The error series are generated from the standard normal distributions with the variance covariance matrix: $\overline{\mathbf{M}} = \mathbf{I}_t - \overline{\mathbf{H}}\left(\overline{\mathbf{H}}^T \overline{\mathbf{H}}\right)^{-1} \overline{\mathbf{H}}^T$ with $\overline{\mathbf{H}} = (\mathbf{1}, \Delta\overline{\mathbf{y}}_t, \overline{\mathbf{y}}_{t-1})$. $\mathbf{I}_t$ is a unit matrix of order $T \times T$ and $\overline{\mathbf{H}}$ is the combination of the dummy variables, average of cross section of the first difference of y_{it} and its first lagged value, y_{it-1} . Here, y_{it} is the RER.

Then, each simulated data is generated from the error series using the parameter estimates with intercept set at zero since the null hypothesis has a unit root. For each series, $T + 50$ are generated for each group, and the first 50 observations are discarded in order to reduce the initial effects of data generation. The remaining T observations are then used to compute the critical values of the unit root test at 0.05 significant levels. The critical values produced in Table 6.15 are based on 5000 replications.

6.3.4 Results and Discussion

First, the pair-wise cross section coefficient of the residuals which consists of $C_2^{13} = 78$ pairs is computed and these coefficients are tabulated in Tables 6.11 and 6.13. From Table 6.11, China (CHN), Hong Kong (HK), India (IND), Pakistan (PAK) and Sri Lanka (LKA) yield small $\hat{\rho}_{ij}$ that is less than 0.2 among the other countries. The PPP of these countries are almost independent of the other ASIAN countries especially China (CHN). The PPP of Malaysia (MYS), Singapore (SGP), Thailand (THA), Indonesia (IDN) and Philippines (PHI) yield strong cross dependency among each other and it is observed that Korea (KOR) and Taiwan (TWN) are also dependent to each other. The absolute pair-wise correlation $|\hat{\rho}| = 0.171$ and cross dependency tests (reported in Table 6.12) provide significant evidence of the CD that is to reject the null cross sectional independence among ASIAN countries.

High cross correlation coefficient is obtained in the CEEC panel with $\hat{\rho}_{ij} > 0.3$ and these results are comparable with $|\hat{\rho}| = 0.570$ (see Tables 6.13 to 6.14). The test statistics of the CD unanimously rejects the null hypothesis of the cross sectional independence among the group of CEEC members at 5% significant levels. This clearly means that the CD among the CEEC countries exists and is strong.

The unit root tests are computed for each ASIAN and CEEC panel and the results are given in Table 6.15. Two bases of critical values are provided at 5% significant levels as a benchmark to compare the unit root test. The ADF unit root test provides significant results in rejecting the presence of the unit root based on the critical values obtained from procedure 1 but fails to reject it based on the second procedure. These results however are unreliable due to the presence of the significant CD (from Tables 6.12 and 6.14). As such, the CIPS and RCIPS unit root tests are employed as the alternative approach which accommodates the CD. Both tests give insignificant results of a unit root test i.e. do not reject the null of the unit root at 5% significant levels, with respect to both the critical values. This means that the PPP hypothesis is no longer valid in the presence of a unit root. Likewise, for the CEEC panels, all the unit root tests fail to reject the null hypothesis of a unit root and therefore, this provides some evidence against PPP.

Table 6.11: The Correlation Results among the $(i, j)^{\text{th}}$ $(\hat{\rho}_{ij})$ Cross Sectional Units of the ASIAN Data[64].

(i, j)	CHN	HKG	IDN	IND	KOR	LKA	MYS	PAK	PHL	WSM	SGP	TWN	THA
CHN	1												
HKG	0.0809	1											
IDN	0.0098	0.0037	1										
IND	0.0245	0.0696	0.0310	1									
KOR	0.0278	0.0386	0.1342	0.2058	1								
LKA	-0.0291	0.0290	-0.2859	-0.0495	-0.1959	1							
MYS	0.0353	0.1053	0.5682	0.0678	0.1339	-0.0965	1						
PAK	0.0236	0.1338	0.1129	0.1629	0.0547	-0.0035	0.1505	1					
PHL	0.0241	-0.0060	0.3615	0.0374	0.3052	0.0664	0.3833	0.1032	1				
WSM	-0.1051	-0.0343	0.1814	0.1122	0.2587	-0.0278	0.1407	0.1097	0.1972	1			
SGP	-0.0212	0.1541	0.5015	0.1687	0.3253	-0.0919	0.6092	0.2157	0.3282	0.2972	1		
TWN	0.0896	0.2209	0.1995	0.1254	0.2586	-0.0457	0.2783	0.0925	0.1197	0.0996	0.3299	1	
THA	0.0050	0.0339	0.5759	0.0386	0.2659	-0.1160	0.7057	0.1340	0.5474	0.1482	0.6186	0.2653	1

Table 6.12 : Cross Dependency Test Results of the ASIAN Data

Test	Test statistics
$\lvert\hat{\rho}\rvert$	0.171
LM	1269.020*
RLM1	335871.716*
RLM2	36970.724*
PCD	21.458*
RPCD1	483.919*
RPCD2	245.817*

*The test is significant when $\text{LM}, \text{RLM1}, \text{RLM2} > \chi^2_{(N(N-1)/2)} = 99.617$ and $\lvert\text{PCD}\rvert, \lvert\text{RPCD1}\rvert, \lvert\text{RPCD2}\rvert > N(0,1) = 1.96$ at 5% significant levels. $\lvert\hat{\rho}\rvert$ is computed as $\lvert\hat{\rho}\rvert = \dfrac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\lvert\hat{\rho}_{ij}\rvert}{N(N-1)}$

[64] Abbreviations of the ASIAN country are given in Appendix D.

Table 6.13: The Correlation Results among the $(i, j)^{th}$, $(\hat{\rho}_{ij})$ Cross Sectional Units of the CEEC Data[65].

(i, j)	AL	BG	CR	CZ	EE	HU	LV	LT	MK	PL	RO	SK	SL
AL	1												
BG	0.3696	1											
CR	0.4711	0.3589	1										
CZ	0.4552	0.2987	0.7447	1									
EE	0.5281	0.4152	0.8928	0.7778	1								
HU	0.4746	0.3084	0.7536	0.6871	0.7990	1							
LV	0.5054	0.3410	0.7720	0.6687	0.8257	0.7091	1						
LT	0.4621	0.2389	0.6893	0.6508	0.7443	0.6650	0.7820	1					
MK	0.3503	0.3591	0.8415	0.6792	0.8374	0.6887	0.7656	0.6592	1				
PL	0.3956	0.2688	0.6112	0.6643	0.6332	0.7616	0.5758	0.5046	0.5576	1			
RO	0.5229	0.3847	0.4338	0.3658	0.4744	0.4308	0.4114	0.3528	0.4145	0.3887	1		
SK	0.3911	0.3127	0.6818	0.6637	0.7463	0.6780	0.6500	0.5900	0.6659	0.5661	0.3734	1	
SL	0.3867	0.3577	0.8214	0.6975	0.8326	0.6251	0.6479	0.6521	0.7708	0.5099	0.3698	0.7420	1

Table 6.14: Cross Dependency Test Results of the CEEC Data

Test	Test statistics
$\lvert\hat{\rho}\rvert$	0.570
LM	4697.555*
RLM1	937523.671*
RLM2	82977.353*
PCD	65.630*
RPCD1	952.565*
RPCD2	720.589*

The test is significant when $\mathrm{LM}, \mathrm{RLM1}, \mathrm{RLM2} > \chi^2_{(N(N-1)/2)} = 99.617$ and $\lvert\mathrm{PCD}\rvert, \lvert\mathrm{RPCD1}\rvert, \lvert\mathrm{RPCD2}\rvert > N(0,1) = 1.96$ at 5% significant level. $\lvert\hat{\rho}\rvert$ is computed as $\lvert\hat{\rho}\rvert = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\lvert\hat{\rho}_{ij}\rvert}{N(N-1)}$

[65] Abbreviations of the ASIAN country are given in Appendix D.

Table 6.15: The Unit Root Test Results

		ADF	CIPS	RCIPS
ASIAN	Critical values[a]	-1.9036	-2.2231	-1.7517
	Critical values[b]	-2.4836	-2.6779	-4.2509
	Test statistics	-1.9658[a]	-0.7164	-0.3772
CEEC	Critical values[a]	-1.8976	-2.2252	-1.7433
	Critical values[b]	-2.1718	-2.1151	-3.9435
	Test statistics	-1.0859	-0.4826	-0.0751

Note: [a] is the critical value obtained from DGP as in Section 5.4 while [b] is computed based on the idea in Breuer et al. (1999) and is discussed in Section 6.3.3.

*: significant and rejected if $|\text{Test statistics}| > \text{Critical values}$, based on its corresponding critical values. Critical values are computed based on 5000 replications at 5% significant levels.

6.4 Discussion

In this chapter, The CD tests discussed in Chapter 2 are employed to investigate the presence of the CD in the gasoline data. There is strong evidence that the CD is present in the gasoline panel and therefore the Generalized M-estimator (named RCMG) is adopted to estimate the parameter of the model. The parameter estimates obtained are in line with the Pooled and CMG estimator. However, as the gasoline panel is contaminated with outliers, the RCMG provides a good fit with larger values of RR^2 and smallest values of RCV, RCV^2 among other estimation procedures. This indicates that the advantage of the RCMG is the ability to handle extreme observations. The unit root tests of PPP data find evidence that the ASIAN and CEEC data are strongly cross correlated among the group members. The unit root results provide some evidence of the non-stationary of the PPP data and conclude that the PPP hypothesis does not hold in both the ASIAN and CEEC countries.

CHAPTER 7

Conclusion

This study propose the method to statistical modeling and inferences in the panel data model when there exist suspicious observations in the series and cross correlation among the group members in the panel. For exploratory purposes, the cross-sectional tests are applied to investigate the presence of cross sectional dependence (CD) in the panel. In Section 2.3 of Chapter 2, the tests by Breusch and Pagan (1980) and Pesaran (2004) are revisited and the residuals obtained by the Least squares estimation are known to be sensitive to the outliers. A robust estimation procedure is considered to capture the outlier effects and those spurious observations are removed when computing pair-wise correlation coefficients, $\hat{\rho}_{ij}$ using robust filter function, such as the Huber function and diagnostic tools. This study supports that the robust version of Pesaran's (2004) (namely RPCD1 and RPCD2) and outperform other tests in detecting the CD especially when mild CD is observed.

In modeling the panel, the Generalized M-estimator is proposed to minimize the objective function of overall error (given in Equation (3.22)) by advocating Pesaran's (2006) estimation procedure named the Common Correlated Effects Mean Group (CMG) in purely static model. The CMG method uses the unobserved factor to explain the cross correlation. The modification of the variance-covariance matrix in the CMG procedure enables the recovery of reliable estimators which limits the influence of both, CD and outliers. This Generalized M-estimator is asymptotically normally distributed as $N \to \infty$ and $T \to \infty$. The asymptotic distribution of the Generalized M-estimator is used to derive the test statistics which is important in the construction of the hypothesis testing and confidence interval of the parameter estimates. Furthermore, this study only focuses on the properties of estimator in term of asymptotic distribution, thus, future studies will be undertaken on qualitative and quantitative robustness such as influence function and breakdown point.

Extensive simulation studies of a simple pure static model with one regressor are chosen to illustrate the estimation techniques, hypothesis testing and confidence interval of parameter estimates. The Pooled, CMG and RCMG procedures are examined and this study concludes that the proposed method, RCMG has small mean square error (MSE) of the residuals and bias of the parameter estimates, $\hat{\beta}_{RCMG}$ in the presence of CD and outliers. The hypothesis test supports the findings in Section 4.2.1.1 that the RCMG is more reliable (powerful) in estimating $\hat{\beta}_{RCMG}$, in terms of accuracy. This can also be observed from the shorter length of CI in Table 4.31.

In the dynamic framework, the reliability of the unit root tests is investigated in the presence of the CD and outliers. An Augmented Dickey Fuller (ADF) unit root test is modified (Im et al., 2003) so that it is suitable with the panel framework; however this test assumes cross sectional independence in the panel. On the other hand, the Pesaran unit root test (2007) which uses the CMG approach is not robust to outliers occurring within the panel. The robust Generalized M-estimator, described in Chapter 3, is used to investigate the stationary of the series in panel. The Monte Carlo simulation studies support the findings of this study that the proposed procedure outperforms the ADF and the Pesaran unit root test in terms of size and power of the test in the presence of the CD and outliers. In testing for a unit root test, the asymptotic distribution of the proposed unit root test is approximated via the Monte Carlo experiment. Future studies will be undertaken on the statistical properties of this method.

For the illustrations, the gasoline data taken from Baltagi's work (2005) are considered in pure static model. The simulation results support the findings of this study that the parameter estimates obtained from RCMG are comparable with uncontaminated panel. The value of RR^2 and RCV and RCV^2 indicate that RCMG is a good fit for the gasoline panel. For the dynamic case, the examples of the PPP data are considered and the results find that the PPP hypothesis does not hold and this clearly indicates that the PPP of ASIAN and CEEC is not mean stationary in the long run. Thus, there is evidence against the PPP and for this; it is left as an open study.

A drawback of the Breusch and Pagan (1980) and Pesaran (2004) CD tests is that they are sensitive to the influence of outliers especially for the case of mild CD in the panel; the presence of the CD being masked by the presence of outliers. Thus, the robust estimation procedures are incorporated to the test with some filtration to remove the outliers' effects; similarly, for the estimation procedure of Pesaran's (2004). As a possible alternative, Pesaran's approach is replaced with the robust Generalized M-estimator and the results provide support that the proposed technique is able to yield good estimates in the presence of outliers and CD. This technique can also be modified to the more complicated panel model such as panel with multiple regressors and unobserved factors. Even though the extended and complicated panel model is not considered in this study due to the complicated nature of the properties of the model, the model is of interest for future work. With the flexibility of the existing software packages that are available today, it is believed that the study can be extended to include complicated panel models.