CHAPTER 5

SYMMETRY PROPERTIES OF LAUGHLIN'S WAVE FUNCTION AND RELATED STATES

5.1 Introduction

We start with the Laughlin's wave function in the complex two-dimensional plane, z=x+iy, and write it in the spinor representation so that the electrons are arranged according to the group theoretical representation so that it is possible to find the ground state energy for a small number of electrons, N=10. We construct another wave function which is appropriate to the quasiparticles of the quantum Hall effect (QHE). A projection of the QHE wave function on the Laughlin's wave function with appropriate product over all of the quasiparticles produces a new wave function. In this way, we can generate a series of wave functions. We also explain the experimental data of the QHE in graphite.

The Laughlin's wave function [1] is defined by using two-dimensions x and y, in a complex plane, z=x+iy. This means that z is completely eliminated from the problem. The electrons can be located in a plane or on a sphere. However, in the case of a sphere also only x and y variables are used. There is an effort to make the two-dimensional sheet as thin as possible so that it is truly a two-dimensional object. The effect on the range is that it becomes too small. The eigen value or the ground state can be solved only when the electrons are arranged according to an irreducible representation of a point group. The wave function is of the form,

$$\Psi_1 = \prod_i z_i|^{\phi}\,|z_i|^n \exp\left[-\frac{|z_i|^2}{4l_0}\right]\prod_{j=1}^{i-1}\left(z_i - z_j\right)^q \tag{5.1}$$

where the wave function is confined to the region $R_1$and z < R_2 with l_0 as the magnetic length. The inner and the outer radii are determined by,

$$\frac{R_2^2}{4l_0^2} - \phi = n + mN \tag{5.2}$$

$$\frac{R_1^2}{4l_0^2} - \phi = n\,. \tag{5.3}$$

The charge is q/m within the annulus. The dimensionless flux ϕ is defined by the unit flux hc/e devided by the magnetic field and the area in which flux is quantized. The electrons on a sphere are described by $r_i(u_i, v_i)$. The spinor representation is described by (u_i, v_i) which are given in terms of (r_i, θ_i, ϕ_i) as,

$$u_i = \cos(\theta_i/2)\exp(i\phi_i/2) \tag{5.4}$$

$$v_i = \sin(\theta_i/2)\exp(-i\theta_i/2)\,. \tag{5.5}$$

The Landau level raising operator is given by,

$$L^+ = \bar{v}\frac{\partial}{\partial u} - \bar{u}\frac{\partial}{\partial v}\,. \tag{5.6}$$

The variables *u* and *v* are defined by equations (5.4) and (5.5). We can eliminate the complex coordinates to study the possible wave functions appropriate to the quantum Hall effect,Ψ_2. In this study, we generate a set of wave functions so that a series of wave functions can be developed. We also see that the quantum Hall effect of graphite is correctly explained.

5.2 Wave functions

Haldane has introduced the idea of generating a series of wave functions which will be similar to a large number of plateaus observed in the experimental data [149,231]. Recently, Bonderson [357] has suggested that the projections of wave functions of the quantum Hall effect, Ψ_2, can be taken with the Laughlin's wave function, Ψ_1, to generate a third wave function. In this way a series of wave functions can be produced. For m=3, the charge of a quasiparticle becomes 1/3 so that they studied only the odd denominators. Actually, the even denominators are also found in the data. We assume that Ψ_1, is a Laughlin's wave function and Ψ_2 is that of the quantum Hall effect. We generate the third wave function Ψ_3 as,

$$\Psi_3 = \int \prod_i^{Nqp} d^2 R_\mu \overline{\Psi_2}(R_\mu) \Psi_1(R_\mu, r_i) \tag{5.7}$$

where the number of quasiparticles is Nqp which are located at $R_1,\ldots, R_{Nqp}$. There are N electrons at the coordinate's $r_1 \ldots r_N$. Once we generate $\Psi_3$with Ψ_1 andΨ_2, we can generate Ψ_4 by using Ψ_3 and Ψ_1 and so on. According to Shrivastava [149,231], the quantum Hall effect wave function should be a hydrogen type wave function with well defined L, S and J and hence Ψ_3 must involve a suitable value of L, S and J. There is a particle-hole symmetry so that$j = l \pm s$ with both of the signs occurs. The usual Lande's formula which gives only one value is not useful at high magnetic fields and hence is replaced by a linear formula.

5.3 Spin and Magnetic Field: Hall Resistance

According to Shrivastava [3] the quantized resistivity may be written as, $\rho = h/e^2$ which may be corrected to, $\rho = h/[(1/2)\mathrm{g}e^2]$ where,

$$\frac{1}{2}\mathrm{g} = \frac{\mathrm{l} + \frac{1}{2} \pm \mathrm{s}}{2\mathrm{l} + 1} \tag{5.8}$$

which makes the quantized value depend on the sign of spin and orbital angular momentum quantum number and the flux quantization becomes, $B.A = \acute{n}\frac{hc}{(1/2)\mathrm{g}e}$ where $A^{1/2}$ is the magnetic length and n is an integer.

The formulas given above are non-relativistic. The electrodynamics effect is included in the value of the charge of the electron. The value of $h/2e^2$ found by using the value of the Planck's constant, h=6.626 068960 $\times 10^{-34}$ Js and that of the electron charge e= 1.602 176 487$\times 10^{-19}$ Coulomb is 12.906 403 783 kΩ. This value neither requires two dimensionality nor it requires Landau levels. According to Shrivastava formula, the positive sign before s gives the resistivity,

$$\rho_+ = \frac{h}{e^2}\frac{2l+1}{l + \frac{1}{2} + s} \tag{5.9}$$

whereas for the negative sign

$$\rho_- = \frac{h}{e^2}\frac{2l+1}{l + \frac{1}{2} - s} \quad . \tag{5.10}$$

For l =0, s=1/2, $\rho_+ = h/e^2$ and $\rho_- = \infty$. Therefore, large changes in the resistivity are possible when the sign of the spin is changed. The above values suggest change in resistivity from 25.8 k Ω to infinity upon spin flip. If we substitute i=0, in $\rho = h/ie^2$, then the resistivity changes from 25.8 k Ω to infinity. Of course, there are other values which show the resistivity as a function of spin. We need not limit to s=1/2 only. Other values of the spin such as 3/2 or 5/2 are also possible due to electron clusters.

5.4 Graphite

According to Shrivastava theory, there are quasiparticles of fractional as well as integer charge and the spin and charge are coupled. Hence, a modified Bohr magneton emerges and resistivity depends on the spin. There are fundamental charges given by e*/e=(1/2)g. where g=(2j+1)/(2l+1) so that the resistivity becomes $\rho = h/[(1/2)\mathrm{ge}^2]$. In heterostructures, the spin need not be 1/2 because there is cluster formation. For example, the spin of a cluster may be 3/2 or 5/2, etc. There are two particle states so that $\omega_1 + \omega_2$ is possible. Similarly, there are resonances so that $\omega_1 - \omega_2$ is also allowed. Hence, quasiparticle charge is determined from (i) spin-charge coupling, (ii) two-particle states, (iii) resonances and (iv) electron clustering. We explain the fractional charges found in graphite. The experimental measurements have been performed by Kopelevich et al. [321] so that we obtain the fractions from their work which are, 2/7, 1/4, 2/9, 1/5, 2/11, 1/6, 2/15, 1/8, 2/17 and 1/9. The energy of a state is given by (1/2)g(n+1/2) so that we consider two oscillators with energies, E_1= (1/2)g_1(n_1+1/2) and E_2= (1/2) g_2(n_2+1/2). The energy difference between these states is (1/2)g(n_1-n_2). For l=3, 2l+1=7 and for positive sign in (1/2)g=[l+(1/2)±s]/(2l+1)=4/7, (1/2)g_1n_1- (1/2)g_2n_2 +(1/2)g_1(1/2)-(1/2)g_2(1/2)= (1/2)(1/2)g_1 for n1=n_2=0, (1/2)g_2=0 for the second oscillator which has l=0, -ve sign and s=1/2 so that (1/2)g_2=0. Hence, (1/2)(1/2)g_1=2/7. The ingredients we put are two oscillators with different values of l and s which are the orbital and spin angular momenta quantum numbers. The effective charge which depends on spin also determines the resistivity. Hence the resistivity depends on spin. In the electron clusters, spin can become zero, so that we put s=0 to obtain (1/2)g= [

l+(1/2)]/(2l+1)=1/2 or g=1. In the formula (1/2)[(1/2)g-0] for g=1 we obtain 1/4. For l=4, 2l+1 =9 and for s=1/2 for negative sign, we obtain (1/2)g=4/9. In the expression, (1/2)[(1/2)g_1-(1/2)g_2] we have g_2=0 and (1/2)g_1=4/9 so that the effective charge becomes (1/2)(1/2)g_1=2/9. For l =2, we have 2l+1=5 and l/2l+1=2/5 and (l+1)/(2l+1)=3/5. We calculate the resonance state at (1/2)g[n_1+(1/2)]-(1/2)g[n_2+(1/2)] at (1/2)g(n_1-n_2) which comes at 3/5 -2/5 =1/5. For l =5 we have l/2l+1 =5/11 and (l+1)/(2l+1)=6/11. The value of (1/2)(6/11-0)=3/11. The resonance state now occurs at 5/11-3/11 =2/11. Let us look at the flux quantization at n'hc/e so that for n'=2, the charge is e/2. Hence for n'=2, the effective value of 1/3 changes to 1/6. The original value for l =1, 2l+1=3 for negative sign is 1/3. For l=7, the two series, l/2l+1 =7/15 and (l+1)/(2l+1)=8/15, (1/2)(1/2)g=4/15 and for n'=2, 4/15 becomes 2/15. We have already obtained 1/4 which for n'=2 becomes 1/8. For l=8, 2l+1=17 and the principal fractions are l/2l+1 =8/17 and 9/17. We have (1/2)(1/2)g =4/17 which for n'=2 gives 2/17. For l=4, l/(2l+1) =4/9 and (l+1)/(2l+1) =5/9. The resonance state of these two occurs at 5/9-4/9=1/9. This explains all of the fractions observed in the fractional quantum Hall effect of graphite experimentally observed by Kopelevich et al. [321]. This confirms that Shrivastava theory provides the correct interpretation of the quantum Hall effect data [2]. Usually, the Hall voltage as a function of magnetic field is a straight line but in graphite for small samples at low temperatures it shows structure. The plateaus or the peaks at fractional flux hc/e are correctly predicted by Shrivastava angular momentum theory.

5.5 Conclusions

We have shown that a large number of wave functions can be generated. All of the observed fractional charges agree with the scheme of a spin-charge relationship. The relationship of resistivity with charge is, $\rho = h/[(1/2)\mathrm{g}e^2]$ which is amply demonstrated [2]. The value of h/e2 was first measured by von Klitzing , Dorda and Pepper [358]. Later work of Tsui, Stormer and Gossard [359] showed that fractional values of the charge also arise. Laughlin [1] showed that there is a gauge invariance problem so that the quantized resistivity measures the charge of the particles. The experimental data shows a large number of plateaus which are largely explained by Shrivastava [3]. An effort was made to attach the flux quanta to the electrons but this theory does not satisfy the electromagnetic character of light correctly [350].Recently Kumar et al. [324] shown that the flux attachment to the electron from the Wilczek's theory [350] is not connected to the experimental data. We see that the spin dependent formula works very well.