CHAPTER 6

CYCLOTRON RESONANCE AND KOHN'S THEOREM

6.1 Introduction

A monolayer of carbon is called graphene. It exhibits unusual properties in the Hall effect and in the cyclotron resonance. It is found that it exhibits fractional charge in the Hall effect. The interactions amongst electrons almost become constant at low temperatures. Hence, the Kohn's theorem, which shows that the interactions do not play much role in determining the cyclotron resonance, becomes operative at low temperatures. The experiments on graphene do not depend on the wave vector dependence of the frequency. Hence whether the dispersion depends on k^2 or on k does not matter. The Hubbard model has been very successful in explaining the ground state of several electron systems. We consider a triangle with three vortices. Each vortex can be occupied by two electrons. By using the spin in a particular way, we can obtain new features in the Hubbard model. There is a doubling in the Peierls-Luttinger phase factor and eigen values acquire higher multiplicities than are known for the usual treatment of spin. The flux is distributed on the area of the triangle. The graphene consists of hexagons of carbon atoms but the Hall effect shows that there are defects on which electrons form clusters so that there is spin wave type behaviour. A cluster of electrons shows spin-waves leading to "spin deviation" of several per cent.

Theorem depends on the k space or due to some reasons such as low temperatures (< 1K), the interaction disappears and there is no k dependence whatsoever. Usually, the A monolayer of carbon atoms with hexagonal structure is called graphene. There is no doubt that it has unusual

quantum Hall effect which has been explained by means of non-relativistic theory [322,323,332,360]. Some authors have suggested to use a charge neutrality point which is located at the center of negatively charged electron band and the positively charged holes. Actually, all of the particles of the Dirac equation must be negatively charged or positively charged so that there are no solutions which predict the electrically neutral particles. Of course, the point in between two bands can exist without the existence of a particle. Hence the point in between electron and the hole bands is called "Dirac point". At this point, the bands apparently "cross" but a careful calculation [332] showed that there is a gap of 27 MeV in graphene. Apparently in 1961, Kohn [361] has shown that the cyclotron resonance is independent of the interaction. This theorem is of interest to understand the experimental data on cyclotron resonance. We wish to understand if this is really true. Some authors [362] are interested in the dependence of the theorem on the dispersion relation. Whether the kinetic energy of the electron has a k^2 dependence in the unperturbed energy, $\hbar^2 k^2 / 2m$. With some stretch of imagination the energy may be a linear function of k. In such a case whether the theorem depends on the wave vector dependence of the kinetic energy? We will see that the correct cyclotron frequency is independent of the wave vector dependence of the kinetic energy and hence the Kohn theorem is well obeyed. We will see that a modification of the unperturbed Hamiltonian leads to the interpretation of the experimental data in graphene. Hence, the data does not depend on the wave vector dependence of the kinetic energy and "Kohn theorem" is obeyed with a modification of the unperturbed Hamiltonian. We will see that such a modification depends on spin and the angular momentum.In this chapter , we report the Kohn's theorem with a modification which explains the graphene data and the activation energies. We also describe the Hubbard model for three sites on a triangle. It is found that Peierls-Luttinger phase factors are affected by spin.

6.2 Kohn's theorem

We first define the cyclotron frequency as,

$$\omega_c = \frac{eB}{mc} \tag{6.1}$$

where e is the charge of the electron, B is the magnetic field in the z direction, m is the mass of the electron and c is the velocity of light. The momentum of the electron along y direction is $p_{i,y} + (eB/mc)mX_i$, hence the components are,

$$\overrightarrow{P_i} = \left[p_{i,x}, p_{i,y} + (eB/mc)mx_i, p_{i,z}\right] \tag{6.2}$$

so that the Hamiltonian is,

$$H = \frac{1}{2m}\sum_{i=1}^{N} P_i^2 + U \quad . \tag{6.3}$$

For a two-dimensional electron gas confined to a rectangle, $L_1 \times L_2$ in the x-y plane, the single-particle Hamiltonian is,

$$H_1 = \frac{1}{2m}\left[p_x^2 + \left(p_y + \frac{eB}{mc}mx\right)^2\right] \tag{6.4}$$

whose eigen functions are,

$$\Psi_{n,k} = \exp(iky)\, u_n(x + k/s_1) \tag{6.5}$$

where

$$s_1 = \frac{eB}{\hbar c} \tag{6.6}$$

is the inverse area. Each level has a degeneracy,

$$g_1 = \frac{s_1}{\pi} L_1 L_2 = \frac{eB}{\hbar c}\frac{L_1 L_2}{\pi} \tag{6.7}$$

so that,

$$\pi\hbar g_1 = eBL_1L_2 \text{ Or } \frac{hc}{e} g_1 = B(2L_1L_2) \quad . \tag{6.8}$$

Hence, g_1 is a number equal to 1 or 2. In general, g_1 is an integer. The n^{th} harmonic oscillator eigen function of the Hamiltonian,

$$\mathcal{H} = \frac{1}{2m}(p_x^2 + \hbar^2 s_1^2 x^2) \tag{6.9}$$

is u_n and the eigen values of $\Psi_{n,k}$ are,

$$E_{n,k} = \left(n + \frac{1}{2}\right)\hbar\omega_c \quad . \tag{6.10}$$

The number of filled levels is ν. The higher levels are empty,

$$\nu g_1 = N \tag{6.11}$$

where N is the total number of particles. The values of the magnetic field for which νlevels are filled are given by,

$$\frac{1}{B_\nu} = \nu\frac{2\pi e}{\hbar cA} \tag{6.12}$$

where,

$$A = \frac{2\pi^2 N}{L_1L_2} \tag{6.13}$$

is the inverse area of the Fermi surface.

Modification: The cyclotron resonance without the interaction can be obtained with fractional charges by a suitable modification of the unperturbed Hamiltonian. In this way the Kohn's theorem is retained, i.e., we can get the correct cyclotron resonance without the interaction. We replace the equation (6.1) by,

$$\omega_c = \frac{1}{2}\, g\frac{eB}{mc} \tag{6.14}$$

where g= (2j+1)/(2*l*+1) so that (6.10) is modified to,

$$E_{n,k} = \left(n + \frac{1}{2}\right)\hbar\omega_c \frac{1}{2}\,\mathrm{g} \tag{6.15}$$

The equation (6.15) explains all of the graphene data correctly [322,323,332,360] and as we see, there is no interaction and the Kohn's theorem is obeyed in spite of the modification. Interactions. As originally suggested by Kohn theorem, the interactions play no role in determining the cyclotron frequency. Then what happened to the interactions? There are two types of cyclotron resonances, (i) $E_\uparrow - E_\downarrow = \hbar\omega$ where $E_\downarrow$ and $E_\uparrow$ are within one band. In that case, the interactions are gone in the g-value which does not constitute an interaction. Hence, the Kohn theorem is well obeyed and (ii) It is possible that interactions should be added as an energy so that one particle is in the valence band and the other is in the conduction band so that the transition requires an extra energy produced by the interaction. The resonance then occurs at a modified frequency, $E_\uparrow - E_\downarrow - \Delta = \hbar\omega$. As far as the experiments are concerned this will give rise to a modified g value, $E_\uparrow - E_\downarrow = \hbar\omega = (1/2)\mathrm{g}\hbar\omega_c$. The gap Δ is involved in the transition so that the resonance may be called the "gap cyclotron resonance". This type of resonances often occur in materials such as GaAs and they are always accompanied with the cyclotron resonance. The wave vector dependence of the dispersion relation does not come into play so that whether the frequency depends on k or on k^2 does not matter. The Kohn's theorem is thus well obeyed in graphene [362].

6.3 Spin Hubbard Model

The Hubbard model has been very successful in explaining the ground state of several electron systems [363]. A lattice of 12×12 sites can be solved to obtain the ground state energy. It permits

the electrons to hop from one site to another and hence it is exact. When the Coulomb interaction is added, the approximations are necessary. Some of the models can be solved exactly when the Coulomb interaction is suitably decoupled. It is customary to ignore the spin and calculate the lowest energy levels. Then electrons can be filled at the rate of one per level and when lowest levels are complete, two electrons, one with spin up and the other with spin down can be filled. A lot of times, this is the only way the spin occurs. We find that when the spin and the orbital angular momenta are taken into account, it is possible to get doubling in the eigen values such that four states occur rather than well-known two states. There is doubling in the Peierls-Luttinger phase factors. Triple dot molecule. In this molecule there are three sites and all of the three sites can be doubly occupied. We consider three dots located on the vortices of an equilateral triangle with a magnetic field directed perpendicular to the plane of the triangle. The three vortices are named 1, 2 and 3. When the electrons jump from site 1 to site 2, they acquire a phase factor ϕ_{12}. The area of the triangle is A_{Δ}. According to the flux quantization,

$$B.A_{\Delta} = \phi \frac{hc}{e} \tag{6.16}$$

The total flux is ϕ which is evenly distributed so that,

$$\phi_{12} = -\frac{1}{3}\phi \tag{6.17}$$

Similarly,

$$\phi_{23} = \phi_{31} = -\frac{1}{3}\phi \tag{6.18}$$

The Hubbard Hamiltonian for this system is,

$$\mathcal{H} = \sum E_{i,\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{\sigma,i,j=1}^{\prime 3} t_{ij}(B) c_{i\sigma}^{\dagger} c_{j\sigma} + U \tag{6.19}$$

This is an exact model [332]. Here U represents the Coulomb interactions. A detailed description of the Hubbard model in one-dimension is given by Essler et al. [364]. In the present example, the three sites are in two dimensions and the Hubbard hopping term is subject to a Peierls-Luttinger phase factor [365,366],

$$t_{ij}(B) = t_{ij}\exp\left(2\pi i\varphi_{ij}\right) \tag{6.20}$$

The current can travel from point 1 to point 2, from point 2 to point 3 and from point 3 to point 1 and also in the reverse direction. We transform the localized basis |j> into a plane wave basis,

$$|k\rangle = \sum_{j=1}^{3} e^{ikj}\left|j\right\rangle \tag{6.21}$$

where$|j>=|1>,|2>or\ |3>$ so that the reduced wave vectors appear as

$$|1'>=\frac{1}{\sqrt{3}}(|1>+|2>+|3>)$$

$$|2'>=\frac{1}{\sqrt{3}}(|1>+\exp(i2\pi/3)\,|2>+\exp(i4\pi/3)\,|3>)$$

$$|3'>=\frac{1}{\sqrt{3}}(|1>+\exp(-2\pi/3)\,|2>+\exp(-i4\pi/3)\,|3>) \tag{6.22}$$

and their eigen values are,

$$E_1 = E - 2|t|\cos(2\pi\varphi/3) + \mathrm{g^*\mu_B H\sigma_z S_z}$$

$$E_2 = E - 2|t|\cos(2\pi(\varphi+1)/3) + \mathrm{g^*\mu_B H\sigma_z S_z}$$

$$E_3 = E - 2|t|\cos(2\pi(\varphi-1)/3) + \mathrm{g^*\mu_B H\sigma_z S_z} \tag{6.23}$$

where,

$$\sigma_z S_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad . \tag{6.24}$$

Usually, this will split E_1, E_2 and E_3 into doublets. However, in the present case, we consider the spin in a more intricate manner so that we get the correct plateaus in the quantum Hall effect [2,323,335,339,367-375]. This theory explains the Stormer's data [322], Pan's data [323] and Tsui's data [374] correctly and all fractions observed [375,376] in the experiments are well explained. The flux is quantized as,

$$B.A = \phi \frac{hc}{e} \quad . \tag{6.25}$$

This in the Hubbard model becomes,

$$B.A = 3\phi_{12} \frac{hc}{e} \tag{6.26}$$

In order to predict the plateaus correctly, we replace g by,

$$\mathrm{g} = 2\frac{l + \frac{1}{2} \pm \mathrm{s}}{2l+1} = \frac{2j+1}{2l+1} \tag{6.27}$$

which due to the charge in the Bohr magneton is equivalent to replacing e by e*=(1/2)ge. The energy levels in the above then split as,

$$E_{1(+)} = E - 2|t|\cos(2\pi\varphi/3) + \frac{1}{2}\mathrm{g}_{\pm}\mu_{\mathrm{B}}H$$

$$E_{1(-)} = E - 2|t|\cos(2\pi\varphi/3) - \frac{1}{2}\mathrm{g}_{\pm}\mu_{\mathrm{B}}H \tag{6.28}$$

due to two values of S_z but there are two values of g due to j=l±s. Here,

$$\mathrm{g}_{+} = 2\frac{l + \frac{1}{2} + \mathrm{s}}{2l+1}\ , \mathrm{g}_{-} = 2\frac{l + \frac{1}{2} - \mathrm{s}}{2l+1} \tag{6.29}$$

so that $E_1(+)$ has two values and $E_1(-)$ also has two values. Hence E_1 splits into 4 values instead of the usual 2, with similar splitting for E_2 and E_3 of (6.23).

6.4 Peierls-Luttinger Phase factor

The phase factor is, $\phi_{12} = \frac{2l+1}{l+\frac{1}{2}\pm s}$. The flux quantizes as,

$$B.A = 3\phi_{12}\frac{hc}{\frac{1}{2}\text{ge}} = 3\phi_{12}\frac{hc(2l+1)}{\left(l+\frac{1}{2}\pm s\right)e} \quad . \tag{6.30}$$

For *l*=0, s= 1/2, there is a divergence so that the area of the quantized flux becomes very large for the negative sign. For, *l*=0, s=1/2 and positive sign,

$$B.\frac{A}{3} = \phi_{12}.hc/e \quad . \tag{6.31}$$

For *l*=1, s=1/2, with positive sign,

$$B.A/3 = \phi_{12}\frac{3}{2}\frac{hc}{e} \tag{6.32}$$

whereas for *l*=1, s=1/2,and negative sign,

$$B.A/3 = \phi_{12}3\frac{hc}{e} \quad . \tag{6.33}$$

Similar relations hold for ϕ_{23} and ϕ_{31} and many more can be generated by varying the value of *l*. We see that the Peierls-Luttinger phase [365,366] splits into two values,

$$\exp\left(2\pi i\phi_{ij}\right) = \exp\left[2\pi i\phi_{ij}\,(2l+1)/(l+\frac{1}{2}\pm s)\right] \tag{6.34}$$

for *l*=0, $\phi_{12} = \infty$; for *l*=1, $(3/2)\phi_{12}$ and $3\phi_{12}$ etc.

6.5 Search for an interaction.

Henriksen et al. [362] made an effort to search for an interaction. It turned out that most of the experiments carried out at low temperatures showed no interaction at all. So if an interaction is assumed and compared with the experiments, it should show that the interaction is absent. All of the observation can be explained on the basis of a suitable unperturbed Hamiltonian. The basic idea of the Kohn's theorem is also that in cyclotron resonance the interaction does not play an important role except for a constant so the cyclotron resonance can be explained by unperturbed Hamiltonian. Hence the problem is reduced to finding the unperturbed Hamiltonian. If we take a bad formula, it should turn out to be incorrect. Let us look for the Coulomb interaction, whether it applies to the data or in what way it enters into the theory. The repulsive Coulomb interaction between electrons is given by,

$$\mathrm{H}_c = \sum_{ij}{}' \frac{e^2}{r_{ij}} \ . \tag{6.35}$$

We replace r_{ij} by $(hc/eB)^{1/2}$ so that the Coulomb potential becomes,

$$\mathrm{H}_c^R = \sum_{ij} e^2 \left(\frac{eB}{hc}\right)^{1/2} \tag{6.36}$$

which varies as $B^{1/2}$. Usually, the distance between two electrons enters the Coulomb interaction rather than the distance upto which flux is quantized. Therefore (6.36) is much smaller than (6.35) but the advantage is that there is a quantized field instead of the Coulomb repulsion. If the cyclotron resonance shift is proportional to the $B^{1/2}$, then we can conclude that electrons are confined within the area in which flux is quantized and there are no electrons outside this area and there is a reduced Coulomb interaction. If the experimental data does not give the square root dependent shift, then it means that there is no Coulomb interaction as such except that which

might be absorbed in the unperturbed Hamiltonian by a transformation. In the case of special properties of spin with both signs in the total angular momentum j = $l \pm$ s, there is no interaction and all the interpretation is based on the unperturbed Hamiltonian [335,339,367-374]. Similarly, Kohn's theorem requires no interaction to appear in the cyclotron resonance. If the Hamiltonian is $H = g\mu_B B.s$, there is no interaction and the unperturbed Hamiltonian is sufficient to explain the data except that some effect of the interaction is absorbed in the g value. It was found [377] by Thomas in 1926, that the derivative of the Coulomb interaction, V_c, appears in the coupling constant of the spin-orbit interaction,

$$\xi(r) = \frac{1}{2m^2c^2}\frac{1}{r}\frac{d}{dr}V_c \tag{6.37}$$

with

$$H_{so} = \sum_k \xi(r_k) L_k . S_k \tag{6.38}$$

which is small compared with the Coulomb interaction, V_c. If the experimental data shows that shift is proportional to $B^{1/2}$, then it means that there is interaction so that Kohn's theorem does not apply. On the other hand, if data did not give $B^{1/2}$, then there is no interaction and Kohn's theorem applies. The shift of the cyclotron resonance measured [362] in a sample of graphene deposited on Si/SiO_2 wafer substrate is shown in Fig 6.1 as a function of B which can fit into a linear behavior. The inset in the same figure shows an effort to fit the data with $B^{1/2}$ dependence. It is quite clear that data is insufficient to favor a $B^{1/2}$ type interaction. The error bars are very large so that it is difficult to obtain any definite conclusion from this data. Therefore, the claim to have found the $B^{1/2}$ dependence of the cyclotron resonance shift is weak and hence there is no interaction so we favor Kohn's theorem. This result requires the definition of the interaction. Any part of the interaction which is diagonalizable is not called an interaction when it leads to non-interacting particles. If a transformation absorbs the interaction into the unperturbed

Hamiltonian, then also we say that there is no interaction. The data shows that the shift can be fitted to a linear dependence on magnetic field, B. Hence, there is no interaction and Kohn's theorem is applicable. It may be noted that (6.35) is better than (6.36).

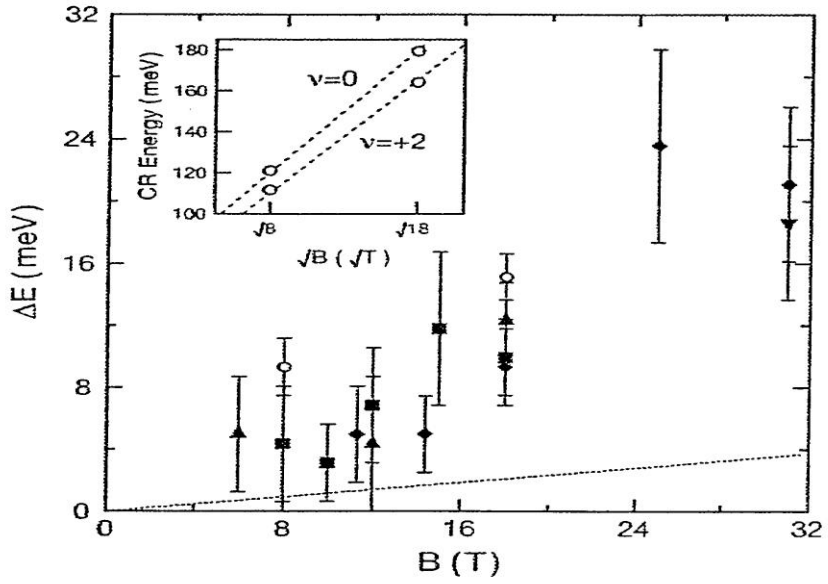

Figure 6.1 The shift of the cyclotron resonance showing linear behavior rather than $B^{1/2}$.

The line through the data can be drawn with a suitable slope.

6.6 Conclusions

We have studied the Kohn theorem and find that the cyclotron resonance does not depend on the interaction. It is also found that the wave vector dependence of the frequency does not affect the theorem and hence it is well obeyed in graphene. We also find that the Peierls-Luttinger phase splits and there are multiplicities in the eigen values of the Hubbard model due to two signs in the spin.