CHAPTER 1

INTRODUCTION

1.1 Background

Electric Power Systems are components that transform other types of energy into electrical energy and deliver this energy from the generating units to the end consumers. Electrical power systems are composed of transmission, sub-transmission, distribution and generation systems. Transmission systems may contain large numbers of substations which are interconnected by transmission lines, transformers, and other devices for system control and protection. Power may be injected into the system by the generators or absorbed from the system by the loads at these substations. At the receiving end, the transmission systems are connected to the sub-transmission or the distribution systems which are operated at lower voltage (Abur and Exposito, 2004).

The distribution systems are typically configured to operate in a radial configuration, where the feeders stretch from the distribution substations and form a tree structure with their roots at the substations and branches spreading over the distribution area (Abur and Exposito, 2004). The production and transmission of electricity is relatively efficient and inexpensive, but unlike other forms of energy, electricity is not easily stored and thus must generally be used as it is being produced. (http://encarta.msn.com/encyclopedia_761566999/ electric_power_systems.html)

Power systems are operated by system operators from the area control centers. The main goal of the system operator is to maintain the system in the normal secured state as the operating conditions vary during the daily operation. Accomplishing this goal requires continuous monitoring of the system conditions, identification of the operating state and determination of the necessary preventive action in case the system state is found to be insecure. This sequence of actions is referred to as the system security analysis. The first step of system security analysis is system monitoring (Abur and Exposito, 2004).

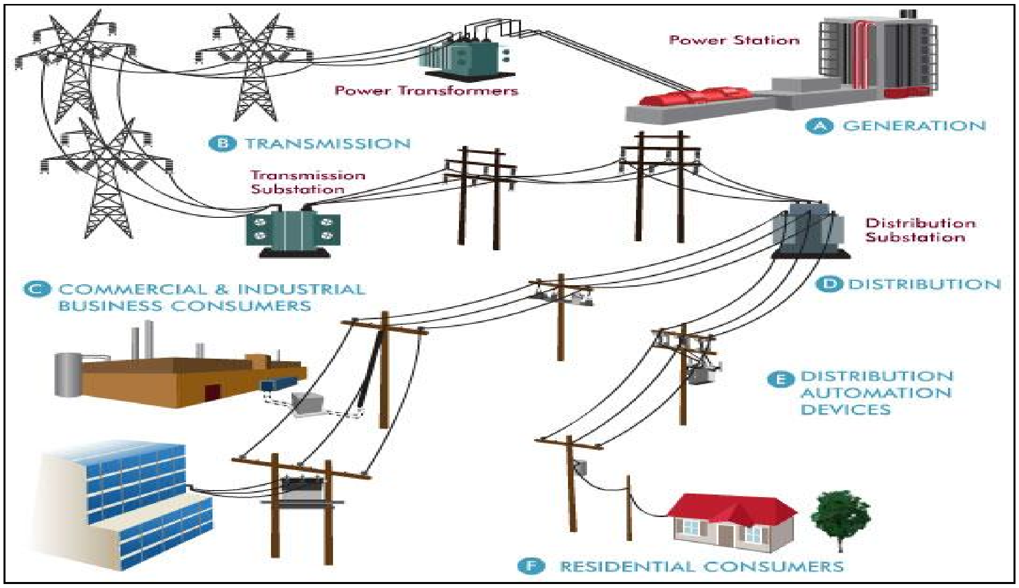

Figure 1.1: Electrical power transmission system

The problem of monitoring the power flows and voltages on a transmission system is very important in maintaining system security. System monitoring provides the operators of the power system with pertinent up-to-date information on the condition on the power system. Many problems are encountered in monitoring a transmission system and these problems come primarily from the nature of the measurement transducers and from communication problems in transmitting the measured values back to the operation control center (Wood and Wollenberg, 1996).

Modern electric power systems use transformers to convert electricity into different voltages. With transformers, each stage of the system can be operated at an appropriate voltage. Transducers used for the power system measurements, like any measurement device, are subject to errors. If the errors are small, they may go undetected but nevertheless can cause misinterpretation in the measured values. Otherwise, transducers may have gross measurement errors that render their output useless. Thus, power system state estimation techniques have been developed.

State estimation is an essential component of an energy management system. It is a process of assigning a value to an unknown system state variable based on measurement from that system. Also, the process of estimating the system state is based on a statistical criterion that estimates the true value of the state variables to maximize or minimize the selected criterion. A commonly used criterion is the weighted least-squares criterion (Wood and Wollenberg, 1996). Further discussions about the statistical criterion are found in chapter 2.

In layman's terms, the state of a power system refers to its operating condition relative to overload, overvoltage and etc., the amount of power flowing through the lines, transformers, substations and etc. and their voltage readings. Mathematically, all these quantities can be computed once the set of bus voltage magnitudes and phase angles is known. Therefore, technically, the state of a power system is defined as the set of bus voltage magnitudes and relative phase angles at the system nodes (Wu, 1990).

The inputs to an estimator are imperfect power system measurements of voltage magnitudes and power, VAR, or ampere-flow quantities. The real-time telemetered measurements are collected through the SCADA (Supervisory control and data acquisition) system (Wood and Wollenberg, 1996). The typical data include active and reactive power injections at buses, bus voltage magnitudes, and real and reactive power line flows. These telemetered data contain errors (Wu, 1990). In addition to the real-time telemetered measurements, there are virtual measurements and weighting factors as well to ensure the observability of the system and enhance the precision of the estimation. Virtual measurements do not require metering, for example, zero injection at a switching station. It represents exact mathematical relationships. The precision of the measurements determines the weighting factors. The more accurate the measurement, the larger is the weighting factor that is assigned (Du et al., 2005).

A state estimator is designed to produce the ‘best estimate’ of the system voltage and phase angles, recognizing that there are errors in the measured quantities and that there may redundant measurements.

A state estimator can ‘smooth out’ small random errors in meter readings, detect and identify gross measurement errors, and ‘fill in’ meter readings that are missing due to communication failures. The outputs are then used in system control centers in the implementation of the security-constrained dispatch and control of the system (Wood and Wollenberg, 2006).

In addition, the outputs also provide the real time database for other advanced computer applications such as security analysis, economic dispatch, optimal power flow and etc (Wu, 1990).

1.2 Literature review

State estimation has been introduced in 1968 by Fred Schweppe. He was the man who led to state estimates and spot pricing, totally new planes of power system engineering. He defined a state estimator as a data processing algorithm for converting redundant meter readings and other available information into an estimate of the state of an electric power system. Today, state estimation is an essential part in every energy management system throughout the world and is a basic tool in ensuring secure operation of a power system (Wu, 1990).

Most state estimation programs in practical use are formulated as overdetermined systems of nonlinear equations and solved as weighted least square problems. The estimate of the state vector **x** is obtained by minimizing the weighted least square function,

$$\min_{\mathbf{x}} J(\mathbf{x}) = [\mathbf{z} - \mathbf{f}(\mathbf{x})]^T [\mathbf{R}^{-1}] [\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

where **z** is the set of measurements, **x** is the vector of state variables, **f** is the mathematical relation between the measured variables and the state variables, and $\mathbf{R}^{-1}$ is a diagonal matrix whose elements are the measurement weighting factors. The measurement weighting factors are the reciprocal of the error variance of the measurement device (Wood and Wollenberg, 1996).

In the last three decades, many state estimation methods have been proposed and some were successfully applied in the electric power industry.

Normal equation method

The normal equation method is the classical and standard approach to the solution of the weighted least square state estimation problem in power system (Schweppe, 1974). The estimate is found iteratively where the corrections $\Delta\mathbf{x}$ will be computed at each iteration by solving the normal equation of the nonlinear weighted least square problem,

$$\mathbf{G}\Delta\mathbf{x} = \mathbf{H}^T\mathbf{R}^{-1}[\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

where $\mathbf{G} = \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$ is the gain matrix, $\mathbf{H}$ is the Jacobian matrix, and $\mathbf{x} = \mathbf{x}^k$ at the k-th iteration. The method starts with first performing sparse matrix triangular factorization on the gain matrix, and then solves the normal equation by back substitutions (Wu, 1990). Large condition number of coefficient matrix may cause ill-conditioning when performing Gaussian elimination or Cholesky decomposition, which will lead to slow convergence or fail to converge at all (Du, 2005).

In this method, different types of measurements are differentiated by the use of different weighting factors in the formulation. There are three types of measurements, which are the telemetered measurements, pseudo-measurements and virtual measurements. Analysis has shown that the assignment of large weighting factors to virtual measurements and small weighting factors to pseudo-measurements may cause numerical ill-conditioning of the system. Another potential source of ill conditioning is the existence of large number of injection measurements in the system (Holten et al., 1988).

All these problems are somehow related to the squared form of the gain matrix. Many other methods developed were partly motivated by the need to unsquare the gain matrix in order to improve the numerical robustness. The proposed orthogonal decomposition method based on Householder transformation avoids the formation of the gain matrix by triangularizing it directly with QR factorization and Householder transformation (refer to section 3.3.2), which is found to be numerically more stable. Details on the proposed method will be presented in Chapter 3.

Hybrid method

To unsquare the gain matrix, the Hybrid method derives the gain matrix:

$$\mathbf{G} = \overline{\mathbf{H}}^T \overline{\mathbf{H}} = (\mathbf{QU})^T (\mathbf{QU}) = \mathbf{U}^T \mathbf{Q}^T \mathbf{QU} = \mathbf{U}^T \mathbf{U}$$

by performing QR decomposition directly on the matrix $\overline{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$. $\mathbf{Q}$ is an orthogonal matrix that has a special property: $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$.Then the normal equation of the nonlinear weighted least squares problem changes to $\mathbf{U}^T \mathbf{U}\, \Delta\mathbf{x} = \mathbf{H}^T \mathbf{R}^{-1}[\mathbf{z} - \mathbf{f}(\mathbf{x})]$, where $\mathbf{U}$ is an upper triangular matrix. The normal equation of the nonlinear weighted least square problem can be solved by performing the forward and backward substitutions (Wu, 1990). Both the proposed method and the Hybrid method utilize numerically stable QR decomposition. Thus, they are well known to be numerically stable compared with other methods. But as the weighting factors of the virtual measurements are set to be very high, the Hybrid method will be numerically unstable (Wu, 1990). The proposed orthogonal decomposition method based on Householder transformation has the advantage that the measurement weights can be adjusted to extreme values as demonstrated in the numerical examples.

Peters Wilkinson method

The Peters Wilkinson method avoids the direct formation of the gain matrix, $\mathbf{G} = \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}$ by performing $\mathbf{LU}$ decomposition directly on matrix $\overline{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$, where $\mathbf{L}$ is a unit lower trapezoidal matrix and $\mathbf{U}$ is a nonsingular upper triangular matrix. Pivoting technique is used due to numerical and sparsity considerations, where a permutation of $\overline{\mathbf{H}}$ is in fact factorized:

$$\mathbf{P}_r \overline{\mathbf{H}} \mathbf{P}_c = \mathbf{LU}$$

where $\mathbf{P}_r$ performs row permutations on $\overline{\mathbf{H}}$ for numerical stability and $\mathbf{P}_c$ performs column permutations on $\overline{\mathbf{H}}$ to reduce fill-in in the sparse case. The normal equation of nonlinear weighted least squares is

$$\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \Delta \mathbf{x} = \mathbf{H}^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

or $$\mathbf{H} \mathbf{R}^{-1/2} \mathbf{R}^{-1/2} \mathbf{H} \Delta \mathbf{x} = \mathbf{H}^T \mathbf{R}^{-1/2} \mathbf{R}^{-1/2} [\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

or $$\overline{\mathbf{H}}^T \overline{\mathbf{H}} \Delta \mathbf{x} = \overline{\mathbf{H}}^T \mathbf{R}^{-1/2} [\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

which can then be written as follows:

$$\mathbf{U}^T \mathbf{L}^T \mathbf{L} \mathbf{U} \Delta \mathbf{x} = \mathbf{U}^T \mathbf{L}^T \mathbf{R}^{-1/2} [\mathbf{z} - \mathbf{f}(\mathbf{x})].$$

Since $\mathbf{U}$ is nonsingular, the result is

$$\mathbf{L}^T \mathbf{L} \mathbf{U} \Delta \mathbf{x} = \mathbf{L}^T \mathbf{R}^{-1/2} [\mathbf{z} - \mathbf{f}(\mathbf{x})].$$

This equation can be solved in two stages, as follows:

$$\mathbf{L}^T \mathbf{L} \mathbf{y} = \mathbf{L}^T \mathbf{R}^{-1/2} [\mathbf{z} - \mathbf{f}(\mathbf{x})],$$

and $$\mathbf{U} \Delta \mathbf{x} = \mathbf{y}.$$

The first stage involves the solution of a transformed normal equation and the second stage involves a simple backward substitution using the triangular factor $\mathbf{U}$. The numerical stability of this method depends on the matrix $\mathbf{L}^T \mathbf{L}$ being well-conditioned.

Normal equations with constraints method

The normal equations with constraints method partitions the measurements into telemetered measurements, $\mathbf{z} = \mathbf{f}(\mathbf{x}) + \mathbf{w}$ where $\mathbf{w}$ is the measurement errors and virtual measurements, $\mathbf{c}(\mathbf{x}) = 0$. Therefore the Jacobian matrix is partitioned into $\mathbf{H}$ and $\mathbf{C}$. Then the normal equations become,

$$\left[\mathbf{H}^T\mathbf{H} + \mathbf{r}\mathbf{C}^T\mathbf{C}\right]\Delta\mathbf{x} = \mathbf{H}^T\left[\mathbf{z} - \mathbf{f}(\mathbf{x})\right] - \mathbf{r}\mathbf{C}^T\mathbf{c}(\mathbf{x})$$

where $\mathbf{r}$ is the ratio between the weighting factors of the virtual measurements and the telemetered measurements.

The virtual measurements represent an exact mathematical relationship incorporated directly in the weighted least square formulation of state estimation by simply assigning large weighting factors.

The second term $\mathbf{r}\ \mathbf{C}^{\mathrm{T}}\mathbf{C}$ in the coefficient matrix dominates for very large $\mathbf{r}$. But due to the network being observable, usually there are not enough virtual measurements to make the matrix $\mathbf{C}$ full rank. Therefore for a large $\mathbf{r}$, the coefficient matrix in the normal equations tends to be singular (the condition number is very large (Arfken, 1985)), thus causing the ill-conditioning problem.

Due to this, the normal equations with constraints method separate the virtual measurements with zero injections from the telemetered measurements and treat them as equality constraints. By treating virtual measurements as equality constraints, the normal equations with constraints method avoid one of the major sources of ill-conditioning in the state estimation, the large weights of zero injections. Hence, it is better than the one without constraints from the viewpoint of numerical stability.

The method of Lagrange multipliers may be applied to solve the state estimation power system problem by minimizing the weighted least squares while equality constraints, $\mathbf{c}(\mathbf{x}) = 0$ are satisfied. The Lagrangian of the problem can be defined as

$$\mathrm{L}(\mathbf{x},\boldsymbol{\lambda}) = \frac{1}{2}\left[\mathbf{z} - \mathbf{f}(\mathbf{x})\right]^T\mathbf{R}^{-1}\left[\mathbf{z} - \mathbf{f}(\mathbf{x})\right] - \boldsymbol{\lambda}^T\mathbf{c}(\mathbf{x}).$$

The estimation of the state variables can be obtained by an iterative procedure where the linearized equation is solved at each iteration:

$$\begin{bmatrix} \mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}\mathbf{H}(\mathbf{x}) & \mathbf{C}^T(\mathbf{x}) \\ \mathbf{C}(\mathbf{x}) & 0 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^T(\mathbf{x})\mathbf{R}^{-1}\Delta\mathbf{z} \\ \Delta\mathbf{c} \end{bmatrix}$$

where $\Delta\mathbf{z} = \mathbf{z} - \mathbf{f}(\mathbf{x})$, $\Delta\mathbf{c} = -\mathbf{c}(\mathbf{x})$ and $\mathbf{x} = \mathbf{x}^k$ at the k-th iteration.

For a symmetric positive definite matrix, the numerical stability is guaranteed when the pivots are taken from the diagonal in any order. Therefore, optimal ordering can be performed symbolically by using only the sparsity criterion. But the coefficient matrix used in this method is no longer positive definite. Ordering and factorization should be carried out simultaneously with special techniques such as 1x1 and 2x2 pivots (Wu, 1987). Thus, the normal equations with constraints method requires more than just triangular factorization. However, the computational implementation complexity does not seem to be very extensive (Holten at el., 1988). The ability of the proposed orthogonal decomposition method based on Householder transformation to handle a wide range of weights obviates the need for special treatment of zero injection equality constraints. This greatly simplifies the state estimator implementation.

Hactel's augmented matrix method

Hactel’s augmented matrix method may solve the constrained minimization problem in the normal equations with constraints method. At each iteration, the following equations are solved

$$\mathbf{K}(\mathbf{x}) \begin{bmatrix} -a^{-1}l \\ a^{-1}\mathbf{R}^{-1}\Delta\mathbf{r} \\ \Delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{c} \\ \Delta\mathbf{z} \\ 0 \end{bmatrix}, \; \mathbf{K}(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \mathbf{C} \\ 0 & a\,\mathbf{R} & \mathbf{H} \\ \mathbf{C}^T & \mathbf{H}^T & 0 \end{bmatrix},$$

where $\mathbf{K}(\mathbf{x})$ is the augmented coefficient matrix, $\mathbf{R}^{-1}$ is a diagonal matrix whose elements are the measurement weighting factors, $\mathbf{H}$ and $\mathbf{C}$ are the Jacobian matrices, $\Delta\mathbf{z} = \mathbf{z} - \mathbf{f}(\mathbf{x})$, $\Delta\mathbf{c} = -\mathbf{c}(\mathbf{x})$, $\Delta\mathbf{r} = \Delta\mathbf{z} - \mathbf{H}\Delta\mathbf{x}$, $\mathbf{x} = \mathbf{x}^k$ at the k-th iteration, l is the Lagrange multiplier, and a is a parameter used to control the numerical stability of the problem (Holten at el., 1988).

The Hactel's augmented matrix method treats $-a^{-1}l$, $a^{-1}\mathbf{R}^{-1}\Delta\mathbf{r}$ and $\Delta\mathbf{x}$ as unknowns and skillfully avoids the cross product of $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$, thus making it numerically stable. But the augmented coefficient matrix used in Hactel's augmented matrix method is sparse, symmetric and indefinite. Ordinary sparsity-oriented pivoting scheme for symmetric matrices may lead to numerical stability problems (Numerical stability refers to the perturbation behavior of an algorithm used to solve that problem on a computer (Trefethen and Bau, 1997).). Thus, it needs more than just the simple ordering and factorization. Additionally, the dimension of the coefficient matrix in this method is large. This influences the efficiency of the factorization (Du, 2005). Hence, the implementation of the Hactel's augmented matrix method is complicated excessively.

From the above literature review, we understand that it is important to have a method with the property of numerical stability, computation efficiency and computation implementation simplicity in order to solve the power state estimation problem efficiently and accurately. Thus, we proposed the orthogonal decomposition method based on Householder transformation in solving the power system state estimation problem.

Orthogonal decomposition method based on Householder transformation

The proposed orthogonal decomposition method based on Householder transformation has a good compromise of numerical stability, computational efficiency and implementation simplicity. The idea of solving state estimator problem via orthogonal decomposition method based on Householder transformation is presented in section 3.3.2.

The proposed method unsquares the gain matrix in order to improve the numerical robustness by triangularizing it directly with numerically stable QR factorization and Householder transformation. Thus, it is a well known numerically stable method.

Besides eliminating sources of numerical ill-conditioning, the orthogonal decomposition method has also simplified the solution process. The numerical robustness of orthogonal decomposition approach allows for zero injection constraints to be modeled as heavily weighted measurements. The ability to handle very wide ranges of weights with an orthogonal decomposition method obviates the need for special treatment of zero injections equality constraints.

From the literature survey, the prevalent approach for orthogonal decomposition system state estimator is the Givens rotation method (Trefethen and Bau, 1997). Instead of the Givens rotation method, this research has proposed the Householder transformation as the ordering method in the QR factorization and applies in the orthogonal decomposition method to solve the power system state estimation problem.

The Householder transformation was introduced in 1958 by Alston Scott Householder. Householder transformation or Householder reflection is a linear transformation that describes a reflection about a plane or hyperplane containing the origin.

The error analysis carried out by Wilkinson showed that the Householder transformation outperforms the Givens rotation method under finite precision computations (Wilkinson, 1965). Additionally, the Householder method is more numerically stable since it uses orthogonal similarity transform (Householder and Bauer, 1959). Straightforward implementation of Givens rotation method requires about 50% more work than Householder method, and also requires more storage. These disadvantages can be overcome, but requires more complicated implementation.

The application of the Householder transformation can cause severe "intermediate" fill-ins (non-zero elements generated by transformation); these fill-ins will be annihilated eventually, but they can cause excessive storage. Details on the Householder transformation is presented in section 3.3.1.

1.3 Layout of the dissertation

This thesis is organized into five chapters. A brief description of the electric power system and the reason why power system state estimation was developed followed by a review of relevant literature was given in this chapter.

Problem formation will be discussed in Chapter 2, where the development of the notions of state estimation and the development of a method for an AC (Alternating Current) network are presented. The development of the notions of state estimation is dependent on the statistical criterion that is selected. Among the three most commonly encountered criteria, the maximum likelihood criterion is utilized since it introduces the measurement error weighting matrix in a straightforward manner. The development of the state formula using the maximum likelihood criterion is by assuming the normal distribution for the measurement errors. Therefore, the result will be a “weighted least square” estimation formula for an AC network. In addition, the general state estimation solution algorithm is also presented.

Chapter 3 describes the method considered in solving the power system state estimation problem. This chapter includes discussion on the two main issues in the power system state estimation problem; the numerical ill-conditioning problem and the convergence problem, and how the proposed method, orthogonal decomposition method based on Householder transformation can effectively solve the estimation problem.

The orthogonal decomposition based power system state estimation implementations have been found to be numerically stable as they use unitary transformations and handle the numerical ill-conditioning encountered in the power system state estimation problem satisfactorily. QR factorization is the heart of the orthogonal decomposition method. Therefore, the efficiency of the whole power system state estimation depends on the efficiency of the QR factorization. Many different methods exist to perform QR factorization, e.g. Householder transformation, Givens rotation, and Gram-Schmidt decomposition. In this research, the Householder transformation is applied in the orthogonal decomposition method to solve the power system state estimation problem. The Householder transformation is a transformation that takes a vector and reflects it about some plane. At the end of Chapter 3, the orthogonal decomposition algorithm that is used to solve the power system state estimation problem is presented.

Results and discussions will be carried out in Chapter 4, following the implementation of the orthogonal decomposition algorithm in Matlab on three test systems. Comparisons with other solving methods will be done to evaluate the performance of the proposed method.

Lastly, Chapter 5 concludes this research, and discusses the implementation of power system state estimation in the electrical power system. Some possibilities for future work are mentioned as well.