

## CONCLUDING REMARKS

The thesis presents a new methodology to find the queue length and waiting time distributions in a system of two or more parallel queues. The proposed method has potential applications in finding the above distributions and other important characteristics in a variety of manufacturing systems, communication systems, computer systems and other related areas.

The results in the literature on the system of dependent queues are usually restricted to the case when the arrival streams are Poisson processes and the service times are exponentially distributed. The present thesis frees the arrival and service times from the above restrictions by assuming that each of them follows a hypoexponential distribution in two stages.

The derivation of the balance equations in the resulting system with more general distributions is made possible by the use of a computer to generate the large amount of codes which represent the balance equations. The solution of the balance equations to the required level of accuracy is made possible by the method proposed in the thesis.

However when the number  $M$  of queues is too large, it would be difficult for a computer to handle the extremely large number of vectors of characteristics of the system. Further research may thus be carried out to solve the related dimensionality problem. Although the thesis concentrates on only two interaction schemes, the methods proposed in the thesis may still be applicable to the cases with other interaction schemes such as the schemes which incorporate features like balking of queue, reneging of queue and jockeying of queue, and switching over to another queue when the customer is still in the process of being served.

# APPENDIX A

Representation of balance equations by codes in a system of two dependent  
Hypo(2)/Hypo(2)/1 queues for  $0 \leq n_1^{(k)} + n_2^{(k)} \leq 2$ .

Constant	h	Power
-1	0 1 0 1 0 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 0 1 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	2 1 0 1 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 2 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	0 1 0 2 0 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 0 2 0 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	2 1 0 2 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 2 2 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	0 1 0 1 0 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	0 1 1 1 0 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 1 1 0 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	0 1 1 1 0 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	2 1 1 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 2 0 1 0 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 2 1 0 2	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	0 1 0 2 0 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	0 1 1 2 0 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 1 2 0 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	0 1 1 2 0 1	0 0 0 0 0 0 0 0 0 1 0 0 0 0
1	2 1 1 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 2 0 2 0 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 2 2 0 2	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	0 1 1 1 0 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	0 1 2 1 0 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 2 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
-1	0 1 2 1 0 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	2 1 2 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 1 1 0 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
1*		
-1	0 1 2 2 0 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	0 1 2 2 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
-1	0 1 2 2 0 1	0 0 0 0 0 0 0 0 0 1 0 0 0 0
1	2 1 2 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 1 2 0 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	0 1 2 1 0 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	0 2 0 1 0 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	0 2 0 1 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	2 2 0 1 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 0 1 0 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	0 2 2 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	0 2 0 2 0 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	0 2 0 2 0 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	2 2 0 2 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0 1 0 2 0 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	0 2 2 2 0 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	0 2 0 1 0 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		

Appendix A, continued

Constant	h	Power
-1	0 2 1 1 0 1	0 0 0 1 0 0 0 0 0 0 0 0
-1	0 2 1 1 0 1	0 0 0 0 1 0 0 0 0 0 0 0
-1	0 2 1 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0
1	2 2 1 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 1 1 0 1	0 0 1 0 0 0 0 0 0 0 0 0
1	0 2 2 1 0 2	0 0 0 0 0 0 1 0 0 0 0 0
1	0 2 0 2 0 0	0 0 0 0 0 0 0 0 1 0 0 0
1*		
-1	0 2 1 2 0 1	0 0 0 1 0 0 0 0 0 0 0 0
-1	0 2 1 2 0 1	0 0 0 0 1 0 0 0 0 0 0 0
-1	0 2 1 2 0 1	0 0 0 0 0 0 0 0 1 0 0 0
1	2 2 1 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 1 2 0 1	0 0 1 0 0 0 0 0 0 0 0 0
1	0 2 2 2 0 2	0 0 0 0 0 0 1 0 0 0 0 0
1	0 2 1 1 0 1	0 0 0 0 0 0 0 0 1 0 0 0
1*		
-1	0 2 2 1 0 1	0 0 0 1 0 0 0 0 0 0 0 0
-1	0 2 2 1 0 1	0 0 0 0 0 1 0 0 0 0 0 0
-1	0 2 2 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0
1	2 2 2 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 2 1 0 1	0 0 1 0 0 0 0 0 0 0 0 0
1	0 2 1 1 0 1	0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	0 2 2 2 0 1	0 0 0 1 0 0 0 0 0 0 0 0
-1	0 2 2 2 0 1	0 0 0 0 0 1 0 0 0 0 0 0
-1	0 2 2 2 0 1	0 0 0 0 0 0 0 0 1 0 0 0
1	2 2 2 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0
1	0 1 2 2 0 1	0 0 1 0 0 0 0 0 0 0 0 0
1	0 2 1 2 0 1	0 0 0 0 1 0 0 0 0 0 0 0
1	0 2 2 1 0 1	0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	1 1 0 1 1 0	1 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 0 1 1 0	0 0 1 0 0 0 0 0 0 0 0 0
-1	1 1 0 1 1 0	0 0 0 0 0 0 1 0 0 0 0 0
1	2 1 0 1 2 0	0 1 0 0 0 0 0 0 0 0 0 0
1	0 2 0 1 0 0	0 0 0 1 0 0 0 0 0 0 0 0
1	1 1 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0
1	0 1 0 2 0 0	0 0 0 0 0 0 0 0 1 0 0 0
1*		
-1	1 1 0 2 1 0	1 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 0 2 1 0	0 0 1 0 0 0 0 0 0 0 0 0
-1	1 1 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0
1	2 1 0 2 2 0	0 1 0 0 0 0 0 0 0 0 0 0
1	0 2 0 2 0 0	0 0 0 1 0 0 0 0 0 0 0 0
1	1 1 2 2 1 1	0 0 0 0 0 0 1 0 0 0 0 0
1	1 1 0 1 1 0	0 0 0 0 0 0 0 0 1 0 0 0
1*		
-1	1 1 1 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 1 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0
-1	1 1 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0
-1	1 1 1 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0
1	2 1 1 1 2 1	0 1 0 0 0 0 0 0 0 0 0 0
1	0 2 1 1 0 1	0 0 0 1 0 0 0 0 0 0 0 0
1	1 2 0 1 1 0	0 0 0 1 0 0 0 0 0 0 0 0
1	1 1 2 1 1 2	0 0 0 0 0 1 0 0 0 0 0 0
1	1 1 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0
1	0 1 1 2 0 1	0 0 0 0 0 0 0 0 0 1 0 0
1*		

Appendix A, continued

Constant	h	Power
-1	1 1 1 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 1 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 1 2 1 1	0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	2 1 1 2 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	0 2 1 2 0 1	0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0
1	1 2 0 2 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0
1	1 1 2 2 1 2	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 1 1 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	1 1 2 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 2 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	1 1 2 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	2 1 2 1 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	0 2 2 1 0 1	0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0
1	1 1 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	0 1 2 2 0 1	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
1*		
-1	1 1 2 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 2 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 1 2 2 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	1 1 2 2 1 1	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	2 1 2 2 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	0 2 2 2 0 1	0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0
1	1 1 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	1 1 2 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	1 2 0 1 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 0 1 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 0 1 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	2 2 0 1 2 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 0 1 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 1 2 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	1 2 0 2 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 0 2 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 0 2 1 0	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	2 2 0 2 2 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 0 2 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 2 2 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 2 0 1 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	1 2 1 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 1 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	2 2 1 1 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 1 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 2 1 1 2	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 2 0 2 1 0	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	0 2 1 2 0 1	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
1*		
-1	1 2 1 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 2 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 2 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	1 2 1 2 1 1	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	2 2 1 2 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 1 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 2 2 1 2	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 2 1 1 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		

Appendix A, continued

Constant	h	Power
-1	1 2 2 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 1 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 1 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 1 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1	2 2 2 1 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 2 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	0 2 2 2 0 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		
-1	1 2 2 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 2 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 2 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	1 2 2 2 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	2 2 2 2 2 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 1 2 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	1 2 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	1 2 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1*		
-1	2 1 0 1 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 0 1 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 0 1 1 0	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 1 0 1 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1*		
-1	2 1 0 2 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 0 2 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	1 1 0 2 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 2 2 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	2 1 0 1 1 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1*		
-1	2 1 1 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	1 1 1 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 2 0 1 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0
1	2 1 2 1 1 2	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
1	2 1 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0
1*		
-1	2 1 1 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 1 2 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	1 1 1 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 2 0 2 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0
1	2 1 2 2 1 2	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
1	2 1 1 1 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1*		
-1	2 1 2 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 1 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 1 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1	1 1 2 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1*		
-1	2 1 2 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 2 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 1 2 2 1 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	1 1 2 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 1 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1*		

**Appendix A, continued**

<b>Constant</b>	<b>h</b>	<b>Power</b>
-1	2 2 0 1 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 0 1 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 0 1 1 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	1 2 0 1 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 0 1 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 2 1 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0
1*		
-1	2 2 0 2 1 0	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 0 2 1 0	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1	1 2 0 2 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 0 2 1 0	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 2 2 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0
1	2 2 0 1 1 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1*		
-1	2 2 1 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 1 1 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	2 2 1 1 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	1 2 1 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 1 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 2 1 1 2	0 0 0 0 0 1 0 0 0 0 0 0 0 0
1	2 2 0 2 1 0	0 0 0 0 0 0 0 0 1 0 0 0 0 0
1*		
-1	2 2 1 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 1 2 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
-1	2 2 1 2 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1	1 2 1 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 1 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 2 2 1 2	0 0 0 0 0 1 0 0 0 0 0 0 0 0
1	2 2 1 1 1 1	0 0 0 0 0 0 0 1 0 0 0 0 0 0
1*		
-1	2 2 2 1 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 2 1 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 2 1 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0
-1	2 2 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	1 2 2 1 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 2 1 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 1 1 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
1*		
-1	2 2 2 2 1 1	0 1 0 0 0 0 0 0 0 0 0 0 0 0
-1	2 2 2 2 1 1	0 0 0 1 0 0 0 0 0 0 0 0 0 0
-1	2 2 2 2 1 1	0 0 0 0 0 1 0 0 0 0 0 0 0 0
-1	2 2 2 2 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1	1 2 2 2 1 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0
1	2 1 2 2 1 1	0 0 1 0 0 0 0 0 0 0 0 0 0 0
1	2 2 1 2 1 1	0 0 0 0 1 0 0 0 0 0 0 0 0 0
1	2 2 2 1 1 1	0 0 0 0 0 0 1 0 0 0 0 0 0 0
1*		

## APPENDIX B

The complete set of feasible events in  $A = \{A_1, A_2, \dots, A_{116}\}$ .

$i$	Component of $A_i$	$i$	Component of $A_i$
1	1 0 0 0 0 0 -1 -1 -1	59	0 0 0 0 -1 1 -1 -1 32
2	0 1 0 0 0 0 -1 -1 -1	60	0 0 0 0 -1 1 -1 -1 33
3	0 1 0 0 0 0 11 -1 -1	61	0 0 0 0 -1 0 -1 -1 -1
4	0 1 0 0 0 0 12 -1 -1	62	-1 1 -1 0 0 0 -1 -1 -1
5	0 1 0 0 0 0 13 -1 -1	63	-1 1 -1 0 0 0 11 -1 -1
6	0 0 1 0 0 0 -1 -1 -1	64	-1 1 -1 0 0 0 12 -1 -1
7	0 0 0 1 0 0 -1 -1 -1	65	-1 1 -1 0 0 0 13 -1 -1
8	0 0 0 1 0 0 -1 21 -1	66	-1 0 -1 1 0 0 -1 -1 -1
9	0 0 0 1 0 0 -1 22 -1	67	-1 0 -1 1 0 0 -1 21 -1
10	0 0 0 1 0 0 -1 23 -1	68	-1 0 -1 1 0 0 -1 22 -1
11	0 0 0 0 1 0 -1 -1 -1	69	-1 0 -1 1 0 0 -1 23 -1
12	0 0 0 0 0 1 -1 -1 -1	70	-1 0 -1 0 1 0 -1 -1 -1
13	0 0 0 0 0 1 -1 -1 31	71	-1 0 -1 0 0 1 -1 -1 -1
14	0 0 0 0 0 1 -1 -1 32	72	-1 0 -1 0 0 1 -1 -1 31
15	0 0 0 0 0 1 -1 -1 33	73	-1 0 -1 0 0 1 -1 -1 32
16	0 0 0 0 0 0 -1 -1 -1	74	-1 0 -1 0 0 1 -1 -1 33
17	-1 1 0 0 0 0 -1 -1 -1	75	-1 0 -1 0 0 0 -1 -1 -1
18	-1 1 0 0 0 0 11 -1 -1	76	-1 1 0 0 -1 0 -1 -1 -1
19	-1 1 0 0 0 0 12 -1 -1	77	-1 1 0 0 -1 0 11 -1 -1
20	-1 1 0 0 0 0 13 -1 -1	78	-1 1 0 0 -1 0 12 -1 -1
21	-1 0 1 0 0 0 -1 -1 -1	79	-1 1 0 0 -1 0 13 -1 -1
22	-1 0 0 1 0 0 -1 -1 -1	80	-1 0 1 0 -1 0 -1 -1 -1
23	-1 0 0 1 0 0 -1 21 -1	81	-1 0 0 1 -1 0 -1 -1 -1
24	-1 0 0 1 0 0 -1 22 -1	82	-1 0 0 1 -1 0 -1 21 -1
25	-1 0 0 1 0 0 -1 23 -1	83	-1 0 0 1 -1 0 -1 22 -1
26	-1 0 0 0 1 0 -1 -1 -1	84	-1 0 0 1 -1 0 -1 23 -1
27	-1 0 0 0 0 1 -1 -1 -1	85	-1 0 0 0 -1 1 -1 -1 -1
28	-1 0 0 0 0 1 -1 -1 31	86	-1 0 0 0 -1 1 -1 -1 31
29	-1 0 0 0 0 1 -1 -1 32	87	-1 0 0 0 -1 1 -1 -1 32
30	-1 0 0 0 0 1 -1 -1 33	88	-1 0 0 0 -1 1 -1 -1 33
31	-1 0 0 0 0 0 -1 -1 -1	89	-1 0 0 0 -1 0 -1 -1 -1
32	1 0 -1 0 0 0 -1 -1 -1	90	1 0 -1 0 -1 0 -1 -1 -1
33	0 1 -1 0 0 0 -1 -1 -1	91	0 1 -1 0 -1 0 -1 -1 -1
34	0 1 -1 0 0 0 11 -1 -1	92	0 1 -1 0 -1 0 11 -1 -1
35	0 1 -1 0 0 0 12 -1 -1	93	0 1 -1 0 -1 0 12 -1 -1
36	0 1 -1 0 0 0 13 -1 -1	94	0 1 -1 0 -1 0 13 -1 -1
37	0 0 -1 1 0 0 -1 -1 -1	95	0 0 -1 1 -1 0 -1 -1 -1
38	0 0 -1 1 0 0 -1 21 -1	96	0 0 -1 1 -1 0 -1 21 -1
39	0 0 -1 1 0 0 -1 22 -1	97	0 0 -1 1 -1 0 -1 22 -1
40	0 0 -1 1 0 0 -1 23 -1	98	0 0 -1 1 -1 0 -1 23 -1
41	0 0 -1 0 1 0 -1 -1 -1	99	0 0 -1 0 -1 1 -1 -1 -1
42	0 0 -1 0 0 1 -1 -1 -1	100	0 0 -1 0 -1 1 -1 -1 31
43	0 0 -1 0 0 1 -1 -1 31	101	0 0 -1 0 -1 1 -1 -1 32
44	0 0 -1 0 0 1 -1 -1 32	102	0 0 -1 0 -1 1 -1 -1 33
45	0 0 -1 0 0 1 -1 -1 33	103	0 0 -1 0 -1 0 -1 -1 -1
46	0 0 -1 0 0 0 -1 -1 -1	104	-1 1 -1 0 -1 0 -1 -1 -1
47	1 0 0 0 -1 0 -1 -1 -1	105	-1 1 -1 0 -1 0 11 -1 -1
48	0 1 0 0 -1 0 -1 -1 -1	106	-1 1 -1 0 -1 0 12 -1 -1
49	0 1 0 0 -1 0 11 -1 -1	107	-1 1 -1 0 -1 0 13 -1 -1
50	0 1 0 0 -1 0 12 -1 -1	108	-1 0 -1 1 -1 0 -1 -1 -1
51	0 1 0 0 -1 0 13 -1 -1	109	-1 0 -1 1 -1 0 -1 21 -1
52	0 0 1 0 -1 0 -1 -1 -1	110	-1 0 -1 1 -1 0 -1 22 -1
53	0 0 0 1 -1 0 -1 -1 -1	111	-1 0 -1 1 -1 0 -1 23 -1
54	0 0 0 1 -1 0 -1 21 -1	112	-1 0 -1 0 -1 1 -1 -1 -1
55	0 0 0 1 -1 0 -1 22 -1	113	-1 0 -1 0 -1 1 -1 -1 31
56	0 0 0 1 -1 0 -1 23 -1	114	-1 0 -1 0 -1 1 -1 -1 32
57	0 0 0 0 -1 1 -1 -1 -1	115	-1 0 -1 0 -1 1 -1 -1 33
58	0 0 0 0 -1 1 -1 -1 31	116	-1 0 -1 0 -1 0 -1 -1 -1



















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