SLATER APPROXIMATION OF COULOMB EXCHANGE ENERGY IN HEAVY NUCLEI

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KUALA LUMPUR

2011

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DISSERTATION SUBMITTED IN FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF PHYSICS
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UNIVERSITY OF MALAYA
KUALA LUMPUR

2011

UNIVERSITI MALAYA
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Registration/Matric No: **SGR 080152**

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Title of Project Paper/Research Report/Dissertation/Thesis ("this Work"):

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Field of Study:
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ABSTRACT

The objective of this research is to study the relevance of the Slater approximation in calculating the exchange term of the Coulomb energy in atomic nuclei. The present motivation for calculating exactly the exchange part of the Coulomb energy was related to the need to obtain an accurate total nuclear energy. This is important, in particular as far as the (relative) deformation energy is concerned. For instance, when calculating the fission barriers, the Coulomb energy term (apart from the surface energy) needs to be calculated as precisely as possible. The Hartree-Fock method was employed to defined self-consistently the mean field together with the Bardeen, Cooper and Schrieffer (BCS) method to treat self-consistently the pairing interaction among nucleons of the same charge. For that purpose, we have used a phenomenological Skryme interaction for the effective strong nucleon-nucleon interaction and a seniority force to generate pairing correlations. The results show that the Slater approximation is quite good in general relative terms. It was found as expected to be less good for light nuclei than for heavy ones. As an important new result, we have shown that it was very significantly better for non closed proton shell nuclei than for proton closed shell ones. This has been readily interpreted as a clean-cut correlation between the proton level density near the Fermi level and the appropriateness of the Slater approximation.

Keywords: Exchange Coulomb, Slater approximation, Hartree-Fock method, pairing correlations, deformation energy, fission barrier.

ABSTRAK

Objektif penyelidikan ini adalah untuk mengkaji kesesuaian penghampiran Slater dalam pengiraan tenaga pertukaran yang terdapat di dalam tenaga Coulomb bagi nukleus atom. Pengiraan sebenar tenaga pertukaran Coulomb ini dijalankan untuk mendapatkan jumlah tenaga nukleus dengan tepat. Hal ini adalah penting khususnya dalam mendapatkan tenaga ubah bentuk (relatif). Contohnya, dalam pengiraan tenaga sawar pembelahan nukleus, tenaga Coulomb (selain daripada tenaga permukaan) perlu dikira dengan setepat yang mungkin. Kaedah Hartree-Fock telah digunakan untuk menentukan purata medan secara swakonsisten bersama-sama dengan kaedah Bardeen, Cooper dan Schrieffer (BCS) untuk merawat interaksi berpasangan di antara nukleon dengan cas yang sama secara swakonsisten. Bagi tujuan itu, interaksi fenomenologi Skyrme digunakan dalam interaksi kuat di antara nukleon dan daya kekananan digunakan untuk menghasilkan korelasi berpasangan. Hasil kajian menunjukkan bahawa penghampiran Slater secara amnya adalah memuaskan. Keputusan yang diperolehi adalah seperti yang dijangka di mana penghampiran Slater adalah kurang baik bagi nukleus yang ringan berbanding dengan nukleus yang lebih berat. Penemuan baru yang penting daripada kajian ini ialah penghampiran Slater didapati adalah jauh lebih baik bagi petala proton yang terbuka berbanding dengan petala proton yang tertutup. Hal ini telah ditafsir sebagai korelasi di antara ketumpatan aras proton di sekitar aras Fermi dengan kesesuaian penggunaan penghampiran Slater.

Kata kunci: Pertukaran Coulomb, penghampiran Slater, kaedah Hartree-Fock, kolerasi berpasangan, tenaga ubah bentuk, sawar pembelahan.

ACKNOWLEDGEMENT

This dissertation comes into fruition as a result of the tremendous support by many people whom I have the honour to have worked with throughout my study period in which I owed them my deepest gratitude.

First and foremost, I would like to express my gratefulness to Associate Professor Ithnin Abdul Jalil who gave me my first exposure into the theoretical work in nuclear physics, for his time and effort in our numerous meetings.

Professor Emeritus Philippe Quentin from University of Bordeaux 1, France, who had played a major role in determining the course of this research. My sincere gratitude to him for introducing me to the research work in nuclear structure, patiently correcting my writing and initiating and arranging my attachment in *Centre d'Etudes Nucléaires de Bordeaux Gradignan* (CENBG), France.

Special thanks also to the research team in CENBG particularly Dr. Ludovic Bonneau and Julien Le Bloas for their assistance during my stay there as well as for allowing me to use their programming code for the present research work. Not forgetting Professor Dr. Bernard Haas, Director of CENBG, for supporting my attachment in CENBG.

I would also like to thank University Technology Malaysia (UTM) for sponsoring my postgraduate study in University of Malaya (UM) and Institute of Postgraduate Study (IPS) of University Malaya for partially sponsoring my attachment in Bordeaux.

Finally, I would like to express my deepest gratitude to my parents and my loved ones for their kind support and understanding throughout my postgraduate study.

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