

ABSTRACT

Differential equations arise in many areas of science and technology. They can be used effectively to analyze the evolutionary trend of such systems, they also aid in the formulation of these systems and the qualitative examination of their stability and adaptability to external stimuli. In recent years, the qualitative theory and asymptotic behavior of differential equations and their applications have been and still are receiving intensive attention. As far as oscillation theory is concerned, most texts in differential equations, both elementary and advanced, deal with second-order equations. In fact, in the last few decades, a great number of research papers concerning oscillation theory have been presented. Although ordinary differential equations of second order have been studied extensively, the study of qualitative behavior of third order ordinary differential equations has received greatly less attention in the literature, however assured results for differential equations of third order have been known for a long time and their applications in mathematical modeling in biology and physics.

In this thesis, oscillation of solutions of ordinary differential equations of second order and third order are considered. The obtained oscillation results are motivated extended and improved many previous oscillation results and new sufficient conditions are established which guarantee that our differential equations are oscillatory. Contradiction concept and Riccati technique are used in proofs of oscillation theorems. A number of theorems and illustrative examples for oscillation differential equation under study are given. Also, a number of numerical examples are given to illustrate the theoretical results. These numerical examples are computed by using Runge Kutta of fourth order

in Matlab. Finally, the obtained results are compared with existing results to explain the motivation of proposed research work.

ABSTRAK

Persamaan pembezaan timbul dalam banyak bidang sains dan teknologi, mereka boleh digunakan dengan berkesan untuk menganalisis trend evolusi sistem itu, mereka juga membantu dalam penggubalan sistem ini dan peperiksaan kualitatif kestabilan dan penyesuaian mereka kepada rangsangan luar. Dalam tahun-tahun kebelakangan ini, teori kualitatif dan tingkah laku asimptot persamaan pembezaan dan permohonan mereka telah dan masih menerima perhatian intensif. Setakat teori ayunan adalah berkenaan, kebanyakan teks dalam persamaan pembezaan, kedua-dua asas dan lanjutan, berurusan dengan persamaan tertib kedua. Malah, dalam beberapa dekad yang lalu, sejumlah besar kertas penyelidikan mengenai teori ayunan telah dibentangkan. Walaupun persamaan pembeza biasa tertib kedua telah dikaji secara meluas, kajian tingkah laku kualitatif persamaan pembeza biasa tertib ketiga telah mendapat perhatian yang amat kurang dalam kesusasteraan, bagaimanapun yakin keputusan untuk persamaan pembezaan tertib ketiga telah dikenali untuk jangka masa yang panjang dan mereka aplikasi dalam pemodelan matematik dalam biologi dan fizik. Dalam tesis ini, ayunan penyelesaian persamaan pembezaan biasa peringkat kedua dan perintah ketiga dipertimbangkan. Keputusan ayunan yang diperolehi bermotivasi, dilanjutkan dan diperbaiki banyak ayunan keputusan sebelumnya dan keadaan yang mencukupi baru ditubuhkan yang jaminan bahawa persamaan pembezaan kami ayunan. Konsep percanggahan dan teknik Riccati digunakan dalam bukti teorem ayunan. Beberapa teorem dan contoh ilustrasi bagi persamaan pembezaan ayunan bawah kajian yang diberikan. Juga, beberapa contoh berangka diberikan untuk menggambarkan keputusan teori. Ini contoh berangka dikira dengan menggunakan Runge Kutta perintah keempat dalam Matlab. Akhirnya,

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LIST OF NOTATIONS AND SYMBOLS

Symbol	Meaning
\mathbb{R}	The set of real numbers.
\mathbb{R}^+	The set of positive real numbers.
\mathbb{R}^-	The set of negative real numbers.
\mathbb{R}^3	The vector space $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$.
$C \in (\mathbb{R}, \mathbb{R}^+)$	The family of continuous functions from \mathbb{R} to \mathbb{R}^+ .
$C^1 \in ([t_0, \infty), \mathbb{R}^+)$	The family of continuous functions and their first derivatives from $[t_0, \infty)$ to \mathbb{R}^+ .
$C^2 \in [t_0, \infty)$	The family of continuous functions and their first and second derivatives on $[t_0, \infty)$.
$L^1_{loc}(D, \mathbb{R})$	The space of locally integrable functions from the domain D to \mathbb{R} .

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