CHAPTER 6: OLG SIMULATIONS OF SOCIAL SECURITY

6.1 Introduction

In this chapter, we would carry out simulations of social security with the overlapping generations (OLG) model. The purpose of doing this is to understand the macroeconomic impacts of social security. More specifically, we are interested to understand how the macroeconomic trends change overtime, when there is no social security, and when there is social security.

6.2 OLG Simulation in Stages

Basically, the simulations were conducted in two main stages: -

1) Testing OLG model without calibration.

2) Calibrating OLG model.

Stage 1 will enable us to test the simulation model for consistency with the theory stated in Chapter 4 & 5. In Stage 2, we will calibrate and simulate the model.

6.2.1 Number of Years in a Period (*t*)

Initially, we assumed **number of years in a period** to be equal to *1* ($t=1$). A 'period' can be meant for a 'generation', a 'decade' or a 'century'. When we assign a period as a generation and yet we set $t=1$, it seems to be not practical and not reasonable, nevertheless this setting will enable us to apply the 1970 data (for capital stock) as the initial value for *k*, in our model, as our data are usually collected annually. Ultimately, we still **test the model using $t=30$, we manage to get a similar path as compared to**

the t=*1*, but the only difference is t=*30* produced smaller value for both vertical (k_{t+1}) and horizontal axis (k_t) from 0 to 0.08 (See Figure 6.3(d) in pg.93) respectively compared to value for both vertical and horizontal axis from 0 to 0.22 (See Figure 6.1 in pg.67) if I assumed $t=1$. Since our OLG model is a two-period life-cycle model, it is more practical to simulate the model with a definition of 30-year-per-generation. However, if we were to compare these simulation results with our real data, we will need to collect a 60-year-data, and this will have some drawbacks due to the limitation of the data collection. Consequently, we will not be able to countercheck the realization of our simulation results.

Box 6.1 Population Growth Rate per Period (Generation)

The value of n in equation (4.1) is the growth rate per period (generation): -

$$X_b = (1+n)^{1/t} X_a$$

t = number of years in a period (generation)

n = population growth rate per period (generation)

X_a = population of last year

X_b = population of this year

Assume N is the annual population growth rate: -

$$N = \frac{X_b - X_a}{X_a}$$

$$= \frac{(1+n)^{1/t} X_a - X_a}{X_b}$$

$$N = (1+n)^{1/t} - 1$$

$$(1+n) = (1+N)^t$$

If the annual population growth rate (N) is 2.7 percent, i.e. $N=0.027$, and a generation is 30 years, $(1+n) = (1.027)^{30} = 2.224$. Thus, $n = 1.224$.

Since we are confident enough with our earlier simulation, this means that our model is feasible, therefore now we can investigate if there is any significant change when $t=30$. Therefore it is more specific to say that n refers to the growth rate per period or per generation and not per year (See Box 6.1 in pg.60).

6.2.2 Capital Income Share (α) and Consumption Preference (β)

In general, **both the parameter α and β lies between 0 to 1** respectively. Parameter α stands for the capital income share and it represents the weightage of capital in contributing to the output in an economy. The nearer the α is to *1*, there is more capital income share in the economy compared to the labour income share. (Refer Box 4.3 in pg.32)

On the other hand, if the goods are normal, young individuals would consume a constant fraction β of his full lifetime income. The nearer the β is to 1,the more the individual would prefer to consume his income in the first-period as compared to second-period and therefore he saves less.[10] (Refer Box 4.1 in pg.28)

We could not find any existing literature, which indicate the suitable value for parameter α and β in the context of Malaysia. **A higher parameter value for α means larger capital income share,** whereas a **higher parameter value for β means the young individual consumes more in the first period. Thus, they will save a smaller fraction of their labor income and the current capital stock will increase.** A value of $\alpha = 0.30$ in our simulation model is obtained literately from Peters (1990),

10 In our base simulation without social security, we also test our model using different value for α and β, i.e. $0.25<\alpha<0.75$ and $0.30<\beta<0.65$. Consequently, we also assigned different value for the parameter α and β into the equations in order to find the most suitable path. However, our simulations assumes $\alpha=0.30$ and $\beta=0.50$.

pg.105. This is quite suitable for the Malaysia context since we **are labor-intensive.** A value of β = 0.50 is obtained literately from Hurd (1997), pg.1003. as we assumed that **young individual consumes half of the fraction of his income in the first period.**

6.2.3 Capital-output Ratio

Capital-output ratio (COR) is measured in terms of the ratio between capital stock (K) and output (Y). It is also the proportion of the real gross domestic product to real fixed capital stock. Gross domestic product (GDP) has been used as a proxy for output (Y). COR refers to a number of unit's value of capital required to generate one unit of GDP. This ratio gives an indication of the amount of capital utilized per unit of output. COR shows whether the economic development strategy during a certain period has emphasized a high capital intensive policy.

.3 Structure of Simulation Model

Before we conduct the OLG model simulation, we need to have a 'bird-eye-view' of the model used in the simulation. Firstly, we have to determine which of the major equations are crucial. Secondly, we must determine which variables are to be made exogenous and endogenous variables. Thirdly, we would need to find an initial value of capital per unit effective labor. Finally, we must find the value of the savings for the initial period, and these savings together with the value of population growth rate per period will determine the initial capital per unit effective labor *for the next period* and so on. In short, we can always denote it as capital stock as cumulated capital per unit effective labor will form capital stock.

The series of steps taken are shown in Box 6.2 below.

Box 6.2 Structure of Simulation Model

Step 1: Exogenous capital per unit of effective labor,

Assume initial k_t at $t=0$ (say in the year 1970), $k_0 = 2$,

Step 2: Find output per capita as explained in pg.32 earlier on,

$f(k_t) = k_t^{\alpha}$ Assume $\alpha = 0.3$. Thus, $f(k_0) = k_0^{\alpha} = 2^{0.3} = 1.231$

Step 3: Find endogenous real rate of return on investment (interest rate)

From my OLG model at the production sector,

$r_t = \alpha k_t^{\alpha-1}$

$r_0 = \alpha k_0^{\alpha-1} = \frac{\alpha k_0^{\alpha}}{k_0} = \frac{0.3\,(1.231)}{2} = 0.1847$

Step 4: Find endogenous wage rate

From my OLG model at the production sector,

$w_t = (1 - \alpha)\, k_t^{\alpha}$

$w_0 = (1 - \alpha)\, k_0^{\alpha} = 0.7\,(1.231) = 0.8618$

Step 5: Determine endogenous consumption of the young

From my OLG model of the household, and assume $\beta = 0.5$

$c_{1t} = \beta\, w_t$

$c_{1,0} = \beta\, w_0 = 0.5(0.8618) = 0.4309$

Step 6: Determine endogenous savings

From my OLG model of the household,

$s_t = w_t - c_{1t}$

$s_0 = w_0 - c_{1,0} = 0.8618 - 0.4309 = 0.4309$

Step 7: Exogenous population growth, n

From Box 6.2, $(1+n) = (1+ 0.027)^{30} = 2.224$.

Thus, $n = 1.224$.

Box 6.2 (cont'd) Structure of Simulation Model

Step 8: Evolution of capital,

$$(1+n)\,k_{t+1} = s_t$$

$$k_{t+1} = \frac{s_t}{(1+n)}$$

Capital at the next period, (in the year of 1971)

$$k_{t+1} = k_1 = \frac{s_0}{(1+n)} = \frac{0.4309}{2.224} = 0.1938$$

Step 9: Using output of 8) as input, repeat Step 2) to 8) to determined

Endogenous variables for iteration *t+1, t+2, t+3, ...*

$f(k_t) = k_t^{\alpha}$ Assume $\alpha = 0.3$.

Thus, $f(k_1) = k_1^{\alpha} = 0.1938^{0.3} = 0.6112$

$$r_{t+1} = r_1 = \frac{\alpha k_1^{\alpha-1}}{k_1} = \frac{\alpha k_1^{\alpha}}{k_1^{2}} = \frac{0.3\,(0.6112)}{(0.1938)^2} = 4.8820$$

Step 10: After we know r_{t+1} *(from step 3 in second iteration)*, we can find **endogenous** consumption of the old,

$$c_{2,t+1} = (1 + r_{t+1})\,s_t$$

$$c_{2,1} = (1 + r_1)\,s_0 = (1 + 4.8820)(0.4309) = 2.5346$$

Continue to find the value for others variables...

Example of Table:

Simulation results - Diamond Model

Time, t	k_t	$f(k_t)$	r_t	w_t	c_{1t}	s_t	n	$c_{2,t+1}$
0	*2.000*	*1.231*	*0.1847*	*0.8618*	*0.4309*	*0.4309*	*1.224*	*2.5346*
1	*0.1938*	*- - all can be determined iteratively --*						
2								
3								
:								

This can be done both for the scenarios of simulation without social security and with social security.

6.4 OLG Simulation Without Social Security

It is essential to recall some of the equations stated in Chapter 4 before we can carry out the simulation of OLG model, in an economy without social security. (See Box 6.3)

Box 6.3 Recall Equations for Simulations Without Social Security

Assume population growth is given exogenously at a fixed rate n.

$$L_t = L_{t-1}(1+n) \tag{4.1}$$

Cobb-Douglas utility function:

$$U = c_{1t}^{\beta} c_{2t+1}^{1-\beta} \tag{4.2}$$

We have to assume the parameter value of β, which is the consumption preference.

Max $c_{1t}^{\beta} c_{2t+1}^{1-\beta}$ subject to,

$$c_{1,t} = w_t - s_t \tag{4.3}$$

$$c_{2,t+1} \leq (1 + r_{t+1}) s_t \tag{4.4}$$

$$c_{1t} = \beta w_t \tag{4.12}$$

$$s_t(r_{t+1}) = w_t - \beta w_t$$

$$= w_t - c_{1t} \tag{4.13}$$

The net aggregate production technology of the economy is,

$$Y_t = F(K_t, L_t), \tag{4.15}$$

Cobb-Douglas production function:

$$F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{4.16}$$

Parameter α stands for the capital income share.

$$f(k_t) = k_t^{\alpha}, \tag{4.19}$$

$$r_t = \alpha k_t^{\alpha-1}, \tag{4.21}$$

$$w_t = (1-\alpha) k_t^{\alpha} \tag{4.24}$$

$$(1+n) k_{t+1} = s_t. \tag{5.15}$$

Assumptions: -

1) Parameter value, $\beta = 0.50$ and $\alpha = 0.30$.

2) Initial capital per unit effective labor for 1970 is 0.018896.

3) Initial GDP per worker (of aged 15-64) is 4724.

4) The capital-output ratio (k/y) is 4 to 1.

5) Population growth per period is constant at 2.7%, $n=0.027$.

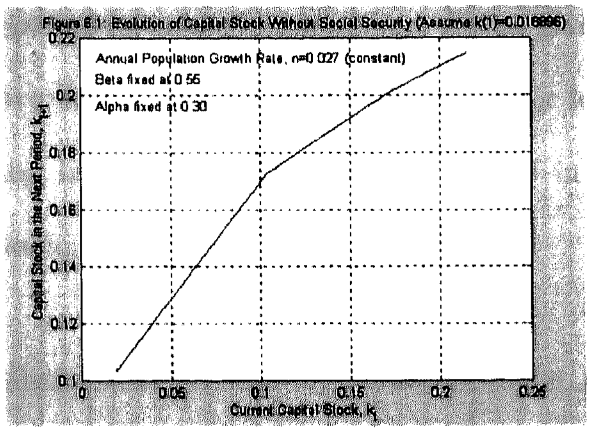
6) No social security, $\theta = 0$.

Basis of Assumptions: -

Initial capital is calculated from equation (4.19) where $y_t = k_t^{\alpha}$. We assigned output per worker at time t (y_t) as Gross Domestic Product (GDP) per worker of aged 15-64, at constant prices (with base year 1987). Taking the GDP per worker (of aged 15-64) for initial year 1970 as RM4724 (RM 26,817 million/5,677,000), assuming that the capital-output ratio (k / y) for Malaysia is 4 to 1 in the 1970. (See Box 6.4) Thus, capital per worker, k_0 is RM18896. (RM4724 x 4). This numerical figure seems to be quite reasonable.

a) Evolution of Capital Stock *(k) when population growth rate (n) is constant, in an Economy Without Social Security*

First, we study the **evolution of capital stock in an economy without social security** by carrying out a simulation. We assumed the initial capital, k_0 (the capital stock at the starting point) to 0.018896, which can represent RM18896 to have a smaller scale to produce more significant and reliable results. Assuming young individuals consume a fraction of 0.5 of his full lifetime income ($\beta = 0.50$) and capital income share is 0.3 ($\alpha = 0.30$). We obtained a nonlinear plot with positive slope and a kink. [See Figure 6.1 for plot].



The **positive slope** of the path **implies that an increase in the current capital stock (k_t) will result the proportionate increase in capital stock in the next period (k_{t+1}).**[11] Savings and interest rates are the major factors that had driven the change in capital stock. If we refer to Appendix Table 1, this kink happened in period 2 where there is a sudden increase in k_t from 0.1036 to 0.1726.

In Figure 6.1, in the absence of social security, in the first period (before the kink), **increase in savings will induce a larger increase in capital stock (the substitution effect)**. The tradeoff between the two periods has become **more favorable for second-period consumption tends to increase saving.** An increase in the interest rate decreases the price of second-period consumption, leading individuals to shift

11 In order to prove whether this statement is correct or not, or if it is only due to the inappropriate use of initial capital or parameter value (α and β) in the plot, we can test the model against a smaller initial capital value and different parameter value. However, we did not find any significant difference in both the path.

consumption from the first to the second period, that is, to substitute second- for first-period consumption.

In the second period (after the kink), the fact that **as given amount of saving yields more second-period consumption tends to decrease saving later on, (the income effect)**. Hence, it resulted in a **smaller increase in capital stock.**

b) Trend for other Macroeconomic Variables *when population growth rate (n) is constant, in an economy Without Social Security*

Apart from evolution of capital stock, we can also study the trend for the macroeconomic variables when the OLG model is simulated in the absence of social security. We have decided to present the graphical plots in a manner where ***the trend without social security is plotted together with the trend with social security in Section 6.5***, for comparison reason. For example, we can easily compare how the savings patterns change overtime, either with or without social security in the economy.

From the Appendix Table 6.1, it is very significant that regardless of its starting point, the economy will converge to a balanced growth path-a situation where each variable of our OLG model is growing at a constant rate. This is also consistent with the implication of Solow model.

Note: Table 6.1 and sample of Matlab program (Basesimulation_table) are presented in Appendix I.

.5 OLG Simulation With Social Security

This section embeds the study of the trend for the macroeconomic variables, which is quite similar to the preceding section but this time, with the presence of social security. In addition to that, as mentioned in the later part of Section 6.4, in order to see a comparable result, we decided to present the graphical plots in a way where the trend without social security is plotted together with the trend with social security. We study four cases here: -

Case 1 : Population growth rate (n) per period ($t=1$) is constant; and social security contribution rate (θ) is constant

Case 2 : Population growth rate (n) per period ($t=1$) is constant; and social security contribution rate (θ) changes from 0.05 to 0.11

Case 3 : Population growth rate (n) per period ($t=30$) changes from 1.037 to 1.978; and social security contribution rate (θ) is constant

Case 4 : Population growth rate (n) per period ($t=30$) changes from 1.037 to 1.978; and social security contribution rate (θ) changes from 0.05 to 0.11

In both Case 1 and Case 2, we assumed $t=1$, i.e. one-year-per period. In Case 3 and Case 4, we assumed $t=30$, i.e. 30-year-period period, in other words this means 30-year-per generation. The time frame of whether $t=1$ or $t=30$ does not affect our simulation results significantly although we already assigned the correct population growth rate per period. In both Case 1 and Case 2, population growth rate ranges from 0.023 to 0.034. However, in both Case 3 and Case 4, population growth rate ranges from 1.037 to 1.978. The reason for this phenomenon has been incorporated in footnote 2, pg. 26.

Given its central importance to the country's growth prospects, we analyzes the macroeconomic variables e.g. capital per unit of effective labor (k), output per capita (y), savings (s), real interest rates (r), wages (w) and consumption (c) in the four cases.

Again, it is essential to recall some of the equations stated in Chapter 5 before we can simulate OLG model, in an economy with social security (See Box 6.4).

Box 6.4 Recall Equations for Simulations With Social Security

Some fundamental equations stated in Box 6.2 and the following equations: -

Max $c_{1\,t}^{\beta}\, c_{2\,t+1}^{1-\beta}$ subject to,

$$\hat{c}_{1,t} = w_t - (\hat{s}_t + d_t). \quad (4.3')$$

$$\hat{c}_{2,t+1} = (1 + r_{t+1})(w_t - c_{1,t} - d_t) + (1+n)\, d_{t+1}. \quad (4.5')$$

where d_t is the contribution per young person at time t :

$$\theta_t w_t = d_t \quad (5.1)$$

$$\theta_{t+1} w_{t+1} = d_{t+1} \quad (5.1')$$

$$\hat{c}_{1\,t} = c_{1\,t} + \lambda \quad (5.5)$$

$$\text{where, } \lambda\ (< 0) = \beta\left[\ d_{t+1}\ \frac{(1+n)}{(1 + r_{t+1})} - d_t\right] \quad (5.6)$$

$$(1 + n)\, k_{t+1} = \hat{s}_t \quad (5.15)$$

Case 1: Base Simulation when population growth rate (*n*) is constant, social security contribution rate (*θ*) is constant

We conduct a base simulation by assuming both population growth rate (n) constant and social security contribution rate (θ) constant. The purpose of this simulation is to compare the impact of social security with various macroeconomics variables, ceteris paribus. When we plot the macroeconomic variable over time, all the plots will reach a balanced growth path steady state.

Assumptions: -

1) Parameter value, β =0.50 and α =0.30.

2) Initial capital with or without social security is 0.018896.

3) Initial GDP per worker (of aged 15-64) for 1970 is 4724.

4) The capital-output ratio (k/y) is 4 to 1.

5) Population growth per period *(t=1)* is constant at 2.7%, *n=0.027.*

6) Social security contribution rate is 11%, θ*=0.11.*

Basis of Assumptions: -

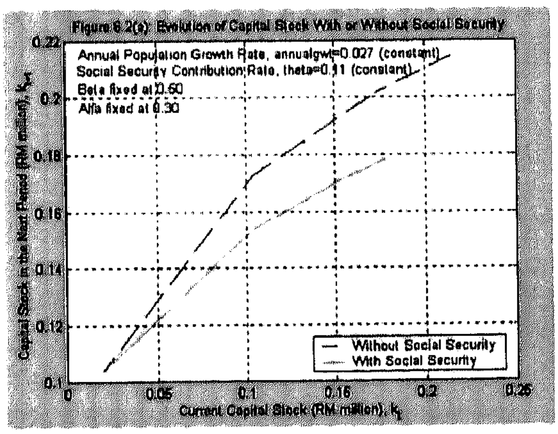
All assumptions are the same as Section 6.4 except there is an additional assumption 6 in this section, which is to incorporate social security into the OLG model. Since we have studied a path of evolution of capital stock without social security, we can simulate the equation (5.15) again, but by incorporating the social security contribution into our OLG model now. Consequently, we expect to see some changes in the value of others variables as compared to the model without social security.

a) Evolution of Capital Stock (k) *when population growth rate (n) is constant, social security contribution rate (θ) is constant.*

We investigate the evolution of capital stock over time by plotting capital stock in the next period against current capital stock, but this time, it is in the presence of social security. The plot has an increasing trend similar to which are consistent with theory in their shapes. [compare Figure 5.5 and 5.6 in Chapter 5 earlier on with

Figure 6.2(a)] There is still income and substitution effect but with the presence of social security, **the evolution of capital stock yields a lower steady state level. This is due to the fact that part of the income had been devoted to social security contribution, hence there is a lesser capital stock in the economy.**

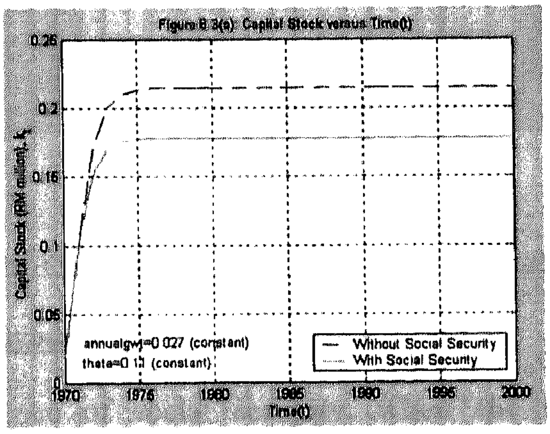
Figure 6.2(a) actually shows the properties of the economy once it converged to a balanced growth path; the population growth rate is constant, the social security contribution rate is constant, output per worker is growing at a constant rate, the capital-output ratio is constant and so on. Therefore, the plots stopped at a point, k^*. When there is social security, the young saves a lesser fraction of their labor income, thus the k_{t+1} function shifts down as depicted in Figure 6.2(a) below.



The downward shift of the k_{t+1} function decreases k^*, the value of k on the balanced growth path. **The impact of social security on the dynamic adjustment of the economy is to slow down the rate of capital accumulation while also reducing the steady state capital stock.**

We shall distinguish between short-run effects, i.e. the effects in the period when social security system is introduced and the long-run effects, i.e. the steady-state effects. The magnitudes of the short-run and long-run effects differ mainly due to the fact that while in the short-run, the wage rate of a decision maker is not affected.

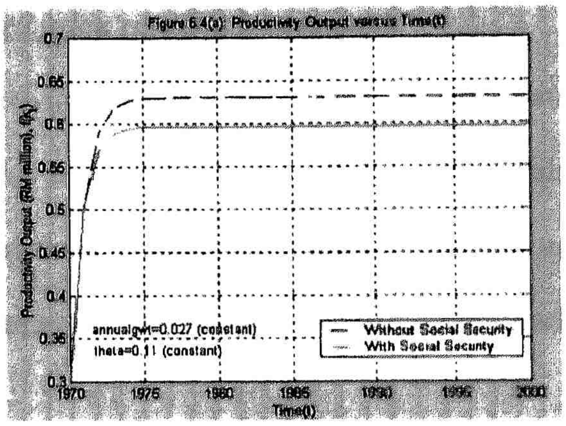
In this case, we adopted a steady state setting with given rate of population growth and social security contribution rate. Generally, capital stock will increase over time, but it will reach a constant level 5 periods later. Social security contribution will shift the curve down and it decrease the capital stock. [See Figure 6.3(a) below] [12]



[12] We found literature for Figure 6.2(a), and only if we get a similar result for this compared to literature, then it would be a wise idea to present the path in the form of time series analysis, as in Figure 6.3(a).

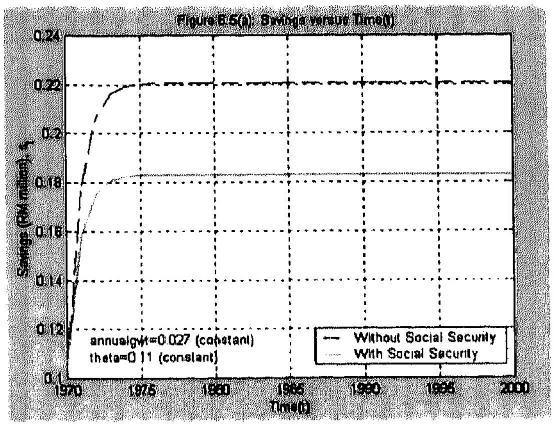
b) Output Per Capita (y) ***when population growth rate (n) is constant, social security contribution rate (θ) is constant.***

Since capital stock increases over time, thus productivity will also increase over time as capital is one of the factor of production. The trend shows that the **productivity path of the firm shifts down if we impose a rate of social security contribution, due to the decrease in the steady state level of capital stock.** [See Figure 6.4(a) below]



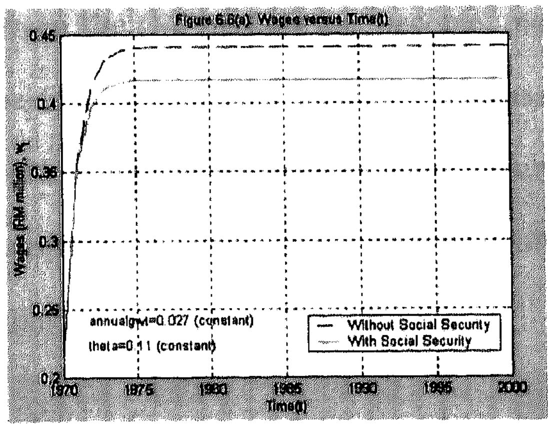
c) Savings (s) ***when population growth rate (n) is constant, social security contribution rate (θ) is constant.***

In the presence of **social security, when the capital stock increases in a slower rate, the private savings will also increase in a slower rate.** This private savings is the result of past decision and the retirees have no control over it. [See Figure 6.5(a) in the next page].



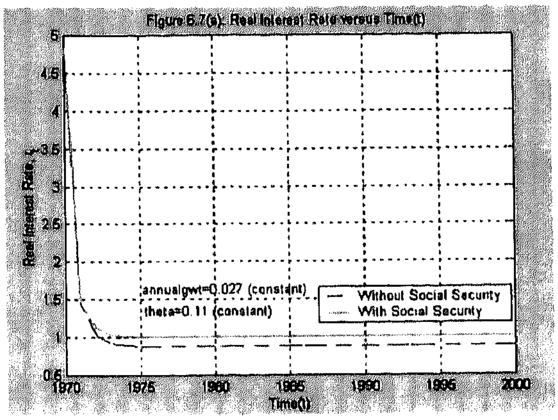
d) Wages (w) *when population growth rate (n) is constant, social security contribution rate (θ) is constant.*

Since productivity increases over time in our assumption, thus wages will also increase until it reaches a steady balanced growth path. **The down-shift in savings, thus in capital accumulation, also shifts wages down to a lower steady state level.** [See Figure 6.6(a) below].



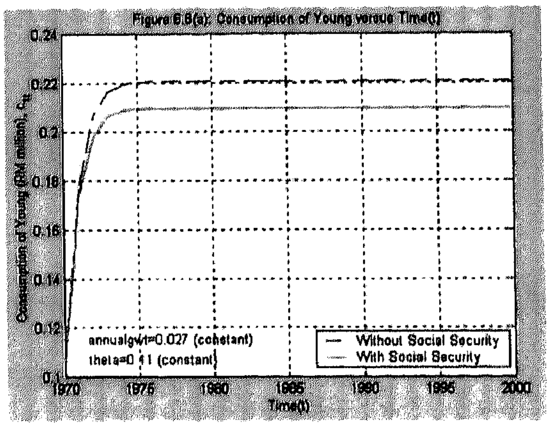
e) Real Interest Rates (r) *when population growth rate (n) is constant, social security contribution rate (θ) is constant.*

Consequently, **a downshift in savings will cause an upward shift in the real interest rates.** [See Figure 6.7(a) in the next page].



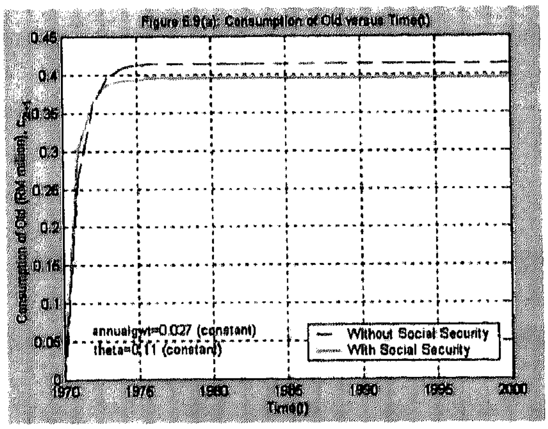
f) Young Consumption ($c_{1,t}$) and Old Consumption ($c_{2,t+1}$) *when population growth rate (n) is constant, social security contribution rate (θ) is constant.*

The decrease in wages further decreases savings. Lifetime consumption increases over time until it reaches a steady state level and this is depicted by an upward-sloping consumption curve. **In the case of without social security system, the fact that the tradeoff between the two periods has become, more favorable for second-period consumption tends to increase saving (the substitution effect). In the presence of social security, the first-period consumption of young over time will shift down.** [See Figure 6.8(a) below].



Theoretically, second-period consumption will increase by b_{t+1} as denoted by equation (4.4'). However, the curve of consumption of old over time shifts down and this violates our theory. [See Figure 6.9(a) below]. The reduction in individual wealth accumulation brought about by social security will induce changes in factor returns, exhibiting both income and substitution effects on consumption. **A higher real interest rates decreases lifetime resources. In addition, a higher rate of interest increases slightly the price of consumption in old age in one period later.**

However, it is not surprising that the introduction of social security **system** further lowers the consumption at old age after 1972 because our OLG model is a two-period model only. In order to show that the result more than two periods, the model has to be extended to a more complex OLG model. (Refer 4.3.2 in pg.27 earlier)



Note: Table 6.2 and sample of Matlab program (BasesimulationSS_table) are presented in Appendix II.

Case 2: Simulation when population growth rate (*n*) *is* constant, social security contribution rate (θ) varies

We can investigate how the social security contribution rate has an impact on those macroeconomic variables. We simply use the contribution rate of EPF in Malaysia from 1970 to 1999 as the social security contribution rate, which are basically varies from 5% to 11% over the 30 year-data.

Assumptions: -

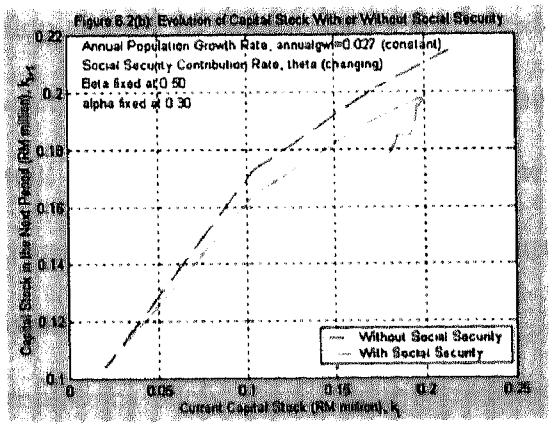
1) Parameter value, β =0.50 and α =0.30.
2) Initial capital with or without social security is 0.018896.
3) Initial GDP per worker (of aged 15-64) for 1970 is 4724.
4) The capital-output ratio (k/y) is 4 to 1.
5) Population growth per period ($t = 1$) is constant at 2.7%, $n=0.027$.
6) Social security contribution rate is varying from 5% to 11%, i.e. from 0.05 to 0.11. (See Appendix VI).

Basis of Assumptions: -

All assumptions are the same as Case 1 except there is a minor modification in assumption 6 in this section, to incorporate different social security contribution rate into the OLG model.

a) Evolution of Capital Stock (k) *when population growth rate (n) is constant, social security contribution rate (θ) changes.*

To our expectation, with the presence of social security, the evolution of capital stock yields a lower steady state level. This is due to the fact that part of the income had been devoted to social security contribution, hence there is a lesser capital stock in the economy. However, in the economy when there is social security with different social security contribution rate at any mean point of time, it seems to produce a downward trend diagram in the later edge of the plot [See Figure 6.2(b) below]

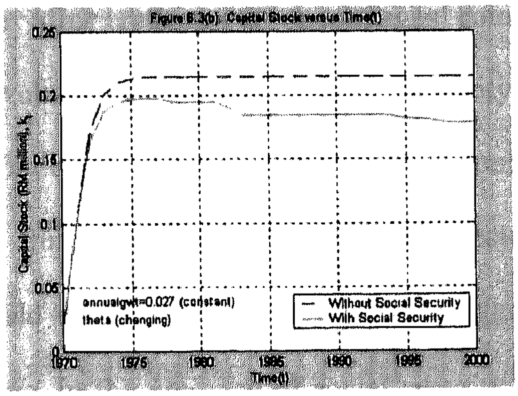


When we refer back to the value for the capital stock next period depicted in Table 6.3 (See Appendix III), we realized that the capital stock first increases, since we assumed there is no capital depreciation, and the productivity increase over time, but in 10 periods later, capital stock starts to decline, until it reaches a balanced growth path as before.

We also noticed that this occur when our social security contribution level is approximately at 0.03972. This means that in the presence of social security, when the contribution amount reaches certain level, it will decapitalize the economy. **This is because social security contribution at this time has decrease private savings to an extent where it is not enough to generate capital stock in the next period, but conversely, it reduces the capital stock in the next period.**

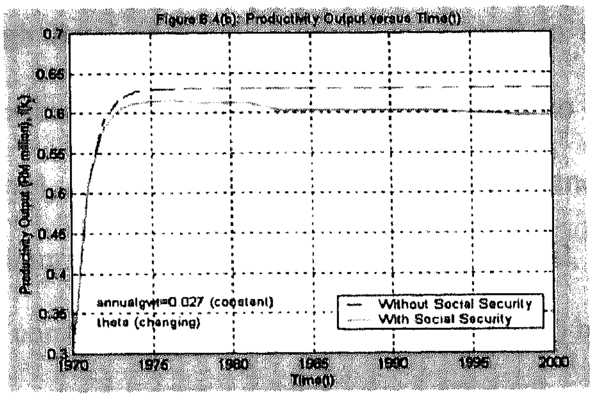
According to Blanchard and Fisher (1989), using a Pareto optimality criterion, whether this is a desirable outcome or not, it will depend on whether the interest rate prevailing before the introduction of social security, *r*, was smaller or larger than the rate of population growth, *n*. **In our case, *r* was greater than *n* before the introduction of social security, then the social security scheme will benefit the first generation, which receives a positive transfer of d_0, at the expense of subsequent generations and this is not Pareto improving.**

Simulations show social security reduce national savings rates and thus endanger the prospects for capital accumulation, which is reflected by the decline in capital stock in the economy [See Figure 6.3(b) below].



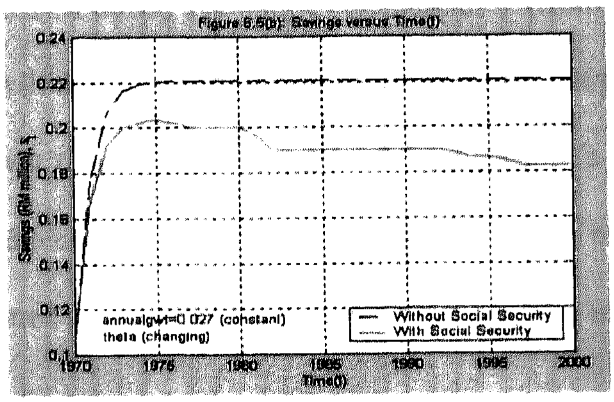
b) <u>Output Per Capita (y) *when population growth rate (n) is constant, social security contribution rate (θ) changes.*</u>

Since there is less capital stock when there is higher social security contribution rate, the productivity curve shifts down. [See Figure 6.4(b) in the next page] This is consistent with our theory which states that social security at certain level will decapitalize the economy.



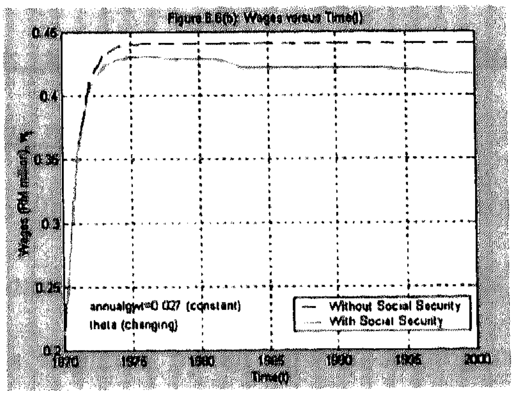
c) **Savings (s) *when population growth rate (n) is constant, social security contribution rate (θ) changing.***

The establishment and expansion of social security are thought to reduce aggregate savings essentially because **the promise of a pension after retirement encourages contributors to reduce their voluntary savings.** Social security plays an income redistribution role whereby the employer contribution redistributes income from the high-saving corporate sector to the low-saving household sector, leading to a decline in aggregate private saving. Therefore, in the presence of social security with different θ, **the higher the θ, the lower the savings** [See Figure 6.5(b) below].



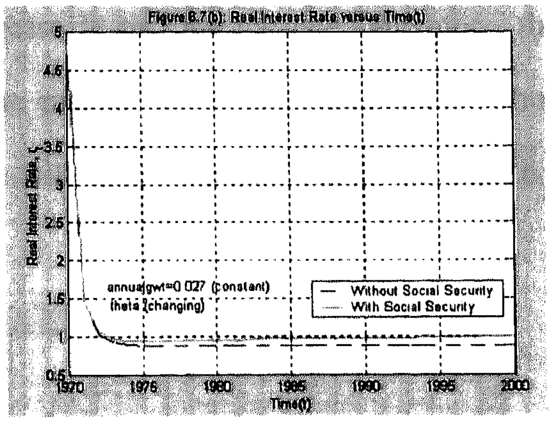
d) Wages (w) ***when population growth rate (n) is constant, social security contribution rate (θ) changes.***

The high level of θ will cause a down-shift in savings, thus in capital accumulation, **also shifts wages down to a lower steady state level.** As mentioned before, firm received income from their own labour as well as returns on their capital, while workers' income consists only of wages. [See Figure 6.6(b) below].



e) Real Interest Rates (r) ***when population growth rate (n) is constant, social security contribution rate (θ) changes.***

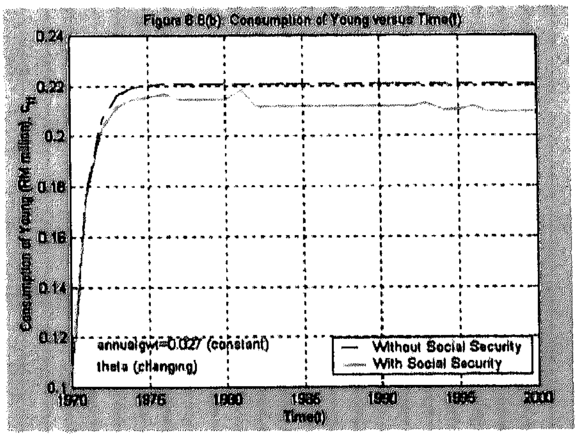
Consequently, a downshift in savings will cause an upward shift in the real interest rates [See Figure 6.7(b) in the next page] to encourage more savings. However, the decrease in wages will further decrease savings and in overall picture, the savings curve will still shift down. The sudden increase in θ (from 0.06 to 0.09 in 1981) will depict a steep down-slope in the plots at period of 1981. Whenever there is an increase in θ, it will show a down-slope in the productivity, capital stock, savings and wages path. When θ is maintained at certain level, it shows a level pattern in the path.



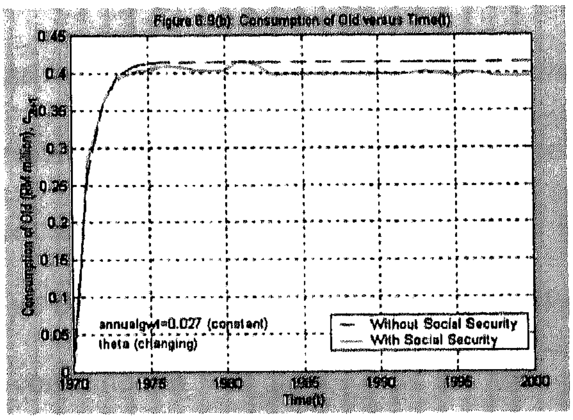
f) Young Consumption ($c_{1,t}$) and Old Consumption ($c_{2,t+1}$) ***when population growth rate (n) is constant, social security contribution rate (θ) changes.***

Similar to the scenario when there is constant θ, lifetime consumption increases over time until it reaches a steady state level and this is depicted by an upward-sloping consumption curve. Consumption curve of the young will shift down where there is social security. When we increase θ from 0.05 to 0.06 in 1976, the consumption of young decreases (from 0.2153 to 0.2167) and the consumption of old increases (from 0.4036 to 0.4172) declined. (Refer Appendix III Simulation Table 6.3)

This is because the social security contribution rate that increased from 0.05 to 0.11 was high enough to decrease the savings, thus decrease wages, increases interest rates and this will ultimately increase the consumption of the old in one period later. When we increase θ from 0.06 to 0.09 in 1981, the young may think that they could have saved enough for their future consumption, hence they will increase their consumption (from 0.2146 to 0.2183). However, a decrease in private savings will drive an increase in real interest rate. Therefore, the young will induce by the rate of return and saves more. Thus, the young consumption will then decreased back to 0.2116 [See Figure 6.8(b)].



These changes in values will ultimately produce a phase diagram. The old consumption will increase (from 0.4036 to 0.4172) in 1981, and then decreased back to 0.3979 [See Figure 6.9(b) below].



As according to the coefficient of lambda, as we recall is:

$$\hat{c}_{1t} = c_{1t} + \gamma \qquad (5.5)$$

$$\lambda = \beta \left[d_{t+1} \frac{(1+n)}{(1 + r_{t+1})} - d_t \right] \qquad (5.6)$$

As for our case as we have mentioned earlier, r_{t+1} ($\cong$ 0.8803) is greater than n (0.027), thus we have negative income effect as stated by equation (5.10) and (5.11) in Box 6.6.

Box 6.5 Individual Lifetime Budget Constraint (With Social Security)

Recall:

If w, r and d (where $d = \theta w$) are held constant and change in savings

$\Delta s = s^{\wedge}_{1t} - s_{1t}$, it implies :

$$-\frac{\partial \Delta s}{\partial d} = 1 + \beta \frac{(n-r)}{1+r} \quad (5.9)$$

$$-\partial \Delta s / \partial d \begin{cases} <1 & \text{if} \quad n < r \quad \text{(negative income effect)} \\ =1 & \text{if} \quad n = r \quad \text{(no income effect)} \\ >1 & \text{if} \quad n > r. \quad \text{(positive income effect)} \end{cases} \quad (5.10)$$

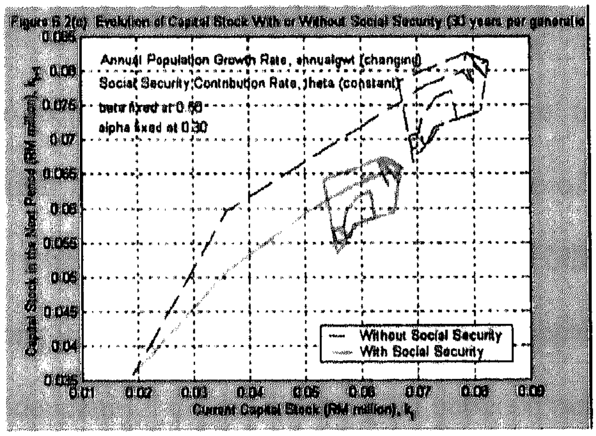
$$\left| \frac{\partial \Delta s}{\partial d} \right| \gtrless 1 \text{ depending on } n \gtrless r \quad (5.11)$$

This means that, **when r is greater than n, the coefficient of $(1+n)/(1+r)$ will be less than one.** If we assumed that $\beta=0.5$, *and* use the same $d_{t}=d_{t+1}$, it will result in $\gamma< 0$. Hence, consumption of the young will drop as depicted by equation (5.5). Additionally, even if we increase $d_{t+1}>d_t$, we will still get $\gamma< 0$, but the negative value of γ is smaller now. Thus, the young and old consumption will increase in the first period, but then decreased back for the next few periods with the higher level of $d_{t}=d_{t+1}$.

Although the results are ambiguous in the case of changing values of social security contribution rates and constant population

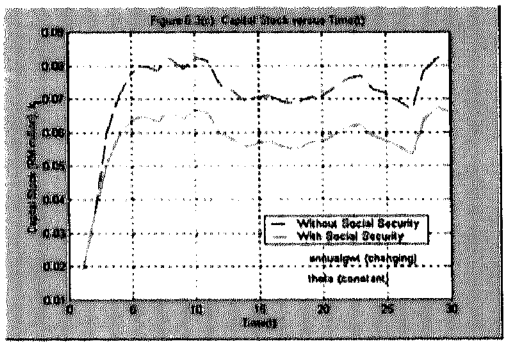
a) Evolution of Capital Stock (k) *when population growth rate (n) changes, social security contribution rate (θ) is constant.*

The phase diagram appeared significantly after the capital stock reaches a value of between 0.055 to 0.075 (which represents RM55,000 to RM75,000). **It shows that k_{t+1} is not uniquely determined by k_t, when k_t is between 0.055 and 0.075.** There are many possible values of k_{t+1}. **When there is a higher rate of population growth, savings is a decreasing function of interest rate *(r)*;** savings would be high if individuals expect a high value of k_{t+1}, and therefore we would expect r to be low, and savings would be low when individuals would be expect a low value of k_{t+1}, hence expect r to be high. [See Figure 6.2(c) below]



According to Romer (1996), **if savings are sufficiently responsive to *r*, and if *r* is sufficiently responsive to *k*, there can be more than one value of k_{t+1} that is consistent with a given k_t.** Thus, **the path of evolution of capital stock in this case is indeterminate.** It means that we cannot fully determine how k evolves over time given its initial value.

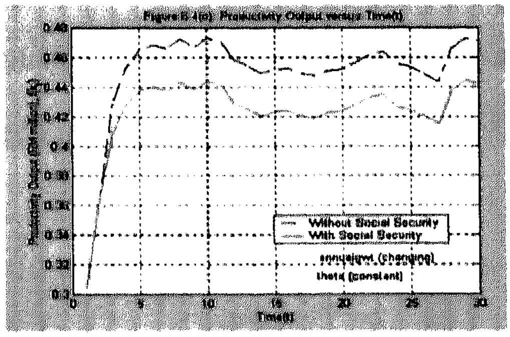
Assuming that the overlapping generations rather than infinitely-lived households, has potentially important implications for the dynamics of the economy. It reveals that the rate of population growth *(n)* affects the steady state equilibrium, quite possibly in the same way as social security taxes. This is due to the reason that **higher rate of population growth raises the per capita transfer to the old for a given lump-sum tax on the young.** [See Figure 6.3(c) below]



If we compared Figure 6.3(c) with Figure 6.2(c), both deliver a similar implication, which is in a high rate of population growth, for example in between period 10 and 13, where *n* increased from 0.023 to 0.028, may well detract, locally at least, from capital deepening. On the contrary, in between period 26 and 29, where *n* decreased from 0.029 to 0.024, the capital stock increased one generation later, and this is depicted in the upward sloping part of Figure 6.3(c) in the period between 27 and 29.

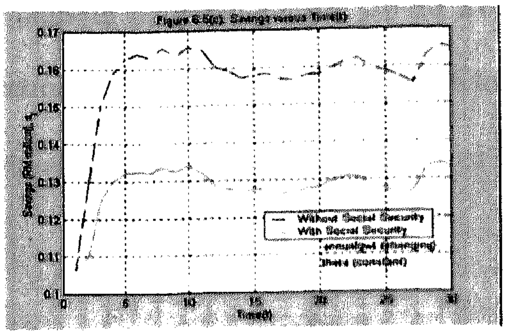
b) Output Per Capita (y) ***when population growth rate (n) changes, social security contribution rate (θ) is constant.***

The decrease in capital stock may lead to the adoption of a less-capital-intensive technology, which also result in a downward shift in the output productivity curve. Overall, the productivity output plot is the same in shape with capital stock plot because we derived productivity from capital (as a form of factor production) [See Figure 6.4(c) below].



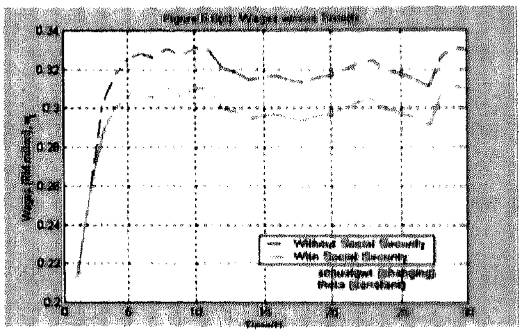
c) Savings (s) ***when population growth rate (n) changes, social security contribution rate (θ) is constant.***

A higher rate of social security contribution and a higher rate of population growth may lead to less savings per head. There is a significant decline in savings in between period 11 to 14 where during this period we have high population growth rate. On the other hand, there is a significant increase in savings in between period 26 to 27 where during this period population growth rate drop from 2.9% to 2.3%. [See Figure 6.5(c)].



d) Wages (w) ***when population growth rate (n) changes, social security contribution rate (θ) is constant.***

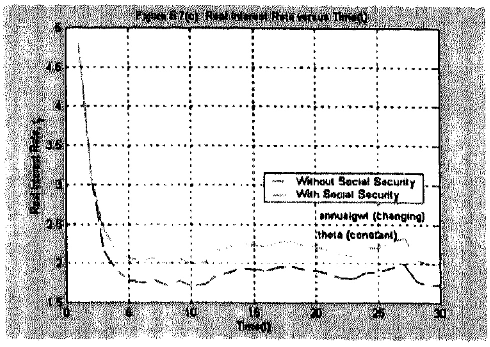
The high level of *n* will cause a down-shift in savings, thus in capital accumulation, **also shifts wages down to a lower steady state level.** [See Figure 6.6(c)].



The wages increase significantly in between period 26 and 27 when there is a drop in population growth rate.

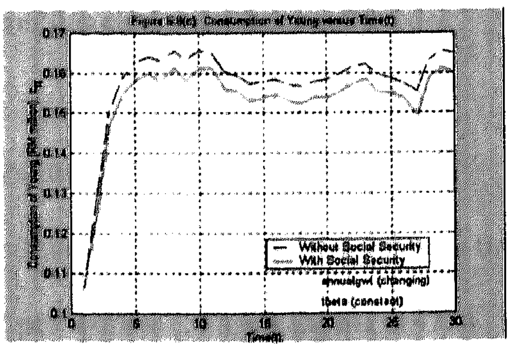
e) Real Interest Rates (r) ***when population growth rate (n) changes, social security contribution rate (θ) is constant.***

Consequently, a **high population growth rate which caused a** downshift in savings will also cause an **upward shift in the real interest rates** [See Figure 6.7(c) in the next page] to encourage more savings. On the contrary, there is a significant decrease in real interest rates in between period 26 to 27 where during this period population growth rate drop from 2.9% to 2.3%.

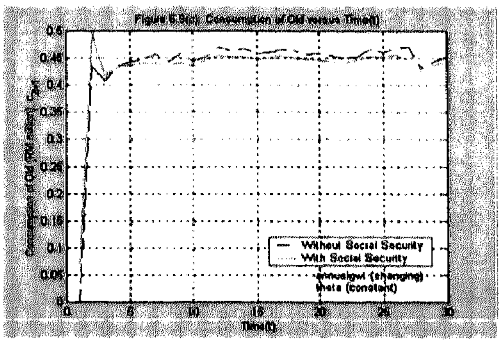


f) Young Consumption ($c_{1,t}$) and Old Consumption ($c_{2,t+1}$) *when population growth rate (n) changes, social security contribution rate (θ) is constant.*

In this case, we restrict our attention at how individuals could exchange a portion of their labor income when young, to smooth consumption in old age. Apparently, when there is high population growth rate, social security reduces national savings. One of the reasons is because that the compulsory plan does not induce households to increase their private savings, in contrast, simply lead them to **reduce their discretionary savings by an offsetting amount.** Ultimately, the consumption of the young will drop as depicted by equation (5.5). [See Figure 6.8(c) below]



The old consumption will increase in the second period, but then it will decrease for the next few periods because the OLG model here is meant for a two-period model. [See Figure 6.9(c)]



Note: Table 6.4 and sample of Matlab program (SSpopchange_table) are presented in Appendix IV.

Case 4: Simulation when population growth rate (*n*) changes, social security contribution rate (θ) changes

Further to our analysis in Case 2 and Case 3, we can also investigate how the combination of both the factors of social security contribution rate and population growth has an impact on those macroeconomic variables. With the contribution rate of EPF (which is basically varies from 5% to 11%), we simply use the population growth rate (which is basically varies from 2.3% to 3.4%) in Malaysia from 1970 to 1999 to investigate how those macroeconomic variables change consequently.

Assumptions: -

1) Parameter value, β=0.50 and α=0.30.

2) Initial capital with or without social security is 0.018896.

3) Initial GDP per worker (of aged 15-64) is 4724.

4) The capital-output ratio (*k/y*) is 4 to 1.

5) Population growth per period *(t=30)* is changing from 103.7% to 197.8%, i.e.1.037 to 1.978 (See Appendix V).

6) Social security contribution rate is varying from 5% to 11%, i.e. from 0.05 to 0.11 (See Appendix VI).

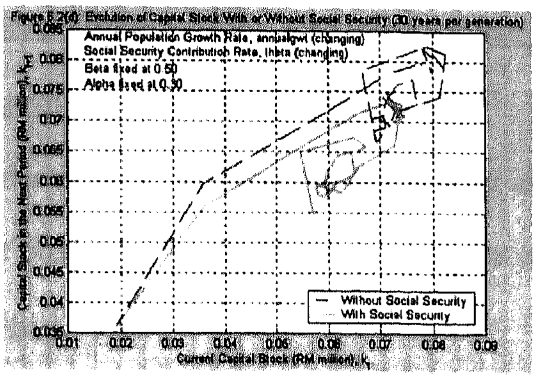
Basis of Assumptions: -

All assumptions are the same as Case 3 except that there is a modification in assumption 5 and 6 in this section, to incorporate different social security contribution rate and population growth rate in the OLG model.

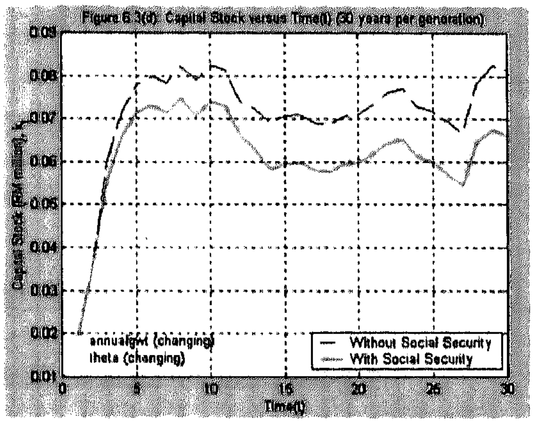
a) Evolution of Capital Stock (k) *when population growth rate (n) changes, social security contribution rate (θ) changes.*

We shall begin by looking at the evolution of capital stock, and it seems to be not consistent all the time [See Figure 6.2 (d)]. We obtained results that are typically the same with Case 3 even though now we are actually using different values for social security contribution rate *(θ)* as compared to Case 3 which we fixed θ at 11%. The phase diagram in Figure 6.2(d) has a evolution of capital which has higher value compared to Figure 6.2(c). This is because here we assumed lower value for social security contribution rate (ranging from 5% to 11%).

As compared to Case 3, say if we have a population growth rate at 2.6% in period 2, but this time we do not fix the social security contribution rate at 11% but let it be 5%.



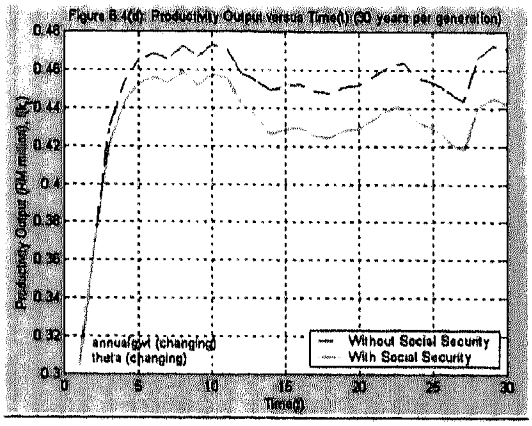
At this lower social security contribution rate, we see the capital per effective labor in the next period has increase from 0.0508 to 0.0556. [See Figure 6.3(d) below]



When there is a combination of factors of population growth rate and social security contribution rate, population growth rate seems to be a dominant factor, which determine how capital per unit effective labor changes.

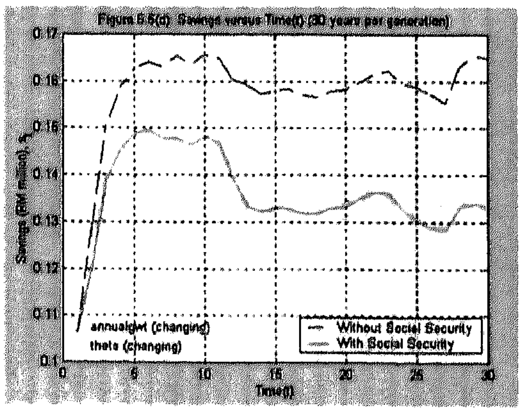
b) Output Per Capita (y) *when population growth rate (n) changes, social security contribution rate (θ) changes.*

Since productivity is driven by the capital stock available, the trend for output productivity will be the same as for capital stock. [See Figure 6.4(d)]



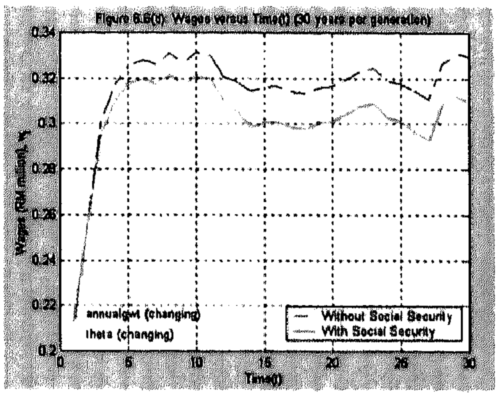
c) Savings (s) *when population growth rate (n) changes, social security contribution rate (θ) changes.*

There is a significant decline in savings in between period 11 to 14 (savings drop from 0.1469 to 0.1322) where during this period we have high population growth rate and high social security contribution rate [See Figure 6.5(c) below].



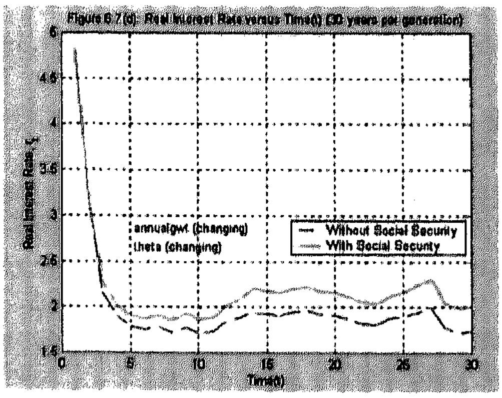
d) Wages (w) *when population growth rate (n) changes, social security contribution rate (θ) changes.*

When population growth changes as in Case 3, within a general equilibrium model with population, whether there is an increase or decrease in social security contribution rate, there will be no impact on the trend of wages as compared to Case 3 [See Figure 6.6(d)].



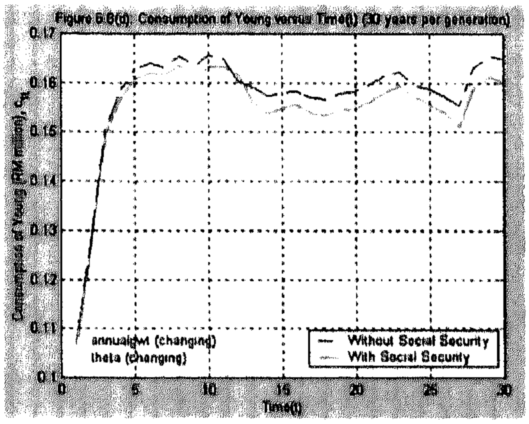
e) Real Interest Rates (r) *when population growth rate (n) changes, social security contribution rate (θ) changes.*

The same explanation applies for the real interest rate. Increase in interest rate will have income and substitution effects while affect savings in the opposite direction [See Figure 6.7(d)].



f) Real Interest Rates (r) ***when population growth rate (n) changes, social security contribution rate (θ) changes.***

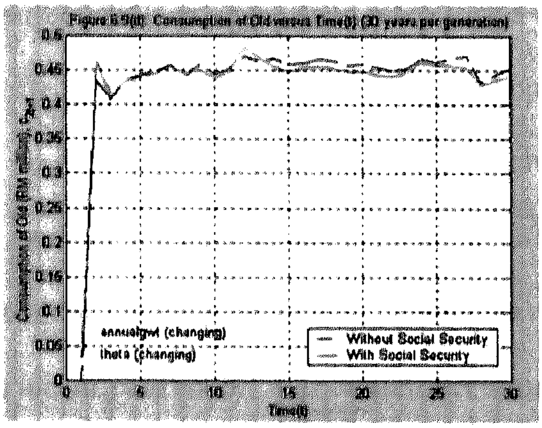
In the presence of social security, the first-period consumption of the young will have to fall by d_t as compared to the case when there is no social security. [See Figure 6.8(d)].



When n suddenly increase from 0.024 to 0.027 in period 11 (11th generation), we noticed that the consumption of the young declined substantially one generation later, which is at 12th generation. The increase in social security contribution rate (θ) from 0.06 to 0.09 in period 12 had further decrease the consumption of young from 0.1617 to 0.1559 one generation later, at period 13.

When n suddenly decreased from 0.029 to 0.023 in period 27, we noticed that the consumption of the young increased tremendously one generation later, which is at 28th generation even though the social security contribution rate (θ) is still maintained at 11% during this period. In short, a higher population growth rate will surely decrease the consumption of young when there is social security.

In the presence of social security, we know that the consumption will increase by b_{t+1}. In Figure 6.9(d), overall there is not much overall variance between the old consumption in the absence of social security and with the presence of social security. When n suddenly increase from 0.024 to 0.027 in period 11, we noticed that consumption of the old increased substantially one generation later, which is at 12th generation, from 0.450 to 0.483, even exceeding the consumption level when there is no social security. However, the increase in social security contribution rate (θ) from 0.06 to 0.09 in period 12 had decrease the consumption of old from 0.483 back to 0.468 one generation later, at period 13, when n is still high at 0.028. In short, a higher population growth rate will increase the consumption of the old.



Note: Table 6.5 and sample of Matlab program (SSpopthetachange_table) is presented in Appendix V.