

THE INTEGRATED INVENTORY AND PRODUCTION  
PLANNING FOR TIME-VARYING DEMAND PROCESS

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THESIS SUBMITTED IN FULFILMENT OF  
THE REQUIREMENTS FOR THE DEGREE  
OF DOCTOR OF PHILOSOPHY

INSTITUTE OF MATHEMATICAL SCIENCES  
FACULTY OF SCIENCE  
UNIVERSITY OF MALAYA  
KUALA LUMPUR

2012

# Abstract

In the literature, integrated inventory model has received a lot of attention. Most previous works on this topic have been based on the assumption of constant demand rate. However this assumption is not reliable in reality; it is either increasing or decreasing with time.

In this thesis, we considered the model which consists of a single vendor who manage the production and deliver to a single buyer with a linearly decreasing demand rate over a finite time horizon. Costs are attached to manufacturing set up, the delivery of a shipment and stockholding at the vendor and buyer. The objective is to determine the number of shipments and size of those shipments which minimize the total system cost - assuming the vendor and buyer collaborate and find a way of sharing the consequent benefits.

We begin this thesis with the integrated inventory policy for shipping a vendor's final production batch to a single buyer under linearly decreasing demand. The first case considered here is the holding cost at the vendor is less than at the buyer. We solve this model with equal shipment sizes policy, equal shipment periods policy and unequal shipment sizes and unequal shipment periods policy.

Then, we develop a mathematical model when the unit holding cost is

higher at the vendor rather than at the buyer (consignment stock problem). For this case, we also consider equal shipment sizes policy, equal shipment periods policy, and unequal shipment sizes and unequal shipment periods as in the previous case policy.

It is followed by an integrated inventory model with  $n$  production batches which consists of the final batch at the end of the production cycle. This model also considers the case of the buyer's holding cost being greater than the vendor's and vice versa. We consider this model with equal cycle time and unequal cycle time for both policies. We show the solution procedure when the shipment sizes are equal and when they are unequal.

We solve all the models in this thesis using Microsoft Excel Solver and illustrate all the policies with numerical examples and sensitivity analysis. Then we make some comparison of the model. Lastly we end the thesis with conclusion and some recommendations for further research.

## Abstrak

Di dalam kesusasteraan, banyak perhatian yang telah diberikan kepada model integrasi inventori. Kebanyakan kerja telah dilakukan ke atas topik ini adalah berdasarkan anggapan bahawa kadar permintaan adalah malar. Walau bagaimanapun, anggapan ini adalah tidak boleh dipercayai kerana realitinya, ianya sama ada menaik atau menyusut dengan masa.

Dalam tesis ini, kami mempertimbangkan model yang mengandungi penjual tunggal yang menguruskan pengeluaran dan menghantar kepada pembeli tunggal dengan kadar permintaan menyusut dengan masa di sepanjang tempoh masa terhingga. Kos yang terlibat adalah kos penyediaan pembuatan, penyampaian penghantaran, pemegangan stok oleh penjual dan pembeli. Objektifnya adalah untuk menentukan bilangan penghantaran dan saiz setiap penghantaran yang meminimakan jumlah kos sistem - dengan anggapan bahawa penjual dan pembeli berkerjasama dan mencari jalan untuk berkongsi manfaat yang diperolehi.

Kami mulakan tesis ini dengan polisi integrasi inventori untuk menghantar kelompok pengeluaran terakhir kepada pembeli tunggal di bawah permintaan menyusut secara linear. Kes pertama yang dibincangkan disini ialah kos pemegangan penjual adalah kurang daripada pembeli. Kami menyelesaikan model ini dengan polisi penghantaran bersaiz yang sama, polisi tem-

poh masa penghantaran yang sama dan polisi saiz penghantaran yang tidak sama dan tempoh masa penghantaran yang tidak sama.

Kemudian, kami membina model matematik apabila kos pemegangan penjual adalah lebih tinggi daripada pembeli (masalah konsainan stok). Untuk kes ini, kami juga mempertimbangkan polisi penghantaran bersaiz yang sama, polisi tempoh masa penghantaran yang sama dan polisi saiz penghantaran yang tidak sama dan tempoh masa penghantaran yang tidak sama seperti kes sebelumnya.

Ianya diikuti oleh model integrasi inventori dengan  $n$  kelompok pengeluaran di mana ianya mengandungi kelompok pengeluaran terakhir di penghabisan kitaran pengeluaran. Model ini juga mempertimbangkan kedua-dua kes dimana kos pemegangan pembeli adalah lebih tinggi dari penjual atau sebaliknya. Kami mempertimbangkan model ini dengan tempoh kitaran masa yang sama dan tidak sama. Kami menunjukkan penyelesaian optimal apabila saiz penghantaran adalah sama dan apabila saiz penghantaran tidak sama dan masa penghantaran tidak sama.

Kami menjelaskan kesemua polisi dengan contoh berangka dan analisis kepekaan. Kemudian kami membuat beberapa perbandingan terhadap model ini. Akhir sekali kami mengakhiri tesis ini dengan beberapa kesimpulan dan cadangan untuk kajian selanjutnya.

## Acknowledgement

First and foremost, I would like to express my sincere gratitude and appreciation to my supervisor, Associate Professor Dr. Mohd bin Omar, for his unlimited and valuable guidance, encouragement and utmost supervision throughout the period of my research. I am deeply indebted to him.

Secondly, I would like to thank the Institute of Mathematical Sciences of the University of Malaya for allowing this research to proceed and for providing computing facilities.

Thanks are also to Dr Rossita, Dr Adriana and Dr Ng Choung Min who always gives me an idea in writing this thesis and guidance in statistics part.

Finally, I would like to express my deepest gratitude to my parents, my husband, Muhammar Yazil and my children, Muhammad Iman Hakimi, Muhammad Iman Danish and Airin Iman Irdina for their understanding and patience throughout the period of this research.

Last but not least, a special thanks to the University of Malaya for giving me a study leave and financial support, which has enabled me to carry out this research.

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