

# CHAPTER 1

## INTRODUCTION

### 1.1 Background

The Hall effect is a physical effect named after Edwin Hall, an American physicist who discovered that when the path of electrons running through a semiconductor is deflected by a magnetic field, a potential difference is induced perpendicular to the direction of the current.

In 1985 (over 100 years after Hall's discovery), a German physicist, Klaus von Klitzing was awarded a Nobel Prize for his work with the quantized Hall effect. The Hall effect is the working mechanism in a wide range of devices and applications including the gauss meter, ammeters, tachometers, spectrum analyzers, paintball guns, and many more electronic devices. Measuring the Hall effect is useful in determining many things, including the type of the semiconductor (p-type or n-type), the charge of the carriers, the concentration, mobility, and band structure of the material.

The Hall effect is quantized due to flux quantization. The quantization occurs at an integer value of the charge of the electron. This is called the integer quantum Hall effect (IQHE). Some of the plateaus in the Hall effect data occur at a fraction multiplied to the charge. Hence it is important to understand the fraction. We calculate several fractions throughout the thesis. The calculated values agree with the experimental data. Hence our work is important for the understanding of the fractional charge which explains the fractional quantum Hall effect (FQHE).

## 1.2 Objectives

The first objective of this thesis is to study Yang's delta potential. The delta function potential can not be transformed into a Coulomb potential. However Yang [5] found a solution of the delta function potential which is an elementary step towards understanding the Laughlin's wave function. When particles move in a circle it is possible to obtain the solutions of the potential. The Young's delta potential will then be inserted into Laughlin's wave function. The investigation of the Laughlin's wave function is then the second objective. In 1983 Laughlin invented a wave function which along with the incompressibility can give fractional charge in the excitations. This is an important step towards understanding of fractional charges. Hence it is important to investigate this wave function. The ground state energy of this wave function is compared with that of one component plasma. It is found that this is not the ground state of the Coulomb Hamiltonian but it has its own potential of which it is the zero-energy ground state. An investigation of this wave function will help understand the potential. After we have obtained the Laughlin's wave function we will move to the third objective which is to investigate the Landau levels. In 1930 Landau wrote the theory of diamagnetism of metals. These days it is believed that this theory is applicable to inert atoms. The original theory of Landau levels showed that electrons in a magnetic field behave like harmonic oscillators in two dimensions. The Landau levels were originally written without spin. Also the levels are not having the particle hole symmetry which occurs at high fields. It will be interesting to look at the Landau levels with spin.

The fourth objective is to investigate the symmetry of the wave functions. It is important to generate a second wave function from a given wave function so that lots of fractional charges can be generated. Therefore we make an effort to generate wave functions. In order to compare

the calculations with experimental data, the fifth objective is to explain the experimental data of GaAs and graphite. Usually the Hall effect resistivity is a linear function of magnetic field. However, in the experimental data there are plateaus. Hence it is important to understand the data. We find that flux quantization is needed to understand the data. Our final objective which is the sixth objective is to modify the Kohn's theorem to explain the graphene data. It is necessary to understand the Kohn's theorem to follow the cyclotron resonance. We wish to check the details of the theorem. There is a lot of work on graphene in recent years. Therefore we wish to check the details.

### **1.3 Methodology**

Mathematical technique is the backbone of the methodology. Many theoretical results are obtained throughout the thesis by analytical methods using quantum mechanics. The Yang potential uses the complex Riemann space. Hence complex algebra is used to understand the Laughlin's wave function. When comparison is made with the experimental data, the idea is to take the experimental value of the fraction from the Hall effect data. Usually, when Hall resistivity is plotted as a function of magnetic field, a straight line should be obtained. However at low temperatures such as a few mK, high magnetic fields such as 10 Tesla and small samples such as in nm, there are plateaus in the resistivity at a fraction of charge in units of  $hc/e^2$ . Here  $h$  is the Planck's constant,  $c$  is the velocity of light and  $e$  is the charge of the electron. We note down the fractions from the experimental data and try to predict the same fraction from our spin dependent theory. In all cases, there is full success in calculating the fractions which agree with the data.

## 1.4 Organization of the Thesis

The thesis is organized as follows. In Chapter 2 we describe the Laughlin's wave function and Landau level where it is defined as for the electrons in a magnetic field the harmonic oscillator type factor  $(n + 1/2)$  appears in the eigen values. The Landau levels and Laughlin's wave function are used several times throughout the thesis. The GaAs data is explained in Chapter 2 also. Several wave functions derived from the Laughlin's wave function are given in Chapters 3 and 4. The symmetry of the wave function is given in Chapter 5. The graphite data is explicitly given in Section 5.4. The modification to Kohn's theorem is given in Chapter 6. The conclusions of the thesis are summarized in Chapter 7.

Since Laughlin was awarded the Nobel prize in 1998, his paper [1] has become very popular which explains the development in more than 600 references. Our original work is largely contained in Chapters 2-6. We are able to understand the fractional charge which can be deduced from the Hall effect data at low temperatures and high fields.