

**ELECTRON AND POSITRON SCATTERING
BY ATOMIC RUBIDIUM**

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ABSTRACT

Electron and positron scattering from the Rubidium atom is studied using the coupled-channel optical method (CCOM). This method incorporates the continuum effect within the close coupling method (CC) by including an optical potential into the calculations. For electron-Rubidium scattering, a 5-state $[\text{Rb}(5s, 5p, 4d, 6s, 6p)]$ and 8-state $[\text{Rb}(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p)]$ CC and CCO calculations are carried out. For positron-Rubidium scattering, a 8-state $[\text{Rb}(5s, 5p, 4d, 6s, 6p) + \text{Ps}(1s, 2s, 2p)]$, 11-state $[\text{Rb}(5s, 5p, 4d, 6s, 6p) + \text{Ps}(1s, 2s, 2p, 3s, 3p, 3d)]$ and $[\text{Rb}(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p) + \text{Ps}(1s, 2s, 2p)]$ and a 14-state $[\text{Rb}(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p) + \text{Ps}(1s, 2s, 2p, 3s, 3p, 3d)]$ calculations are carried out. The elastic, excitation, differential and total cross sections are reported and compared to other theoretical data and experimental measurements.

ABSTRAK

Penyerakan elektron dan positron pada Rubidium atom dikaji dengan menggunakan kaedah “coupled-channel optical” (CCOM). Kaedah ini mengaplikasikan pendekatan keupayaan optikal ke dalam kaedah “close-coupling” (CC). Bagi sistem penyerakan elektron-Rubidium, pengiraan dijalankan dengan menggunakan 5-saluran dan 8-saluran [Rb(5s, 5p, 4d, 6s, 6p) dan Rb(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p)]. Bagi sistem penyerakan positron-Rubidium, pengiraan 8-saluran [Rb(5s, 5p, 4d, 6s, 6p) + Ps(1s, 2s, 2p)], 11- saluran [Rb(5s, 5p, 4d, 6s, 6p) + Ps(1s, 2s, 2p, 3s, 3p, 3d) dan Rb(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p) + Ps(1s, 2s, 2p)] dan 14-saluran [Rb(5s, 5p, 4d, 6s, 6p, 5d, 7s, 7p) + Ps(1s, 2s, 2p, 3s, 3p, 3d)] telah dijalankan. Keratan rentas elastik dan bukan-elastik telah dipaparkan. Pembeza dan jumlah keratan rentas juga akan dipaparkan. Semua hasil pengiraan akan dibandingkan dengan hasil pengiraan teori lain dan ukuran ujikaji.

LIST OF PUBLICATIONS

1. Chin, J. H., Ratnavelu, K., & Zhou, Y. (2011). Optical Potential Study of Electron Scattering by Rubidium. *Journal of Korean Physical Society*, 59(4), 2877-2879.

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