Chapter 2

Literature review

2.1 Integrated inventory model

In this chapter we will review some of the literature that focussing in the integrated inventory model for constant and varied demand. We begin with some explanation regarding the separate and integrated inventory policy. The inventory policy where the vendor and buyer do not choose to cooperate, particularly in terms of sharing parameter is known as a separate policy or an independent policy. The buyer plans their own ordering policy and inform their minimum ordering size to the vendor. Then, the vendor plans their own optimal production policy and delivers the respective amount of finished product to the buyer with their optimal number of shipments.

As we discussed in the first chapter, the EOQ model is the simplest inventory model which is consider the constant and deterministic demand rate. This model is the example of the separate model. It has been modified to make it applicable under a variety of conditions. Synder [48] demonstrated that the constant demand assumed in EOQ model can be relaxed to allow for uncertain demands without changing the basic nature of this formula. Then, Pan and Liao [40] developed an EOQ-based inventory model to merge the Just-In-Time, (JIT) purchasing concept. Larson [34] make a note on where Pan and Liao model's can only yield one optimal solution that is one blanket order covering annual requirement, delivered in lots of one unit at a time. Larson [34] also comment that Pan and Liao [40] assumption where the delivery costs are zero is false. Since the delivery costs are seldom zero, it seems that useful JIT blanket order/split delivery inventory model must include ordering cost, inventory carrying cost, and delivery cost. However, regardless the approach considered in these model, the major problem is that the concept of independent decision behavior in EOQ model cannot assure that the two parties as a whole can reach the optimal state. Seeking an optimal solution from only the buyer's point of view, would be neither effective nor feasible in the long run [20]. Therefore, due to this need, the concept of integrated policy is introduced to refine the well known classical EOQ model.

Recently, researchers found that the coordination between vendor and buyer enhances the profitability of both parties. The collaboration between them which allows for information sharing and negotiation such as the issues of price, lot sizing and etc. often results in advantageous benefits. A number of authors including Monahan [36], Banerjee [3], Joglekar [32] and Banerjee and Burton [4] reinforced that the coordinated or integrated replenishment policy is more desirable than the individual optimal policies.

To develop a collaboration between the vendor and the buyer, considerable changes in behaviour and attitude are required in both of them. For this to work, three important conditions must exist.

1. Both vendor and buyer must have a shared understanding of the long term nature of what constitutes a mutually beneficial relationship.

2. A vendor and a buyer are interlinked, and as such form of the same business, they must each achieve the level and nature of the commitments involved. The vendor have to accept full responsibility for the quality of their shipped product, and not rely on the buyer's receiving inspection to verify that the product is up to specification. On the other hand, the buyer will have to be prepared to develop plans and procedures for working with the vendors, and to commit resources to this. As a prerequisite of the new relationship, both parties have to reach an agreement on how they will work together.

3. Finally, the vendor and the buyer must recognize that not only is the quality of the supplied product ensured, but also their system cost minimized.

Based on the coordination between vendor and buyer approach, now, the integrated inventory policy has received a great deal of attention in order to improve the lack in the separate policy. In this approach, the vendor and buyer decide to co-operate, share information between them and agree to follow the agreement made by both parties. The vendor's and buyer's total costs are determined simultaneously.

The integrated inventory policy is initiated by Goyal [14]. Banerjee and Kim [5] developed both the separate and integrated model, and also showed that the separate model always gives the suboptimal solution. Ha and Kim [20] proposed an integrated inventory model by implementing Just-In-Time (JIT) purchasing. Their approach showed that cooperation between both parties at the outset of the long-term contract can provide them with a better opportunity of increasing their mutual benefit.

The total system cost for the integrated inventory policies is always superior than the independent policy. However, it may initially appear to involve some cost sacrifice on the part of only one party. Nevertheless, in view of the greater joint total cost efficiency of the integrated policy, the total cost savings can and should be shared in some equitable manner through the mechanism of a side payment to the other parties or price discount scheme. Goyal and Gupta [15] reviewed the literature on the models which provide a coordinating mechanism between buyers and vendors and not the price discrimination.

Yang and Wee [53] consider the integration of producer, distributers and retailers with a constant demand. They have shown that the integrated approach results in a significant cost reduction compared to that of independent decision making by each individual entity of the supply chain.

Sarmah [45] concluded in their paper, since different parties are involves in coordination process, to make coordination successful, faith between the parties and true revelation of information is necessary which model builder should take into consideration in their model in future.

As integrated inventory model is always better than the separate model, we will only consider the integrated inventory policy in the whole of this thesis. We develop an integrated final production batch and extend it to the n production batch.

In the next section, we will present some of the research which have been done to the integrated inventory policy under constant and varied demand. We divided these policies into two section where the holding cost of the vendor less than the buyer and vice versa.

2.2 Integrated inventory model for constant demand

The integrated inventory model with constant demand rate has been widely discussed in the literature. We divide it into two sections based on the characteristic of the holding cost. The first section will be the integrated inventory model with constant demand which consider the holding cost of the vendor is less than the holding cost of the buyer. Whereas in the second section the integrated inventory model with constant demand which consider the reverse situation will be presented.

2.2.1 Vendor's holding cost is less than the buyer's

The determination of economic production-shipment policy which consider the coordination between the vendor and the buyer has been studied by many researchers since the past four decades.

Goyal [13] initiated the idea of integrated policy where the objective is to minimize the total relevant costs for both the vendor and the buyer. This model is suitable when a collaborative arrangement between the buyer and the vendor is enforced by some contractual agreement. This situation is not uncommon in organization which have implemented JIT purchasing.

Monahan [36] suggested an approach to estimate the buyer's costs by using the buyer's earlier lot size decisions including changes in interest rates, tax rates, and etc. He presented an analytical approach to designing the terms of a vendor oriented, optimal quantity discount pricing schedule. His approach describes how a seller can improve his financial position by properly anticipating his major customer's reaction to any proposed discount plan. He admitted that there is probably no best method to estimate the buyer's costs accurately.

Banerjee [3] was one of the first person who analysed the integrated vendor-buyer model with the vendor manufacturing at a finite rate. The collaboration of two parties is taken into account to determine a more profitable economic lot size and move away from the adversarial bargaining process. He examined a lot-for-lot model in which the vendor manufactures each buyer's shipment as a separate batch. He also presented the independent model for comparison purposes and concluded that the integrated models not only reduced both vendor and buyer total cost substantially but it is also more conducive towards the creation and fostering of a business climate in which JIT and other related manufacturing philosophies are implemented.

Goyal and Szendrovits [19] developed a constant lot size model with equal and unequal sized batch shipment. This equal or unequal sized batch can be shipped from one stage to the next, and the total number of batches may differ across stages. They found that the cost resulting from their model is lower than the cost generated by a model that requires equal batch size at particular stage.

Goyal [14] generalized Banerjee's model by relaxing the assumption of lot-for-lot policy for the vendor, and showed that his model provides a lower or equal joint total relevant cost as compared to Banerjee's model. He developed a joint total relevant cost model for a single-vendor single-buyer production inventory system assuming that the vendor's lot size is an integer multiple of the buyer's order size. His model was derived based on the assumption that the vendor can supply to the buyer only after completing the entire lot size. He assumed that each shipment from the vendor to the buyer is of equal size.

Goyal and Gupta [15] assumed an infinite replenishment rate for the vendor (i.e the vendor does not manufacture the items himself but in turn buys it from his vendor) and ignored the effect of finite production rate in computing his inventory carrying costs. He also assumed that the vendor inventory holding costs are independent of the inventory items.

Lu [35] relaxed the assumption implied in Goyal's model and assumed that the vendor can supply to the buyer even before completing the entire lot. He suggested a solution for determining the optimal policy for the single-vendor single-buyer case and presented a heuristic approach for the single-vendor multi-buyer case.

Goyal [16] suggested an approach which is capable of obtaining lower total joint relevant cost of the single-vendor single-buyer production-inventory system. His model demonstrated how lower cost policies sometimes can be obtained when successive shipment size increase by a ratio which is equal to production rate divided by the demand rate that is

$$q_i \cdot (\tfrac{P}{D})^{i-1}, \qquad i = 1, 2 \ldots, k$$

where q_i is the size of the ith shipment, k is the number of shipment, P is the production rate for the vendor, and D is the demand rate for the buyer.

Hill [26] presented a more general class of policy for determining the vendor's production batch and successive shipment size. He suggested that the ith shipment sizes should be determined by evaluating

$$q \cdot \lambda^{i-1}, \text{ where } 1 \leq \lambda \leq \tfrac{P}{D}.$$

He showed that his model could provide a lower total cost policy as compared to the policies obtained by Lu [35] and Goyal [16].

Viswanathan [49] proposed two different strategies: one where the delivered quantity to the buyer at each replenishment is identical and another strategy where at each delivery, all the inventory available with the vendor is supplied to the buyer. He showed that no one strategy could obtain the best solution for all possible problem parameters.

Goyal [18] suggested a simple procedure for modifying shipment sizes as obtained on applying the procedure given by Hill [26]. He compared the result from his proposed procedure with result from Goyal [16] and Hill [26] and showed that his procedure produce the minimum total cost.

Goyal and Nebebe [17] further showed that the shipment policy where the first shipment is followed by $(n-1)$ equal-sized shipment of :

$$\text{First shipment size} \cdot \left(\frac{\text{Rate of production}}{\text{Rate of demand}}\right)$$

often achieves lower cost than the shipment policy adopted in Hill's model [26].

Hill [27] built a model based on the assumption that the buyer's inventory holding cost per unit per unit time, h_2 is always bigger than the vendor's, h_1. Although Hill ([26], [27]) listed some reasons that such an assumption is reasonable, they are not very intuitively obvious. As matter of fact, if the vendor and the buyer belong to the same firm, the opportunity costs involved in both sides' inventory holding costs will be the same. The difference between the two parties' inventory holding costs in this case will mainly come from other costs related to storage space and facility, loading and unloading, safekeeping, and etc. Hence, the buyer's inventory holding cost will probably be less than the vendor's if the buyer represents a bulk storage point or warehouse. The question that arises from Hill's model is : What is the structure of the optimal shipment policy should the vendor's unit holding cost is bigger than the buyer's ?.

Zhou and Wang [58] reconsidered Hill's [27] model by removing the above assumption made by Hill. They showed that their model can always generate the lowest average total cost among the hitherto existing vendor-buyer integrated models no matter whether the $h_1 < h_2$ or not. Their model makes us understand that if $h_1 > h_2$, the optimal shipment policy only consists of unequal sized shipments with successive shipment sizes increasing by a fixed factor $\frac{P}{D}$. But their model permits shortages to occur.

Hill and Omar [29] extend the work of Hill [27] to the case where holding cost decreases as the stock moves down the supply chain ($h_1 > h_2$). They also considered the case where the vendor's unit holding cost exceeds the buyer's unit holding cost ($h_1 < h_2$). They found that a non-trivial cost reduction can be achieved if the shipment sizes are permitted to vary.

Ben-Daya, Darwish, and Ertogral, [6] also discussed the integrated inventory model with single-vendor single-buyer. They compared seven policies proposed in the literature and concluded that the geometric-then-equal policies provided solutions very close to the optimal ones.

All the reviewed models in this section assumed that the demand rate is deterministic and constant. However, in reality, this assumption is not realistic. The demand is either increasing or decreasing with time. The literature of some works on integrated inventory model with time varying demand will be discussed in Section 2.3.

2.2.2 Vendor's holding cost is greater than the buyer's

In the literature, the policy where the holding cost of the vendor greater than the buyer is referred as the Consignment Stock, (CS). The CS policy is becoming an important strategy that companies adopt to face the manufacturing challenges. It implies great collaboration between vendor and buyer which is allows them to exchange their information about production, demand and shipments data.

In CS approach, goods will be supplied to the buyer as soon as there is enough to make up the batch-size, thus reducing the inventory cost during the production period. In other words, the vendor will keep their inventory level as low as possible by delivering all their finished product to the buyer's warehouse. The buyer pays the physical storage costs but the ownership of goods is transferred to the buyer only after the goods are used by the buyer for production and the buyer pays for the goods only upon use.

Hill ([26], [27]) found that the cooperation between the vendor and buyer gives a far greater benefit than a non-collaborative relationship. The most radical application of CS may lead to the suppression of the vendor's inventory, as this party will use the buyer's warehouse to stock material. This warehouse is close to the buyer's production line so that the material may be picked up when needed [9].

Braglia and Zavanella [9] proposed a method that is useful in identifying those productive situation where CS might be implemented successfully. They developed an analytical model which consider the proper minimum stock level, s and maximum stock level, S. The analytical results obtained allow the identification of the benefits and profitability that the CS approach determines in environments affected by uncertain demand. They also consider the case where delayed deliveries are allowed and examined a frequent and realistic situation, that is the case of stochastic demand.

CS policy gives a benefit to both parties. Persona, Grass and Catena [41] listed several benefit for the vendor and buyer by implementing CS policy.

The benefits for the buyer:

1. Suppression of the inventory holding costs, because the buyer only pays when drawing on the raw material.

2. Higher service level compared with a traditional policy, because the vendor guarantees a minimum stock level s in the buyer's warehouse.

3. Reduction in administrative costs, because the purchase order processing is lower than that of a traditional policy.

4. Application of just-in-time procurement, without high replenishment costs.

<u>The benefits for the vendor:</u>

1. Access to the final demand without the filter of the buyer's order policy.

2. Reduction in warehouse dimensions.

3. Reduction in production costs for the increased batch production size and a reduction in setup numbers per period.

4. Reduction in shipping costs compared with a just-in-time procurement.

Piplani and Viswanathan [44] assuming a one-buyer multi-vendor approach, proposed a numerical study that confirmed the lower joint costs of a CS policy in comparison with a continuous review inventory policy. This reduction increased with the increasing share of total demand from the buyer that employed a CS policy. They extend Braglia and Zavanella [9] approach in order to take into account the obsolescence of product. Their results shows that the presence of obsolescence reduces the optimal inventory level, particularly in the case of a short period of life. They found that the ratio between the vendor's production rate and the buyer's demand rate is another important parameter to take into account. They concluded that their work provides the analyst a robust methodology and some important insights in how to manage the effects of obsolescence so as to reduce total costs in supply chain managed with a CS policy.

Hill and Omar [29] considered the CS policy with identical and different shipment sizes. His numerical solution showed that the different shipment sizes case give the better solution compared to the identical shipments size case.

Zavanella and Zanoni [56] developed a model which considered a single vendor and multiple buyer integrated production inventory model with CS policy. They showed that the integrated inventory policy gives rise to economic benefits, which, however, may be modest or relevant according to the structure of the chain.

However all the previous consignment policy are implemented under constant demand.

2.3 Integrated inventory model for time-varying demand

When the demand rate varies with time, we can no longer assume that the best strategy is always to use the same replenishment quantity. In fact, this will seldom be the case. Donalson [12] was among the first to treat the inventory model with the case of linearly increasing demand pattern. He use calculus considerations to establish the appropriate number of replenishment for a given demand pattern and horizon length.

Silver [46] considered the Silver-Meal heuristic which is developed for the general case of a deterministic, linearly increasing demand pattern. He also determined each lot size sequentially, one at a time, by finding the local minimum of the inventory cost per unit.

Deb and Chaudhuri [11] extended Silver's heuristic for item with linearly increasing demand and shortages are allowed.

Hariga [23] developed two simple heuristic procedures for lot-sizing problem with continuous time-varying demands. These are the generalized Silver-Meal and the Modified Least-cost approaches.

Ben-Daya and Hariga [7] relaxed the assumption that demand is deterministic and assume that it is stochastic and tackle the lead time issue. They assume a linear relationship between lead time and lot size but take into consideration also nonproductive time in the lead time expression.

In the next section we will discuss some of the research which consider the integrated inventory problem (two level inventory problem) where the demand varies with time for both cases (vendor's holding cost is less than the buyer's holding cost and vice versa).

2.3.1 Vendor's holding cost is less than the buyer's

The inventory researchers realized that the constant demand rate is not realistic with the real world situation. Hariga [21] developed an efficient procedure that determines the optimum operating schedule for both decreasing and increasing demands. In contrast to the existing analytic methods in the literature, his procedure applies to both growing and declining markets. He illustrated the procedure with two examples from Donalson [12] and Phelps [42].

Hariga and Ben-Daya [24] extended the proposed model by Hariga [21] by considering the effect of inflation and the time value of money. Their model generates an optimal replenishment schedules when shortages are not allowed, and optimal ordering policies when shortages are permitted and completely backordered.

Benkherouf [8] presented an optimal procedure for finding the replenishment schedule for an inventory model with deteriorating items and decreasing time-varying demand and shortages. He concluded that the result obtained from their paper are applicable in an obvious way to inventory systems with deterioration and no shortages, to system without deterioration and with shortages, and to systems without deterioration and no shortages.

Balkhi [2] presents the integrated inventory model with a single product. The demand, production and deterioration rates of raw material are assumed to be a function of time. The underlying problem has been converted to an equivalent unconstrained minimization problem. It is shown that when the solution to the underlying inventory system exists, then it is the unique global solution.

Hill, Omar and Smith [30] developed an integrated model which considering the exponentially declining demand process. They used a dynamic programming formulation in continuous time to determine the replenishment policy.

Omar [37] developed an integrated inventory model for shipping a vendor's final production batch to a single buyer with equal lots policy under a linearly decreasing demand rate. He took an example of a company maintaining stocks of spare parts for products which are no longer being manufactured. The rationale behind his idea of treating the final production batch is that there is only one opportunity to make enough stock to meet all the remaining demand just before the equipment for manufacturing the product is dismantled. So, the vendor needs to make sure that no stock out will be occur otherwise they have to reassemble the equipment for manufacturing the product. If this happens, the cost will be increased.

Motivated from Omar's model, because of the production system seldom involved only one batch, we would like to extend his model which considers the n batch production cycle that includes the final batch at the end of the cycle.

2.3.2 Vendor's holding cost is greater than the buyer's

In this research, we also consider the consignment model with the integrated approach under time varying demand process.

Valentini and Zavanella [50] studied an industrial case and performance analysis of CS policy. They highlighted several potential benefits and pitfalls by implementing CS. They compared to the Hill model and showed that their policy reduce the joint total cost and stockout risks while assuring a higher service level in the case of fluctuating demand.

Zanoni and Grubbstrom [55] extend the theoretical solution proposed by Braglia and Zavanella [9]. They give an explicit analytical expressions for the three decision variables; q (quantity transported per delivery), n (number of transports per production batch) and k (number of deliveries). Whereas only q is considered analytically while n and k are analysed numerically. Hence, Zanoni and Grubbstrom [55] derived a formulae which is provide a quick method for calculating the optimal total number of deliveries and the number of deliveries to be delayed.

Recently, Bylka and Gorny [10] proposed the consignment stock of inventories in vendor buyer coordinated model where the demand is a continuous function of time. They solved the open problems of optimality of policies in Zanoni and Grubbstrom [55] for the case of the shipment sizes preferred by the vendor's is similar with the shipment sizes preferred by the buyer's.

2.4 Conclusion

In this chapter, we have presented some of the relevant literature for single and two level (inventory model) with constant and time varying demand rate. In the next chapter, we will discuss further the integrated inventory model for final production batch, and consider several possible policy.