SLATER APPROXIMATION OF COULOMB EXCHANGE ENERGY IN HEAVY NUCLEI

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ABSTRACT

The objective of this research is to study the relevance of the Slater approximation in calculating the exchange term of the Coulomb energy in atomic nuclei. The present motivation for calculating exactly the exchange part of the Coulomb energy was related to the need to obtain an accurate total nuclear energy. This is important, in particular as far as the (relative) deformation energy is concerned. For instance, when calculating the fission barriers, the Coulomb energy term (apart from the surface energy) needs to be calculated as precisely as possible. The Hartree-Fock method was employed to defined self-consistently the mean field together with the Bardeen, Cooper and Schrieffer (BCS) method to treat self-consistently the pairing interaction among nucleons of the same charge. For that purpose, we have used a phenomenological Skryme interaction for the effective strong nucleon-nucleon interaction and a seniority force to generate pairing correlations. The results show that the Slater approximation is quite good in general relative terms. It was found as expected to be less good for light nuclei than for heavy ones. As an important new result, we have shown that it was very significantly better for non closed proton shell nuclei than for proton closed shell ones. This has been readily interpreted as a clean-cut correlation between the proton level density near the Fermi level and the appropriateness of the Slater approximation.

Keywords: Exchange Coulomb, Slater approximation, Hartree-Fock method, pairing correlations, deformation energy, fission barrier.
ABSTRAK


Kata kunci: Pertukaran Coulomb, penghampiran Slater, kaedah Hartree-Fock, kolerasi berpasangan, tenaga ubah bentuk, sawar pembelahan.
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