CHAPTER 4: OLG MODEL WITHOUT SOCIAL SECURITY

4.1 Introduction

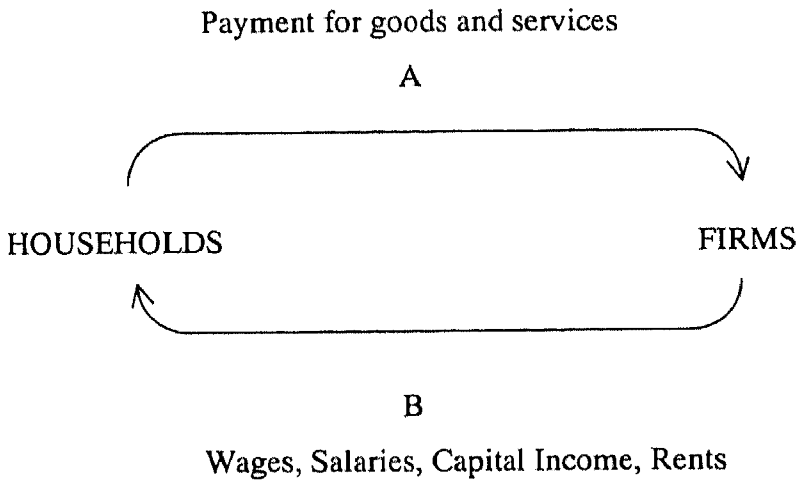
This chapter mainly discusses how useful is the OLG model in explaining some macroeconomic impacts in an economy, with the absence of social security. These impacts include consumption, savings, wages, and interest rates and capital accumulation. The OLG model is used to represent the economy at three levels; households, the production sector and the goods market equilibrium. This can be illustrated with a circular flow diagram.

4.2 Circular Flow Diagram Without Social Security

We begin the analysis by investigating the relationships among the different parts of the economy as illustrated by a circular flow diagram. The circular flow diagram is not merely useful in keeping track of how funds flow through the economy, but it also enables us to focus on certain **balance conditions**, which must always be satisfied. The **income of households (the flow of funds from firms) must equal the spending of households (the flow of funds to firms).** Thus, we can easily study the social security effects on national savings by incorporating social security transfer payments (in later part) into the circular flow diagram.

A circular flow can be analyzed from any starting point. Let us start at point A depicted by Figure 4.1, moving from left to right. Households buy goods and services from firms and they supply labor *(L)* and capital *(K)* to firms and make payment to firms. At any point of time *(t)*, this income flows back to households at B in the form of wages *(w)*, rents or interest rates *(r)* and capital income, to buy goods and services from firms again. The firms receive income from selling their products and part of this income is paid to their workers. Since households ultimately own the firms, thus anything left over is paid out to households as profits. (For simplicity, we assume firms earn zero profits in OLG model)

Figure 4.1 A Simple Circular Flow Diagram without Social security (no savings, no government and no foreign trade)



4.3 Overlapping Generations (OLG) Model Without Social Security

One of the frameworks for undertaking simulations of the social security system is the general-equilibrium, **overlapping-generations model (OLG)** originally presented by Samuelson (1958) and developed further by Diamond (1965).

The simple OLG model of **Diamond** is a basic model used in macroeconomics. Individuals at different stages of their life cycles interact in markets, when young, dealing mainly with older people and later with mostly younger people.

4.3.1 Two-period Lives

Figure 4.2 Relation between Time and Period of Life

(Generation Type in Box)

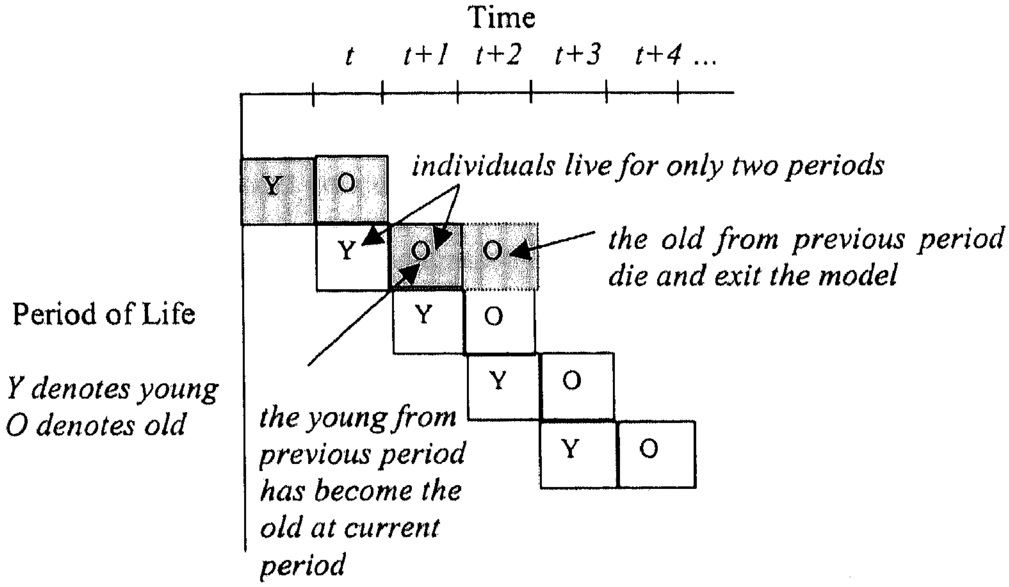


Figure 4.2 shows the relation between time (defined in terms of generation) and period of lives and indicates when each generation lives. An '*O*' signifies that the individual is old, and a '*Y* ' signifies that the individual is young. Horizontally, two blocks connected from left to right is a life. Vertically, two blocks connected top to bottom shows two generations are alive in the same period. We construct a simple OLG model in which *individuals live for only two consecutive periods*, which do not distinguish between sexes. Each group of individuals born during the same period is called a cohort, or generation.

At each period of life, a young generation is born at the beginning of each period and lives till the next period and become the old generation. **Each generation consists of a large number of identical individuals within as well as across time,** which means in economic terms, they have the **same lifetime pattern** in their endowment of goods and the **same preferences** - likes and dislikes - about their consumption. Individuals at different stages of their life cycles interact in markets, when young, dealing mainly with older people and later with mostly younger people.

There is **turnover in the population** and with this, it turns out to be simpler to assume that **time is discrete** rather than continuous; that is, the variables of the model are defined for $t = 1,2...$ rather than for all values $t \geq 0$. In other words, time is comprised of **distinct periods**, which we can think of these periods as years or as generations.[1] In each period L_t $(L_t > 0)$ individuals are born. If we assumed individuals live for two periods, at time t, there are L_t individuals in the first period of their lives and $L_{t-1} = L_t /(1+n)$ individuals in their second periods. Assume population growth is given exogenously at a fixed rate n. Thus, the number of individuals **working** in period t is: [2]

$$L_t = L_{t-1}(1 + n) \tag{4.1}$$

At any point in time, the economy consists of two generations, **the young as working population (age 15-64 years old) and the old as the retired group (age 65 years old and above),** with only the young owning goods. That is, the working life starts at the age of 15, younger children were assumed to be fully dependent on their parents on consumption of

1 However, Romer, D. (1996) commented that it is the general assumption of turnover in the population and not the specific assumptions of discrete time and two-period lifetimes that is crucial to the model's results.

2 To provide a numerical illustration of these values, it is important to recognize that n refers to the growth rate per period and not per year. In other words, the value of n is the growth rate per generation. If the annual population growth rate is 2.7 percent, and a generation is 30 years, (1+n) = (1.027)30 = 2.224. Thus, n = 1.224.

goods and services to which they do not constitute extra burden and do not provide any utility. Thus the **unproductive** non-working population (age 0 to 14 years old) is not included in our two-period OLG model.

The aggregate supply is based on the **size of the working-age population** (age 15-64 years old) while social security expenditures are dependent on the **size of retired age groups** (age 65 years old and above).[3]

4.3.2 Three-period Lives

We can also extend the model by assuming that the economy lives for three periods -: age 0 to 14 years old **(young),** age 15-64 years old **(adult)** and age 65 years old and above **(old).** However, in this research, since we assumed that the working age only starts from age 15 to 64 years old, thus we only construct the two-period lives OLG model.

4.4 Existing OLG Modeling without Social Security

We begin from constructing an OLG model without Social Security into three different categories; individuals, the firms as the production sector and the goods market as the whole.

4.4.1 Households

Households consist of different individuals. Initially, we assume that individuals live for two periods. Let $c_{1,t}$ and $c_{2,t}$ denote the consumption in period t of young and old individuals respectively. The first subscript denotes 'young' (if it is numbered '1') and 'old' (if it is numbered '2'). The

---

3 In Malaysia, we assume that there are some individuals at age 55 years and over still working until they reach aged 65 years old, thus we still categorized age 55 to 64 years old into the working population

second subscript denotes the period of time, t, $t+1$, $t+2$, ... As referring to Figure 4.2, at current period $t = 1$, $c_{1,t}$ is denoted by ☐ , $c_{2,t}$ is denoted by ▩ . The lifetime utility of a young individual born at t is depends on $c_{1,t}$ and $c_{2,t+1}$. That is, in Figure 4.2, individual born in period $t = 1$ has the utility depends on ☐ and ▩ .

The maximization problem of lifetime utility is based on a specified utility function. (See Box 4.1)

Box 4.1 Cobb-Douglas Utility Function (Without Social Security)

More specifically, let **utility** from consumption be the **Cobb-Douglas** type:

$$U = c_{1t}^{\beta} c_{2t+1}^{1-\beta}. \quad (4.2)$$

Where $\beta \in [0,1]$ if goods are normal. In equation (4.2), a young individual consume a constant fraction β of his full lifetime income. The nearer the β to 1,the individual prefers to consume his income in the first-period as compared to second-period.[4] Thus, his maximization problem is:

Max $c_{1t}^{\beta} c_{2t+1}^{1-\beta}$ subject to,

$$c_{1,t} = w_t - s_t \quad (4.3)$$

$$c_{2,t+1} \leq (1 + r_{t+1})\, s_t \quad (4.4)$$

Equation 4.4 says that in the period t (first period) or at any point of time, the young individual works full time, each young individual inelastically supplies one unit of labor ($L_t=1$) and earns a real wage of w_t, divides his resulting labor income between first-period consumption, $c_{1,t}$ and savings, s_t.

4 In our simulation later, we assumed β = 0.50, which is obtained literately from Hurd, M.D. (1997), pg.1003.

The individual will consume part of their first-period income and save the rest, s_t to finance their second-period retirement consumption, $c_{2,t+1}$. Assume that in period $t+1$ (second period), the individual just has just become old and is unable to work anymore. Let r_{t+1} be the next period real interest, paid on savings held from period t to period $t+1$. The young individual is uncertain about the future price of capital. He simply consumes all his existing wealth; both his savings from first period (principal) and any interest earned as capital income and leave no bequests. He then dies and exits the model. We can represent this phenomenon by using a lifetime budget constraint. (See Box 4.2)

Box 4.2 Lifetime Budget Constraint (Without Social Security)

Assuming no depreciation of capital, substitute equation (4.3) into (4.4) or s_t:

$$c_{2,t+1} = (1 + r_{t+1})(w_t - c_{1,t}) \quad (4.5)$$

It implies that the consumption of the old in period $t+1$ is equal to his savings when young (in period t) plus accumulated interest.

Dividing both sides of this expression by $1+r_{t+1}$ and bringing $c_{1,t}$ over to the left-hand side yields the individual lifetime **budget constraint**:

$$c_{1,t} + \frac{1}{1 + r_{t+1}} c_{2,t+1} = w_t \quad (4.6)$$

The individual maximizes utility subject to budget constraint.

Assume a discrete time version, the Lagrangian is:

$$\mathrm{L} = c_{1t}^{\beta} c_{2t+1}^{1-\beta} + \lambda [w_t - (c_{1t} + \frac{1}{1 + r_{t+1}} c_{2t+1})]. \quad (4.7)$$

The first-order conditions for a maximum are:

$$\beta \left(\frac{c_{2t+1}}{c_{1t}} \right)^{1-\beta} = \lambda \quad (4.8)$$

$$(1 - \beta) \left(\frac{c_{2t+1}}{c_{1t}} \right)^{-\beta} = \frac{1}{1 + r_{t+1}} \lambda \quad (4.9)$$

Box 4.2 (cont'd) **Lifetime Budget Constraint (Without Social Security)**

Substitute the equation (4.8) into the equation (4.9) and rearranging yields:

$$\frac{c_{2t+1}}{c_{1t}} = \left(\frac{1}{\beta} - 1\right) (1 + r_{t+1}) \tag{4.10}$$

It implies that whether an individual's consumption is increasing or decreasing over time depends on both the real rate of return (r_{t+1}) and the parameter value of β, where the young consumes a fraction (β) of their full lifetime income. Specifically, multiplying both sides of equation (4.10) by c_{1t} and substituting into the budget constraint in equation (4.6) and rearranging yields:

$$C_{1,t} + \frac{1}{1 + r_{t+1}}\left(\frac{1}{\beta} - 1\right) (1 + r_{t+1})\, c_{1t} = w_t \tag{4.11}$$

$$c_{1t} = \beta\, w_t \tag{4.12}$$

Thus, how much an individual consumes in the first period will depend on how much he earns from working when he is young.

From equation (4.3), $c_{1,t} = w_t - s_t$, let $s(r_{t+1})$ denote the fraction of income saved, substitute equation (4.12) into equation (4.3) yields:

$$S_t = w_t - c_{1,t}$$

$$S_t\,(r_{t+1}) = w_t - \beta\, w_t$$

$$= (1 - \beta)\, w_t \tag{4.13}$$

Equation (4.13) can be represented by a function s_t^*, where

$$S_t^* = s\,(w_t, r_{t+1}) \tag{4.14}$$

This is the **life-cycle hypothesis** in which we can observe how the savings of individuals smooth consumption over the path of their lives in the face of varying income. Consequently, this will lead to the accumulation of wealth at the individual level and hence of the capital stock at the national level. It implies that consumption is a function only of lifetime wealth. It is not affected by changes in the **pattern of income over time.** [5]

From the above assumptions, the optimal savings decision $s_t^* = s(w_t, r_{t+1})$ depends on the current total income w_t of the young household, and on the probability measure r_{t+1} which describes the subjective expectations concerning the distribution of next period's real interest rate. Hence, in the absence of social security, the nearer the β to 1, the individual prefers to consume his income in the first-period as compared to second-period, and thus the individual can save less for the second-period. In other words, if $\beta = 1$, the individual consumes all his income in the first period and saves nothing for the second period, vice versa.

4.4.2 The Production Sector

In order to examine the general equilibrium effects of social security, we must introduce firms as the production sector. Our model of an economy with stochastic aggregate production originates from the Diamond's overlapping generations model. At any period of time, a single good, which can be either consumed or used as capital input, is produced using a linearly homogeneous production function $F(K_t, L_t)$ consisting of two factors, capital and labor, are needed in the production process, where $K_t \geq 0$ is the capital stock at the beginning of period t, and $L_t \geq 0$ is the labor

5 Provided that wealth - defined to include not only financial and real assets but also the expected value of future income from labor (human capital wealth) does not change, neither should the pattern of consumption over time.

supply at time t. For simplicity, we assume that there is no productivity shock and thus no technological shock, and there are no fluctuations in total hours worked and the employee supplies one unit of labor. We adopt the framework of a neoclassical economy with production and storable output. The specification of Cobb-Douglas production function is denoted in Box 4.3.

Box 4.3 Cobb-Douglas Production Function (Without Social Security)

The net aggregate production technology of the economy is,

$$Y_t = F(K_t, L_t), \tag{4.15}$$

Where Y_t is the real output. We consider that this output is represented by a simple Cobb-Douglas production **function, with no depreciation.**

$$F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1 \tag{4.16}$$

Parameter α stands for the capital income share.[6]

Output per capita is given by the production function:

$$y_t = f(k_t), \quad f' > 0, \; f'' < 0, \tag{4.17}$$

where y_t is output per unit of labor, k_t is capital per unit of effective labor (capital-labor ratio), and w_t is the wage rate per unit of effective labor, both at time t. We assume that there are no installation costs and no technological change.

$$F(K,L) = 0 \quad \text{if } K \text{ or } L = 0. \; f' > 0, \; f'' < 0, \tag{4.18}$$

That is, it satisfies Inada conditions where $F(.)$ is twice continuously differentiable and strictly quasi-concave.

$$\begin{aligned} f(k) &= F(K/L, 1) \\ &= (K/L)^{\alpha} \\ &= k^{\alpha} \end{aligned} \tag{4.19}$$

---

6 The value for $\alpha = 0.30$ is obtained from Peters, W. (1990), pg.105.

In each period, the production function instantaneously transforms real value of the capital stock *(K)* and effective labor force *(L)* into a flow of consumer goods. Capital lasts for one period and has zero scrap value. The per capita production function is denoted $f(k_t)$, where k_t is the capital-labour ratio in period *t*. Assuming there is some initial capital stock K_0 that is owned equally by all old individuals. The firms choose non-negative K_t and L_t to maximize profit. (See Box 4.4)

Box 4.4 Profit Maximization (Without Social Security)

Factor demand and output are determined by the two familiar first-order conditions for profit maximization,

$$r_t = f'(k_t), \quad (4.20)$$

$$= \alpha k_t^{\alpha-1}, \quad (4.21)$$

$$= \alpha(K_t/L_t)^{\alpha-1}, \quad \textit{[From equation (4.19)]}$$

$$= \alpha K_t^{\alpha-1} L_t^{1-\alpha}. \quad \textit{[Cobb-Douglas production function]} \quad (4.22)$$

$$w_t = f(k_t) - k_t f'(k_t), \quad (4.23)$$

$$= k_t^{\alpha} - k_t \alpha k_t^{\alpha-1}$$

$$= (1-\alpha) k_t^{\alpha} \quad (4.24)$$

$$= (1-\alpha)(K_t/L_t)^{\alpha} \quad \textit{[From equation (4.19)]}$$

$$= (1-\alpha)K_t^{\alpha}L_t^{-\alpha} \quad \textit{[Cobb-Douglas production function]} \quad (4.25)$$

where a prime (') refers to a derivative with respect to the capital-labor ratio. The profit-maximization problem is:

$$\Pi_t = F(K_t, L_t) - r_t K_t - w_t L_t. \quad (4.26)$$

If the markets are perfectly competitive, then $\Pi_t = 0$, for all $t \geq 0$. Thus, the firms earn zero profit. Therefore, *F(.)* exhibits constant returns to scale,

$$F(K_t, L_t) = r_t K_t + w_t L_t \quad (4.27)$$

Substitute equation (4.15) into (4.27) yields,

$$Y_t = r_t K_t + w_t L_t. \quad (4.28)$$

We assumed that he firms are perfectly competitive markets, capital and labor earn their marginal products in all states of nature. Thus, the firm hires labor to the point where the marginal product of labor is equal to the wage, and renting capital to the point where the marginal product of capital is equal to its rental rate. The firms choose non-negative K_t and L_t to maximize profit.

Let r_t and w_t represent respectively the present value of capital and labor available in period t, $t \geq 0$. Thus, it implies that regardless of the functional forms of production, the productivity of a firm is reflected by both its factor of production, marginal product for capital and labor respectively.

In short, several implications of economic behavior had to be made in order to make the model tractable.

a) The production technology is given by a standard Cobb-Douglas function;
b) The one-good economy is assumed to be closed; and all investment has to be financed out of domestic savings.
c) The households or individuals are rational, have perfect foresight, forward-looking and have no liquidity constraints, embodying the life cycle hypothesis of consumption behavior and continuous market clearing;
d) Individual labor supply is fixed (exogenous).

4.4.3 Goods Market Equilibrium

In the case of without social security, the young owns everything, all of the goods; the old owns nothing. The old has nothing to give up in order to gain some goods for consumption. This generates equilibrium without social security in which all individuals consume all of their endowment (denoted as *g*) when young and consume nothing when old. (Refer Figure 4.3)

Figure 4.3 [7] Relation between Time and Generation (Consumption in Box) Without Social Security System

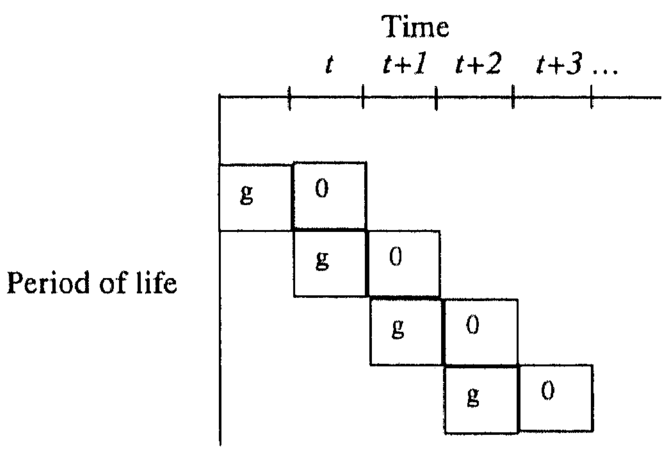


Figure 4.4 was modified from Klaus (1999) where OLG model is used to illustrate the properties of aggregate output, consumption, capital stock, savings and wages in the absence of social security.

Consider *t-1* as the first period (for the young), and *t* as the second period (for the old). Thus, there are L_t individuals in the first period and L_{t-1} individuals in the second period. Begin from the households sector, the

[7] Figure 4.3 is same with Figure 4.2 but with consumption amounts listed in the boxes instead of age.

consumption of an old individual will depend on the fraction of how much he can save from his productivity, that is, $c_{2\,t} = F\,(K_t\,,L_t\,)\; s(\,w_{t-1}\,,\,r_t)$. This amount of savings, $L_{t-1}\;\; s(\,w_{t-1}\,,\,r_t)$ will generate capital K_t to the production sector, which will implicitly define output productivity of the firm as denoted by $Y_t = F\,(K_t\,,L_t\,)$. The young individuals will supply labor to the production sector and have their budget constraint to maximize their utility. Hence, from the wages they received, they generate private savings of $L_t\,s(\,w_{t-1}\,,\,r_t)$ and good market equilibrium of $(\,1\,+\,n\,)\;k_{t+1}\;=\;s_t^*$. This will generate the capital stock in the next period and the process continues. (Refer Figure 4.4)

Figure 4.4 Illustration of the Time Structure of the Overlapping Generations Economy with Stochastic Production (Without Social Security).