Freeness of Certain Torsion-free Abelian Groups

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Abstract

One of the old problems in abelian group theory is the following: When is a Torsion-free Abelian Group Free Abelian? This problem has been attacked by many mathematicians. However, until now, we can only have certain criteria under which a torsion-free abelian group is free abelian. A satisfactory answer to this problem has yet to emerge. In this thesis, we will study this problem. We will start with a literature review on the recent development of this problem and present some new results.

Abstrak

Salah satu masalah tertua dalam teori kumpulan abelan adalah seperti yang berikut: Bilakah Kumpulan Abelan Torsi Bebas adalah Bebas Abelan ? Masalah ini telah dikaji oleh ramai ahli matematik tetapi penyelesaiannya tidak begitu mudah. Sehingga sekarang kita hanya mampu menunjukkan beberapa kumpulan abelan torsi bebas yang tertentu adalah bebas abelan. Dalam tesis ini, kita akan mengkaji masalah ini, memberikan sorotan kajian untuk perkembangan terkini bagi masalah ini dan memberikan hasil-hasil kajian yang diperolehi.

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