CHAPTER 3

PFAFFIAN, GAFFNIAN AND HAFFNIAN STATES:

QUANTUM COMPUTATION

3.1 Introduction

The number of particles in a cluster of electrons determines the angular momentum which is used to define the filling factor of the lowest Landau level. A fuzz factor is added to it so that it gives for the desired denominators. Laughlin state is obtained for any value of the fuzz factor, p. For g=2, p=4 we get the half filled Landau level, the state of which is called Haffnian. For p=2 there are always two particles which come from non-abelian determinant called pFaffian. For g=2, p=3 the state is called Gaffnian. In all cases, the ground state belongs to non physical many body Hamiltonians. The composite fermion wave function is not the ground state of any known single Hamiltonian. From the angular momentum, we construct the filling factor and then look for the wave function and then for the Hamiltonian. In this approach the Hamiltonian found are unrealistic. It is possible to make the filling factors agree with the experimental value but then Hamiltonians are not the usual type.

In 1983, Laughlin proposed a wave function and calculated the ground state energy [1]. For certain value of a parameter called m, the ground state energy is claimed to be lower than that of one component plasma charge – density wave. It was interpreted that 1/m becomes the effective charge of the quasiparticles. The Hamiltonian consists of electron-nuclear attractive potential and the usual Coulomb repulsive interaction between electrons. When m=3, the effective charge of the quasiparticles is e/3. Hence, there is a fractionalization of charge due to Coulomb interactions. This theory gives the fractions less than 1 so that the values of 1/3 and 1/5 emerge well. However, if the theory has to be sufficient for the interpretation of the experimental data of the quantum Hall effect it will have to explain many other fractions such as 2/5, 12/5 or 1/4 , 1/2, etc. There has been a lot of effort to obtain all of the experimentally observed fractions. One of the suggestions is called the composite fermions (CF) which attaches the flux quanta to the electrons. It is clear that such a CF theory does not treat the Schrödinger equation with electromagnetic field correctly. The problem is that if the wave functions are written, the Hamiltonian becomes nonphysical and if the Hamiltonian is physical then the ground state is not obtained. Recently a non–abelian determinant has been tried. It is called the pFaffian wave function which treats the two- particle states. It is possible to define the fractional charge in terms of the difference of the angular momenta of the two particles. When this difference is odd, the particles become fermions and when it is even, they become the bosons. These fermions do not have spin. It is possible to generate the fractional charge by means of fuzzy factor and the values seen in the experiment can then emerge from the theory. When the number of particles is changed, the pFaffian can be changed to Gaffnian and Haffnian [351,352]. A whole series of fractions can be generated. In this work, we calculate the effective fractional charge of the bosons as well as the fermions which appear in the algebra.

3.2 Calculations

The relative angular momentum of two particles is L_2. It's minimum value $L_2^{min} = 1$ determines the fermions and $L_2^{min} = 0$ for bosons. The operator $P^p{}_2$ projects any state where any two particles have relative angular momentum less than $L_2^{min} + p$. The number p is the fuzz factor and the effective filling factor which also determines the fractional charge becomes,

$$\upsilon = \frac{1}{L_2^{min} + p} \quad . \tag{3.1}$$

This expression given the various wave functions,

$$\psi = (z_1 - z_2)^m \tag{3.2}$$

where $m < L_2^{min} + 1$. The relative angular momentum of $g + 1$ particle is L_{g+1}. For electrons in lowest Landau level (LLL), the minimal value would be $L_{g+1}^{min} = 0$. The fractional factors are given in table 1.

Table 3.1: The value of $1/\left(L_2^{min} + p\right)$ for fermions as well as bosons.

P	**Fermions** **L_2=odd , L_2^{min}=1**	**Boson** **L_2=even , L_2^{min}=0**
2	1/3	1/2
3	1/4	1/3
4	1/5	1/4

The clusters are important because we do not know the number of electrons in a cluster. Any cluster has relative angular momentum $L_{g+1}=L_{g+1}^{min}$. The minimum relative angular momentum of $g + 1$ particles be $L_{g+1} \geq L_{g+1}^{min}$ + p. For bosons, L_{g+1}^{min} = 0, the wave function does not need to vanish as g particles approach a given position $\tilde{z}$. As $g+1$ particle arrives, the wave function must vanish as $\left(z_{g+1} - \tilde{z}\right)^p$. To determine highest density zero energy state of the prepared Hamiltonian P_{g+1}^p, we find that a single solution does not exist for arbitrary g and p. For g = 1, any p, Laughlin state should be obtained. Then, υ = 1/m. Here, it may be noted that the usual kinetic energy plus the Coulomb potential forms the physical Hamiltonian but $\mathcal{H} = \mathrm{P}_{g+1}^p$ is not a physical Hamiltonian. Laughlin is not a minimum in the energy for a physical Hamiltonian. It may be that the energy is very close to that of charge–density wave. For g = 2, p= 4 we get the Haffnian which gives the half filled Landau level. For p=2 we always have two particles and pFaffian. For g = 2, p = 3, the state is called Gaffnian. Hence we get g= any, 1 or 2 and p=1,2,3 and 4. The values higher than p=4 can be tabulated. The wave function has one more factor. For LLL wave function,

$$\varphi(r) = \psi(r)\mu(r) \ . \tag{3.3}$$

For disk geometry,

$$\mu(r) = \exp(-|z|^2/4) \tag{3.4}$$

and for spherical geometry,

$$\mu(r) = \frac{1}{(1+|z|^2)^{1+N_\varphi/2}} \tag{3.5}$$

Here N_φ is the total flux penetrating the sphere. The degree of the polynomials $\psi(z)$ is determined from z^0 to z^{N_φ}. The filling factor for fermions can be written in terms of those of bosons

$$\upsilon_f = \frac{\upsilon_b}{\upsilon_b+1} \quad . \tag{3.6}$$

The wave function should vanish when g+1 particles are brought to the same point, $\tilde{z}$. This limiting behavior is described by

$$\lim_{z_1,\dots z_{g+1},\tilde{z}} \psi(z_1, \dots z_N) \cong f\left(z_1, \dots, z_{g+1}\right)\tilde{\psi}\left(\tilde{z};\ z_{g+2}, \dots, z_N\right) \tag{3.7}$$

where f are the polynomials. The g+1 particles are not allowed to have relative angular momentum$< p$.

On the sphere, each particle has the angular momentum $N_\varphi/2$. The g+1 bosons have the angular momentum $(g+1)N_\varphi/2$.

3.3 Discussion

As long as the Hamiltonian contains the usual kinetic energy and the Coulomb potential, the Laughlin's wave function gives the ground state energy very closed to that of charge –density waves. It does not have a minimum at 1/3. The charge of the quasiparticles is 1/m. Hence it can be 1/L but not 1/J, (J= L+S). There is no spin in the theory. As a modification of Laughlin's theory, the charge can become.

$$\frac{e^*}{e} = \frac{1}{2L+1} \ . \tag{3.8}$$

But still there is no spin. If we relax the Hamiltonian and look for another Hamiltonian for which the Laughlin's wave function will be the ground state, the Hamiltonian becomes nonphysical. Sometimes it has only the projection operators. Similarly, the composite fermion (CF) wave function is found not to be the ground state of the physical Hamiltonian. Hence, if the CF series is accepted, it does not give a physical Hamiltonian and it's flux attachment formula is incomplete. The state with $\upsilon = \kappa/(\kappa + 2)$ is known as "parafermion ", (PF) [353] and it's ground state is represented by the Hamiltonian,

$$\mathcal{H} = \sum_{i<j<k} \delta(z_i - z_j)\delta(z_j - z_l)\,\delta(z_l - z_m) \ldots .\, k + 1 terms \quad (3.9)$$

which is quite unrealistic. Actually, the quasiparticles represented by Laughlin, CF or L_2 do not have spin. Hence, this is an effort to obtain "spinless" particles which may exhibit fractional charge. As far as the experimental values of the quantum Hall effect are concerned they agree with the theories because the possibility of comparison is taken into account.

3.4 Conclusions

The wave function of Laughlin does not have a ground state when Hamiltonian is physical. The pFaffian, Gaffnian and Haffnian are the ground state of nonphysical Hamiltonians. The CF is algebraically not complete and its electromagnetic field cannot be treated. If only fractions are considered, it is the ground state of a nonphysical Hamiltonian. The fractions which look like experimental values of quantum Hall effect are produced but not all values are predicted correctly. Some ground states are belonging to nonphysical Hamiltonians.